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# An efficient algorithm by employing neutrosophic Weber hybrid aggregation operators in decision-making analysis

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**Abstract**

This study aims to explore Weber aggregation operations by utilizing Weber t-norms and t-conorms of single-valued neutrosophic sets (SVNS). We define and elucidate single-valued (SV) neutrosophic Weber weighted average (SVNWWA) Aggregation Operators and SV neutrosophic Weber weighted geometric (SVNWWG) Aggregation Operators and their characteristics. An algorithm was developed to authenticate the importance of the aggregation operators by implementing them into an application to tackle multiattribute decision-making challenges by employing SV neutrosophic data. Finally, a comparison is

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presented with existing SVneutrosophic aggregation operators to assess the suggested techniques performance.

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## 1. Introduction

Classical or crisp sets have well-defined and precise boundaries whereas, fuzzy sets (FS) allow for a more flexible representation of uncertainty by assigning degrees of membership to elements. Fuzzy logic is a useful tool introduced by Zadeh [1] that can help us deal with uncertainty and imprecision information. It gives us a framework for handling ambiguous data and leads us to arrive at more realistic and efficient decisions. FS have been extensively investigated in many different fields including AI, Computer Science. Pattern recognition, Decision Analysis, and many more [2, 3, 4]. Although fuzzy set theory was effective at dealing with uncertainty, it did not address an element's non-membership degree.

As a generalization of FS, Atanassov [5] proposed a notion of intuitionistic fuzzy sets (IFS) in which each element has a degree of membership and non-membership. IFS finds applications in various sectors, including expert systems, pattern detection, clustering, and managerial decision-making. Several articles [6, 7, 8] provide a comprehensive overview of IFS, their mathematical foundations, and specific applications. To obtain a unified result from multiple IFS, their membership, and non-membership degrees are aggregated. Some of the operators are the Intuitionistic Fuzzy Weighted Averaging (IFWA) operator, the Intuitionistic Fuzzy Ordered Weighted Geometric (IFOWG) operator, and the Intuitionistic Fuzzy Hybrid Weighted Geometric (IFHWG) operator. There are extensive descriptions of these aggregation operations within the context of intuitionistic fuzzy set theory in literature [9, 10]. In [11], Zeshui outlines techniques for analyzing ordered weighted aggregation operators. Further research demonstrates the applicability of these operators for IFS, as illustrated in [12, 13, 14], offering perspectives to overcome decision-making challenges.

To handle real-world problems in decision-making and problem-solving, which include indeterminacy, inconsistency, ambiguity, and imprecision, a powerful conceptual framework was developed. That is the development of NS as an extension of IFS by Smarandache [15, 16]. T-norms (triangular norms) and t-conorms (triangular co-norms) are mathematical operators used to model the intersection (AND operation) and union (OR operation) of NS, respectively. T-norms and t-conorms play crucial roles in neutrosophic logic operations and are fundamental components of neutrosophic systems.The articles [17, 18, 19] provide detailed explanations of t-norms and t-conorms for SVNS. Norms developed by Dombi [20, 21], Frank [22], Hamacher [23], Mayor-Torrens [24] and Schweizer-Sklar [25] have been elucidated by many authors. Hezam et al. [26] provide an overview of SVNSto solve transport problems. The practical application of NS in transportation problem was discussed in [27]. Rani et al. [28] provide integrated compromise solution techniques and a step-by-step weighted assessment ratio analysis approach for SVNSRecently, a theoretical framework for NS has been established by examining the connections between continuous, open, and strongly open maps, [29] explores new classes of neutrosophic (1,2)-maps within neutrosophic bi-topological spaces. Neutosophic D-metric space was introduced in [30], which explores features such as Hausdorff and completeness. Neutosophic crisp open set theory is extented with new notions in [31]. In 1982, Weber [32] pioneered the introduction of the Weber t-norm and t-conorm operations within the realm of FS. The primary strength of Weber t-norms lies in their adherence to copulas [33] conditions, endowing them with enhanced capabilities in MCDM.

### 1.1 *Contributions*

The main objective of this research is to extend Weber operations and develop new aggregation operators (AgOs) for NS. In this study,

1. the operational principles for SVNSusing Weber t-norm and t-conorm are defined.
2. an aggregation operator called SV neutrosophic Weber weighted averaging (SVNWWA) and a SV neutrosophic Weber weighted geometric (SVNWWG) aggregation operator are explored.
3. we have developed an algorithm to validate the significance of the newly introduced aggregation operators.
4. we provide an example to illustrate SVNWWA and SVNWWG aggregation operators in multiattribute decision-making problems.
5. a comparative analysis with established operators [34, 35, 36, 37, 38, 39, 40] is presented to illustrate the suggested operator's effectiveness.

This article is systematized as follows: Section [2] delivers an overview of concepts allied to SVNS that will be applied in the study. Section [3] introduces Weber operating principles for SV neutrosophic data, and section [4] analyzes the properties of the SV neutrosophic Weber averaging (SVNWWA) aggregation operator and the SV neutrosophic Weber geometric (SVNWWG) aggregation operator. In the section [5], we have developed an algorithm for MCDM problems utilizing the SVNWWA and SVNWWG aggregation operators, and a numerical example was given. Finally, a comparative analysis with other aggregation operators is given in section [5.3]. The article's main conclusions are outlined in the conclusion section [6], which also offers suggestions for further investigation.

## 2. Preliminaries

**Definition 1:** [1] Let $\mathbb{E}$ denote the universal set, and consider a neutrosophic set $\Delta$ within $\mathbb{E}$ defined by three functions: $\Delta = \{\langle e, j(e), i(e), \ell(e)\rangle | e \in \mathbb{E}\}$, where

$j, i,$ and $\ell$: $\mathbb{E} \rightarrow ]0^-, 1^+[$ represent the degrees of truth, indeterminacy, and falsity membership grades of $\Delta$ in $\mathbb{E}$, respectively. These functions satisfy the condition that for all $e \in \mathbb{E}$, $0^- \leq j(e) + i(e) + \ell(e) \leq 3^+$.

**Definition 2:** [5] Let $\mathbb{E}$ denote the universal set, and consider a SV neutrosophic (SVN) set $\Delta$ within $\mathbb{E}$. There are three functions in this SVN set, which are defined as follows: $\Delta = \{\langle e, j(e), i(e), \ell(e)\rangle | e \in \mathbb{E}\}$. Here, $j$, $i$, and $\ell$ are functions mapping elements from $\mathbb{E}$ to the interval [0,1]. They represent the degrees of truth, indeterminacy, and falsity membership of the element in $\Delta$ such that

$$0 \leq j(e) + i(e) + \ell(e) \leq 3.$$

**Definition 3:** [18] A function $T_N$ :[0,1] × [0,1] → [0,1], is termed as t-norm if the conditions given below are satisfied:

1. $T_N(0, \zeta) = \zeta$ and $T_N(1, \zeta) = 1$,
2. $T_N(\xi, \zeta) = T_N(\zeta, \xi) \quad \forall\, \xi, \zeta$,
3. $T_N(\xi, T_N(\zeta, \eta)) = T_N(T_N(\xi, \zeta), \eta) \quad \forall\, \xi, \zeta, \eta$,
4. If $\xi \leq \xi^*$ and $\zeta \leq \zeta^*$, then $T_N(\xi, \zeta) \leq T_N(\xi^*, \zeta^*)$.

**Definition 4:** [18] A function $T_C$ :[0,1] × [0,1] → [0,1], is termed as t-conorm if the conditions given below are satisfied:

1. $T_C(\xi, 0) = 0$ and $T_C(\xi, 1) = \xi$,
2. $T_C(\xi, \zeta) = T_C(\zeta, \xi) \quad \forall\, \xi, \zeta$,
3. $T_C(\xi, T_C(\zeta, \eta)) = T_C(T_C(\xi, \zeta), \eta) \quad \forall\, \xi, \zeta, \eta$,
4. If $\xi \leq \xi^*$ and $\zeta \leq \zeta^*$, then $T_C(\xi, \zeta) \leq T_C(\xi^*, \zeta^*)$.

**Definition 5:** [18] Let $\mathcal{N}_1 = (\theta_1, \iota_1, \phi_1)$, $\mathcal{N}_2 = (\theta_2, \iota_2, \phi_2) \in$ SVNS($\mathcal{N}$) then the neutrosophic t-norm is denoted by S and S is a function from, $(]^-0, 1^+[\times ]^-0, 1^+[\times]^-0, 1^+[)^2$ to $]^-0, 1^+[\times]^-0, 1^+[\times]^-0, 1^+[$ such that S( $\mathcal{N}_1, \mathcal{N}_2$) = ($T_N(\theta_1, \theta_2)$, $T_C(\iota_1, \iota_2)$,$T_C(\phi_1, \phi_2)$).

**Definition 6:** [18] Let $\mathcal{N}_1 = (\theta_1, \iota_1, \phi_1)$, $\mathcal{N}_2 = (\theta_2, \iota_2, \phi_2) \in$ SVNS( $\mathcal{N}$ ) neutrosophic t-conorm is denoted $\tilde{S}$ is a function from $(]^-0, 1^+[\times]^-0, 1^+[\times ]^-0, 1^+[)^2$ to $]^-0, 1^+[\times]^-0, 1^+[\times]^-0, 1^+[$ such that $\tilde{S}(\mathcal{N}_1, \mathcal{N}_2)$ = ($T_C(\theta_1, \theta_2)$, $T_N(\iota_1, \iota_2)$,$T_N(\phi_1, \phi_2)$).

**Definition 7:** [2] Let $\breve{p}$: [0,1] → [0,1] and $\breve{p}(x(j_1, i_1, \ell_1)) = (\check{c}j_1(x), \check{c}i_1(x), \check{c}\ell_1(x)))$. $\breve{p}$ is defined as neutrosophic compliment if $\forall\ x, y$ in the neutrosophic set of the universal set $\mathbb{E}$, must adhere following axioms:

1. $\breve{p}(0) = 1$ and $\breve{p}(1) = 0$,
2. For all x,y $\varepsilon$ [0,1], if $x \leq y$, then $\breve{p}(x) \geq \breve{p}(y)$.

**Definition 8:** [32] Weber's norms for any $\phi, \psi \in \mathbb{R}$ and $\beta > -1$:

1. $\grave{O}(\phi,\psi) = max\left(\frac{\phi+\psi-1+\beta\phi\psi}{1+\beta},0\right)$ [t-norm],
2. $\acute{O}(\phi,\psi) = min((\phi+\psi+\beta\phi\psi),1)$ [t-conorm].

They have the decreasing generator as $\mu(\phi) = 1-\frac{ln(1+\beta\phi)}{ln(1+\beta)}$ for t-norm and increasing generator as $\xi(\psi) = \frac{ln(1+\beta\psi)}{ln(1+\beta)}$ for t-conorm respectively.

## 3. Weber Operating Laws for SVNs

This section introduces the Weber neutrosophic t-norm and t-conorm for Weber NS with a single value. The operating laws are then dveloped.

**Definition 9:** Let $\Delta_1$, $\Delta_2$ $\varepsilon$ SVNS($\mathbb{E}$). Then the Weber t-norm and t-conorm are defined as:

1. The Neutrosophic Weber t-norm of $\Delta_1$ and $\Delta_2$,

$$\Delta_1 \Cap \Delta_2 = \left\{max\left(\left(\frac{j_1+j_2-1+\beta j_1 j_2}{1+\beta}\right),0\right), min((i_1+i_2+\beta i_1 i_2),1), min((\ell_1+\ell_2+\beta\ell_1\ell_2),1)\right\},$$

2. The Neutrosophic Weber t-conorm of $\Delta_1$ and $\Delta_2$,

$$\Delta_1 \Cup \Delta_2 = \left\{min((j_1+j_2+\beta j_1 j_2),1), max\left(\left(\frac{i_1+i_2-1+\beta i_1 i_2}{1+\beta}\right),0\right), max\left(\left(\frac{\ell_1+\ell_2-1+\beta\ell_1\ell_2}{1+\beta}\right),0\right)\right\}.$$

**Definition 10:** Let $\Delta_1$ , $\Delta_2$ $\varepsilon$ SVNS($\mathbb{E}$). The Weber Operating Laws (WOLs) are $\Delta = (j(x), i(x), \ell(x))$ :

1. $\Delta_1 \Cap \Delta_2$,
2. $\Delta_1 \Cup \Delta_2$,
3. $(\Delta_1)^{\varsigma}$=$\{max\left(\varsigma\left(\frac{j_1-1+\beta j_1}{1+\beta}\right),0\right), min(\varsigma(i_1+\beta i_1),1), min(\varsigma(\ell_1+\beta\ell_1),1)\}$,
4. $\varsigma(\Delta_1)$=$\left\{min(\varsigma(j_1+\beta j_1),1), max\left(\varsigma\left(\frac{i_1-1+\beta i_1}{1+\beta}\right),0\right), max\left(\varsigma\left(\frac{\ell_1-1+\beta\ell_1}{1+\beta}\right),0\right)\right\}$,
5. $\check{c}(\Delta_1) = \left\{min\left(1,\left(\frac{1-j_1}{1+\beta j_1}\right)\right), max\left(0,\left(\frac{1-i_1}{1+\beta i_1}\right)\right), max\left(0,\left(\frac{1-\ell_1}{1+\beta\ell_1}\right)\right)\right\}$.

**Theorem 1:** *Let $\Delta_1, \Delta_2 \in SVNSet$. Then,*

1. $\tilde{f}(\Delta_1 \Cup \Delta_2) = \tilde{f}(\Delta_1) \Cap \tilde{f}(\Delta_2)$,
2. $\tilde{f}(\Delta_1 \Cap \Delta_2) = \tilde{f}(\Delta_1) \Cup \tilde{f}(\Delta_2)$.

***Proof:*** (i) We prove the equality for the truth membership function.

The complement operation is given by:

$$T(\tilde{f}(\Delta_1 \Cup \Delta_2)) = \min\left[1, \frac{1-\min(j_1+j_2+\beta j_1 j_2,1)}{1+\beta\min(j_1+j_2+\beta j_1 j_2,1)}\right].$$

On the other hand, the truth membership function of the intersection of complements is:

$$T(\tilde{f}(\Delta_1) \Cap \tilde{f}(\Delta_2)) = \max\left[0, \frac{\min(1,\frac{1-j_1}{1+\beta j_1})+\min(1,\frac{1-j_2}{1+\beta j_2})-1+\beta\min(1,\frac{1-j_1}{1+\beta j_1})\min(1,\frac{1-j_2}{1+\beta j_2})}{1+\beta}\right].$$

We must show that:

$$\min\left[1, \frac{1-\min(j_1+j_2+\beta j_1 j_2,1)}{1+\beta\min(j_1+j_2+\beta j_1 j_2,1)}\right] = \tag{1}$$

$$\max\left[0, \frac{\min(1,\frac{1-j_1}{1+\beta j_1})+\min(1,\frac{1-j_2}{1+\beta j_2})-1+\beta\min(1,\frac{1-j_1}{1+\beta j_1})\min(1,\frac{1-j_2}{1+\beta j_2})}{1+\beta}\right]. \tag{2}$$

**Case (i):** Suppose $j_1 + j_2 + \beta j_1 j_2 < 1$. Then,

$$\min(j_1 + j_2 + \beta j_1 j_2, 1) = j_1 + j_2 + \beta j_1 j_2.$$

Thus, the left-hand side (LHS) of 1 simplifies to:

$$\min\left[1, \frac{1-(j_1+j_2+\beta j_1 j_2)}{1+\beta(j_1+j_2+\beta j_1 j_2)}\right].$$

Since $0 \leq j_1, j_2 \leq 1$ and $\beta > -1$, we obtain:

$$0 < j_1 + j_2 + \beta j_1 j_2 < 1, \quad 1 + \beta > 0.$$

Hence,

$$\frac{1-(j_1+j_2+\beta j_1 j_2)}{1+\beta(j_1+j_2+\beta j_1 j_2)} < 1.$$

For the right-hand side (RHS) of 1, using similar simplifications, we find:

$$\max\left[\frac{1-(j_1+j_2+\beta j_1 j_2)}{1+\beta(j_1+j_2+\beta j_1 j_2)}, 0\right].$$

Since $\frac{1-(j_1+j_2+\beta j_1 j_2)}{1+\beta(j_1+j_2+\beta j_1 j_2)} > 0$, the RHS simplifies to the same expression as the LHS, proving the equality.

**Case (ii):** Suppose $j_1 + j_2 + \beta j_1 j_2 > 1$. Then, the LHS of 1 simplifies to:

$$\min\left[1, \frac{1-1}{1+\beta}\right] = 0.$$

For the RHS, using the same simplifications, we find:

$$\max\left[\frac{1-(j_1+j_2+\beta j_1 j_2)}{1+\beta(j_1+j_2+\beta j_1 j_2)}, 0\right] = 0.$$

Thus, the equality holds. Similar arguments apply for indeterminacy and falsity membership grades.

(ii) The proof follows analogously to (i).

## 4. Neutrosophic WAO

4.1 *Weber Weighted Averaging Aggregation Operators*

**Definition 11:** Let $\Delta_q$=$(j_q(x), i_q(x), \ell_q(x))$ ∈ SVN norms and $q \in \mathbb{N}$. Then, Weber weighted averaging aggregation operators for SVN norms is described as

$SVNWWA(\Delta_1, \Delta_2, \ldots, \Delta_n) = \delta_1\Delta_1 \Cup \delta_2\Delta_2 \Cup \ldots \Cup \delta_n\Delta_n$, where the weights $(\delta_1, \delta_2, \ldots, \delta_q)$ of $\Delta_q$ have $\delta_q \geq 0$, $\sum_{q=1}^{n} \delta_q = 1$.

**Theorem 2:** *When* $\Delta_q$=$(j_q(x), i_q(x), \ell_q(x)) \in$ *SVN norms and* $q \in \mathbb{N}$ *and the weights* $(\delta_1, \delta_2, \ldots, \delta_q)$ *of* $\Delta_q$ *having* $\delta_q \geq 0$ *and* $\sum_{q=1}^{n} \delta_q = 1$. *The SVNWWA aggregation operators are a function from* $\mathcal{V}^n \to \mathcal{V}$ *such that* $SVNWWA(\Delta_1, \Delta_2, \ldots, \Delta_n) = S_{1,2,3,\ldots,n}$

$$\{min[(\textstyle\sum_{q=1}^{n} \delta_q(j_q + \beta j_q) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\delta_{q_1}(j_{q_1} + \beta j_{q_1})\delta_{q_2}(j_{q_2} + \beta j_{q_2}))$$
$$+\ldots+\textstyle\sum \prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\delta_{q_1}(j_{q_1} + \beta j_{q_1})\ldots\delta_{q_n}(j_{q_n} + \beta j_{q_n})),1],$$
$$max[(\textstyle\sum_{q=1}^{n} \delta_q(\frac{i_q-1+\beta i_q}{1+\beta}) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\delta_{q_1}(\frac{i_{q_1}-1+\beta i_{q_1}}{1+\beta})\delta_{q_2}(\frac{i_{q_2}-1+\beta i_{q_2}}{1+\beta}))$$
$$+\ldots+\textstyle\sum \prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\delta_{q_1}(\frac{i_{q_1}-1+\beta i_{q_1}}{1+\beta})\ldots\delta_{q_n}(\frac{i_{q_n}-1+\beta i_{q_n}}{1+\beta})),0],$$
$$max[(\textstyle\sum_{q=1}^{n} \delta_q(\frac{\ell_q-1+\beta \ell_q}{1+\beta}) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\delta_{q_1}(\frac{\ell_{q_1}-1+\beta \ell_{q_1}}{1+\beta})\delta_{q_2}(\frac{\ell_{q_2}-1+\beta \ell_{q_2}}{1+\beta}))$$
$$+\ldots+\textstyle\sum \prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\delta_{q_1}(\frac{\ell_{q_1}-1+\beta \ell_{q_1}}{1+\beta})\ldots\delta_{q_n}(\frac{\ell_{q_n}-1+\beta \ell_{q_n}}{1+\beta})),0]\},$$

where $\mathbb{J}_n$=$(1,2,3,\ldots,n)$.

***Proof:*** Inductive mathematical reasoning on n is used to provide the proof for each q.,

$$\Delta_q = (j_q(x), i_q(x), \ell_q(x)) \in \text{SVN norms} \Rightarrow j_q, i_q, \ell_q \in [0,1] \text{ and}$$
$$j_q + i_q + \ell_q \leq 3.$$

When $n = 2$, we obtain, $S_{1,2} = \delta_1\Delta_1 \Cup \delta_2\Delta_2$

$$= \{min(\delta_1(j_1 + \beta j_1),1), max(\delta_1(\tfrac{i_1-1+\beta i_1}{1+\beta}),0), max(\delta_1(\tfrac{\ell_1-1+\beta \ell_1}{1+\beta}),0)\}$$
$$\Cup \{min(\delta_2(j_2 + \beta j_2),1), max(\delta_2(\tfrac{i_2-1+\beta i_2}{1+\beta}),0), max(\delta_2(\tfrac{\ell_2-1+\beta \ell_2}{1+\beta}),0)\}$$
$$= \{min[(\textstyle\sum_{q=1}^{2} \delta_q(j_q + \beta j_q) + \beta\delta_{q_1}(j_{q_1} + \beta j_{q_1})\delta_{q_2}(j_{q_2} + \beta j_{q_2})),1],$$
$$max[(\textstyle\sum_{q=1}^{2} \delta_q(\frac{i_q-1+\beta i_q}{1+\beta}) + \beta\delta_{q_1}(\frac{i_{q_1}-1+\beta i_{q_1}}{1+\beta})\delta_{q_2}(\frac{i_{q_2}-1+\beta i_{q_2}}{1+\beta})),0],$$
$$max[(\textstyle\sum_{q=1}^{2} \delta_q(\frac{\ell_q-1+\beta \ell_q}{1+\beta}) + \beta\delta_{q_1}(\frac{\ell_{q_1}-1+\beta \ell_{q_1}}{1+\beta})\delta_{q_2}(\frac{\ell_{q_2}-1+\beta \ell_{q_2}}{1+\beta})),0]\}.$$

Suppose if it is true when $n = k$,

$$S_{1,2,3,\ldots,k} = \delta_1\Delta_1 \Cup \delta_2\Delta_2 \Cup \ldots \Cup \delta_k\Delta_k$$
$$= \{min[(\textstyle\sum_{q=1}^{k} \delta_q(j_q + \beta j_q) + \sum \prod_{q_1,q_2 \in \mathbb{J}_k} \beta\delta_{q_1}(j_{q_1} + \beta j_{q_1})\delta_{q_2}(j_{q_2} + \beta j_{q_2}))$$
$$+\ldots+\textstyle\sum \prod_{q_1,q_2,\ldots,q_k \in \mathbb{J}_k} \beta\delta_{q_1}(j_{q_1} + \beta j_{q_1})\ldots\delta_{q_k}(j_{q_k} + \beta j_{q_k})),1],$$
$$max[(\textstyle\sum_{q=1}^{k} \delta_q(\frac{i_q-1+\beta i_q}{1+\beta}) + \sum \prod_{q_1,q_2 \in \mathbb{J}_k} \beta\delta_{q_1}(\frac{i_{q_1}-1+\beta i_{q_1}}{1+\beta})\delta_{q_2}(\frac{i_{q_2}-1+\beta i_{q_2}}{1+\beta}))$$
$$+\ldots+\textstyle\sum \prod_{q_1,q_2,\ldots,q_k \in \mathbb{J}_k} \beta\delta_{q_1}(\frac{i_{q_1}-1+\beta i_{q_1}}{1+\beta})\ldots\delta_{q_k}(\frac{i_{q_k}-1+\beta i_{q_k}}{1+\beta})),0],$$

$max[(\sum_{q=1}^{k} \delta_q(\frac{\ell_q-1+\beta\ell_q}{1+\beta}) + \sum \prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{\ell_{q_1}-1+\beta\ell_{q_1}}{1+\beta})\delta_{q_2}(\frac{\ell_{q_2}-1+\beta\ell_{q_2}}{1+\beta}))$

$+\ldots+\sum \prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{\ell_{q_1}-1+\beta\ell_{q_1}}{1+\beta})\ldots\delta_{q_k}(\frac{\ell_{q_k}-1+\beta\ell_{q_k}}{1+\beta})),0]\}$.

Now we prove for $n = k + 1$.

$S_{1,2,3,\ldots,k+1} =$

$= \{min[(\sum_{q=1}^{k} \delta_q(j_q + \beta j_q) + \sum \prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(j_{q_1} + \beta j_{q_1})\delta_{q_2}(j_{q_2} + \beta j_{q_2}))$

$+\ldots+\sum \prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(j_{q_1} + \beta j_{q_1})\ldots\delta_{q_k}(j_{q_k} + \beta j_{q_k})),1]$,

$max[(\sum_{q=1}^{k} \delta_q(\frac{i_q-1+\beta i_q}{1+\beta}) + \sum \prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{i_{q_1}-1+\beta i_{q_1}}{1+\beta})\delta_{q_2}(\frac{i_{q_2}-1+\beta i_{q_2}}{1+\beta}))$

$+\ldots+\sum \prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{i_{q_1}-1+\beta i_{q_1}}{1+\beta})\ldots\delta_{q_k}(\frac{i_{q_k}-1+\beta i_{q_k}}{1+\beta})),0]$,

$max[(\sum_{q=1}^{k} \delta_q(\frac{\ell_q-1+\beta\ell_q}{1+\beta}) + \sum \prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{\ell_{q_1}-1+\beta\ell_{q_1}}{1+\beta})\delta_{q_2}(\frac{\ell_{q_2}-1+\beta\ell_{q_2}}{1+\beta}))$

$+\ldots+\sum \prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{\ell_{q_1}-1+\beta\ell_{q_1}}{1+\beta})\ldots\delta_{q_k}(\frac{\ell_{q_k}-1+\beta\ell_{q_k}}{1+\beta})),0]\}$

$\Cup \{min(\delta_{k+1}(j_{k+1} + \beta j_{k+1}),1), max(\delta_{k+1}(\frac{i_{k+1}-1+\beta i_{k+1}}{1+\beta}),0)$,

$max(\delta_{k+1}(\frac{\ell_{k+1}-1+\beta\ell_{k+1}}{1+\beta}),0)\}$

$= \{min[(\sum_{q=1}^{k+1} \delta_q(j_q + \beta j_q) + \sum \prod_{q_1,q_2\in\mathbb{J}_{k+1}} \beta\delta_{q_1}(j_{q_1} + \beta j_{q_1})\delta_{q_2}(j_{q_2} + \beta j_{q_2}))$

$+\ldots+\sum \prod_{q_1,q_2,\ldots,q_{k+1}\in\mathbb{J}_{k+1}} \beta\delta_{q_1}(\frac{\ell_{q_1}-1+\beta\ell_{q_1}}{1+\beta})\ldots\delta_{q_{k+1}}(\frac{\ell_{q_{k+1}}-1+\beta\ell_{q_{k+1}}}{1+\beta})),0]\}$.

Hence, for any n, the equation holds.

**Theorem 3:** *When* $\Delta_q$=$(j_q(x), i_q(x), \ell_q(x))$, $\Delta_q^-$ = $(min(j_q(x)), max(i_q(x)), max(\ell_q(x)))$ *and* $\Delta_q^+$ = $(max(j_q(x)), min(i_q(x)), min(\ell_q(x)))$ ∈ *SVN norm (*$q \in \mathbb{N}$*), Then*

$$\Delta_q^- \leq SVNWWA(\Delta_1, \Delta_2, \ldots, \Delta_n) \leq \Delta_q^+.$$

***Proof:*** We know that, $\Delta_q$=$(j_q(x), i_q(x), \ell_q(x))$, $\Delta_q^-$=$(j_q^-(x), i_q^-(x), \ell_q^-(x))$ and $\Delta_q^+$ = $(j_q^+(x), i_q^+(x), \ell_q^+(x))$, where $j_q^-(x) = min_q(j_q(x))$, $i_q^-(x) = max_q(i_q(x))$, $\ell_q^-(x) = max_q(\ell_q(x))$ and $j_q^+(x) = min_q(j_q(x))$, $i_q^+(x) = max_q(i_q(x))$, $\ell_q^+(x) = max_q(\ell_q(x))$. And where $\mathbb{J}_n$=$(1,2,3,\ldots,n)$. Now,

$min[(j_q^- + \beta j_q^-) + \prod_{q_1,q_2\in\mathbb{J}_n} \beta\delta_{q1}\delta_{q_2}(j_q^- + \beta jq^-)^2+\ldots$

$+\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n} \beta\delta_{q1}\delta_{q_2}\ldots\delta_{q_n}(j_q^- + \beta jq^-)^n, 1] < min[(j_q + \beta j_q)$

$+\prod_{q_1,q_2\in\mathbb{J}_n} \beta\delta_{q1}\delta_{q_2}(j_q + \beta jq)^2+\ldots+\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n} \beta\delta_{q1}\delta_{q_2}\ldots$

$\delta_{q_n}(j_q + \beta jq)^n, 1]$

$< min[(j_q^+ + \beta j_q^+) + \prod_{q_1,q_2\in\mathbb{J}_n} \beta\delta_{q1}\delta_{q_2}(j_q^+ + \beta jq^+)^2+\ldots+\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}$

$\beta\delta_{q1}\delta_{q_2}\ldots\delta_{q_n}(j_q^+ + \beta jq^+)^n, 1]$

Also, $max[(\frac{i_q^- -1+\beta i_q^-}{1+\beta}) + \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}(\frac{i_q^- -1+\beta i_q^-}{1+\beta})^2+\ldots$
$+\prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}\ldots\delta_{q_n}(\frac{i_q^- -1+\beta i_q^-}{1+\beta})^n, 0] < max[(\frac{i_q -1+\beta i_q}{1+\beta})$
$+\prod_{q_1,q_2 \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}(\frac{i_q -1+\beta i_q}{1+\beta})^2+\ldots+\prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}\ldots\delta_{q_n}(\frac{i_q -1+\beta i_q}{1+\beta})^n, 0]$
$< max[(\frac{i_q^+ -1+\beta i_q^+}{1+\beta}) + \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}(\frac{i_q^+ -1+\beta i_q^+}{1+\beta})^2+\ldots$
$+\prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}\ldots\delta_{q_n}(\frac{i_q^- -1+\beta i_q^-}{1+\beta})^n, 0]$

Also, $max[(\frac{\ell_q^+ -1+\beta \ell_q^+}{1+\beta}) + \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}(\frac{\ell_q^+ -1+\beta \ell_q^+}{1+\beta})^2+\ldots$
$+\prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}\ldots\delta_{q_n}(\frac{\ell_q^- -1+\beta \ell_q^-}{1+\beta})^n, 0] < max[(\frac{\ell_q -1+\beta \ell_q}{1+\beta})$
$+\prod_{q_1,q_2 \in \mathbb{J}_n} \beta\ell_{q_1}\delta_{q_2}(\frac{\ell_q -1+\beta \ell_q}{1+\beta})^2+\ldots+\prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}\ldots\delta_{q_n}(\frac{\ell_q -1+\beta \ell_q}{1+\beta})^n, 0]$
$< max[(\frac{\ell_q^+ -1+\beta \ell_q^+}{1+\beta}) + \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}(\frac{\ell_q^+ -1+\beta \ell_q^+}{1+\beta})^2+\ldots$
$+\prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\delta_{q_1}\delta_{q_2}\ldots\delta_{q_n}(\frac{\ell_q^+ -1+\beta \ell_q^+}{1+\beta})^n, 0].$

Then $\Delta_q^- \leq SVNWWA(\Delta_1,\Delta_2,\ldots,\Delta_n) \leq \Delta_q^+$.

**Theorem 4:** *Suppose* $\Delta_q = (j_q(x), i_q(x), \ell_q(x))$, $\Delta_q^* = (j_q^*(x), i_q^*(x), \ell_q^*(x))$, $\in$ *SVN norms* ($q \in \mathbb{N}$). *If* $j_q \leq j_q^*$, $i_q \geq i_q^*$, $\ell_q \geq \ell_q^*$, *Then*

$$SVNWWA(\Delta_1,\Delta_2,\ldots,\Delta_n) \leq SVNWWA(\Delta_1^*,\Delta_2^*,\ldots,\Delta_n^*).$$

***Proof:*** The proof has resemblance to Theorem 3. 3.

**Definition 12:** Let $\Delta_q = (j_q(x), i_q(x), \ell_q(x)) \in$ SVN norms ($q \in \mathbb{J}_n$). Then, Weber ordered weighted averaging aggregation operators for SVN norms is defined as:

$$SVNWOWA(\Delta_1,\Delta_2,\ldots,\Delta_n) = \delta_1\Delta_{x(1)} \Cup \delta_2\Delta_{x(2)} \Cup\ldots\Cup \delta_n\Delta_{x(n)}$$

where $\Delta_{x(q-1)} \geq \Delta_{x(q)}$ for every q, and the permutation of (1,2,3,...,n) is notated by $(x(1), x(2),\ldots,x(n))$. Moreover, the weights $(\delta_1,\delta_2,\ldots,\delta_q)$ of $\Delta_q$ with $\sum_{q=1}^n \delta_q = 1$ and $\delta_q \geq 0$.

**Theorem 5:** *Let* $\Delta_q = (j_q(x), i_q(x), \ell_q(x)) \in$ *SVN norms* ($q \in \mathbb{N}$) *& weights* $(\delta_1,\delta_2,\ldots,\delta_q)$ *of* $\Delta_q$ *having* $\delta_q \geq 0$ & $\sum_{q=1}^n \delta_q = 1$. *Then, the SVNWOWA aggregation operators is a function from* $\mathcal{V}^n \to \mathcal{V}$ *such that* $SVNWOWA$ $(\Delta_1,\Delta_2,\ldots,\Delta_n) =$
$\{min[(\sum_{q=1}^n \delta_q(j_{x(q)} + \beta j_{x(q)}) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\delta_{q_1}(j_{x(q_1)} + \beta j_{x(q_1)})$
$\delta_{q_2}(j_{x(q_2)} + \beta j_{x(q_2)}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}\beta\delta_{q_1}(j_{x(q_1)}+\beta j_{x(q_1)})\ldots\delta_{q_n}(j_{x(q_n)}+\beta j_{x(q_n)})),1],$
$max[(\sum_{q=1}^{n}\delta_q(\frac{i_{x(q)}-1+\beta i_{x(q)}}{1+\beta})+\sum\prod_{q_1,q_2\in\mathbb{J}_n}\beta\delta_{q_1}(\frac{i_{x(q_1)}-1+\beta i_{x(q_1)}}{1+\beta})\delta_{q_2}$
$(\frac{i_{x(q_2)}-1+\beta i_{x(q_{n2})}}{1+\beta}))+\ldots+\sum\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}\beta\delta_{q_1}(\frac{i_{x(q_1)}-1+\beta i_{x(q_1)}}{1+\beta})\ldots\delta_{q_n}$
$(\frac{i_{x(q_n)}-1+\beta i_{x(q_n)}}{1+\beta})),0],max[(\sum_{q=1}^{n}\delta_q(\frac{\ell_{x(q)}-1+\beta\ell_{x(q)}}{1+\beta})+\sum\prod_{q_1,q_2\in\mathbb{J}_n}\beta\delta_{q_1}$
$(\frac{\ell_{x(q_1)}-1+\beta\ell_{x(q_1)}}{1+\beta})\delta_{q_2}(\frac{\ell_{x(q_2)}-1+\beta\ell_{x(q_2)}}{1+\beta}))$
$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}\beta\delta_{q_1}(\frac{\ell_{x(q_1)}-1+\beta\ell_{x(q_1)}}{1+\beta})\ldots\delta_{q_n}(\frac{\ell_{x(q_n)}-1+\beta\ell_{x(q_n)}}{1+\beta})),0]\},$
where $\mathbb{J}_n=(1,2,3,\ldots,n)$.

***Proof:*** Analogous to theorem 2.

**Theorem 6:** *When $\Delta_q=(j_q(x),i_q(x),\ell_q(x))$, $\Delta_q^-=(min(j_q(x)),max(i_q(x)),max(\ell_q(x)))$ and $\Delta_q^+=(max(j_q(x)),min(i_q(x)),min(\ell_q(x)))$ $\in$ SVN norm ($q\in\mathbb{N}$), Then*

$$\Delta_q^-\leq SVNWOWA(\Delta_1,\Delta_2,\ldots,\Delta_n)\leq\Delta_q^+.$$

**Theorem 7:** *When $\Delta_q=(j_q(x),i_q(x),\ell_q(x))$, $\Delta_q^*=(j_q^*(x),i_q^*(x),\ell_q^*(x))$, $\in$ SVN norms ($q\in\mathbb{N}$). If $j_q\leq j_q^*$, $i_q\geq i_q^*$, $\ell_q\geq\ell_q^*$, Then,*

$$SVNWOWA(\Delta_1,\Delta_2,\ldots,\Delta_n)\leq SVNWOWA(\Delta_1^*,\Delta_2^*,\ldots,\Delta_n^*).$$

**Definition 13:** Let $\Delta_q=(j_q(x),i_q(x),\ell_q(x))$ $\in$ SVN norms ($q\in\mathbb{N}$). The following is a description of Weber hybrid weighted averaging aggregation operations for SVN norms:

$$SVNWHWA(\Delta_1,\Delta_2,\ldots,\Delta_n)=\kappa_1\widetilde{\Delta}_{x(1)}\acute{O}\kappa_2\widetilde{\Delta}_{x(2)}\acute{O}\ldots\acute{O}\kappa_n\widetilde{\Delta}_{x(n)}$$

where the weights $(\delta_1,\delta_2,\ldots,\delta_q)$ associated with $\Delta_q$ are subject to the conditions $\delta_q\geq 0$, $\sum_{q=1}^{n}\delta_q=1$, and the $k$-th largest weighted value is denoted as $\widetilde{\Delta}x(q)$ (where $\widetilde{\Delta}x(q)=n\delta_q\Delta_{x(q)}$ for $q=1,2,\ldots,n$), arranged according to the total order $(x(1),x(2),\ldots,x(n))$.

**Theorem 8:** *Let $\Delta_q=(j_q(x),i_q(x),\ell_q(x))$ $\in$ SVN norms ($q\in\mathbb{N}$) & weights $(\delta_1,\delta_2,\ldots,\delta_q)$ of $\Delta_q$, $\delta_q\geq 0$ & $\sum_{q=1}^{n}\delta_q=1$. SVNWHWA aggregation operator is a function from $\mathcal{V}^n\to\mathcal{V}$ with associated weights $(\kappa_1,\kappa_2,\ldots,\kappa_q)$ of $\Delta_q$ having $\kappa_q\geq 0$ and $\sum_{q=1}^{n}\kappa_q=1$, and $SVNWHWA(\Delta_1,\Delta_2,\ldots,\Delta_n)=$*
$\{min[(\sum_{q=1}^{n}\kappa_q(\tilde{j}_q+\beta\tilde{j}_q)+\sum\prod_{q_1,q_2\in\mathbb{J}_n}\beta\kappa_{q_1}(\tilde{j}_{q_1}+\beta\tilde{j}_{q_1})\kappa_{q_2}(\tilde{j}_{q_2}+\beta\tilde{j}_{q_2}))$
$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}\beta\kappa_{q_1}(\tilde{j}_{q_1}+\beta\tilde{j}_{q_1})\ldots\kappa_{q_n}(\tilde{j}_{q_n}+\beta\tilde{j}_{q_n})),1],$
$max[(\sum_{q=1}^{n}\kappa_q(\frac{\tilde{\imath}_q-1+\beta\tilde{\imath}_q}{1+\beta})+\sum\prod_{q_1,q_2\in\mathbb{J}_n}\beta\kappa_{q_1}(\frac{\tilde{\imath}_{q_1}-1+\beta\tilde{\imath}_{q_1}}{1+\beta})\kappa_{q_2}(\frac{\tilde{\imath}_{q_2}-1+\beta\tilde{\imath}_{q_2}}{1+\beta}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}\beta\kappa_{q_1}(\frac{\tilde{\imath}_{q_1}-1+\beta\tilde{\imath}_{q_1}}{1+\beta})\ldots\kappa_{q_n}(\frac{\tilde{\imath}_{q_n}-1+\beta\tilde{\imath}_{q_n}}{1+\beta})),0],$

$max[(\sum_{q=1}^{n}\kappa_q(\frac{\tilde{\ell}_q-1+\beta\tilde{\ell}_q}{1+\beta})+\sum\prod_{q_1,q_2\in\mathbb{J}_n}\beta\kappa_{q_1}(\frac{\tilde{\ell}_{q_1}-1+\beta\tilde{\ell}_{q_1}}{1+\beta})\kappa_{q_2}(\frac{\tilde{\ell}_{q_2}-1+\beta\tilde{\ell}_{q_2}}{1+\beta}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}\beta\kappa_{q_1}(\frac{\tilde{\ell}_{q_1}-1+\beta\tilde{\ell}_{q_1}}{1+\beta})\ldots\kappa_{q_n}(\frac{\tilde{\ell}_{q_n}-1+\beta\tilde{\ell}_{q_n}}{1+\beta})),0]\},$

where $\mathbb{J}_n$=$(1,2,3,\ldots,n)$.

**Theorem 9:** *Let* $\Delta_q=(j_q(x),i_q(x),\ell_q(x))$, $\Delta_q^-=(min(j_q(x)),max(i_q(x)),max(\ell_q(x)))$ *and* $\Delta_q^+=(max(j_q(x)),min(i_q(x)),min(\ell_q(x)))\in$ *SVN norm (*$q\in\mathbb{N}$*), Then*

$$\Delta_q^-\leq SVNWHWA(\Delta_1,\Delta_2,\ldots,\Delta_n)\leq\Delta_q^+.$$

**Theorem 10:** *When* $\Delta_q=(j_q(x),i_q(x),\ell_q(x))$, $\Delta_q^*=(j_q^*(x),i_q^*(x),\ell_q^*(x))$, $\in$ *SVN norms (*$q\in\mathbb{N}$*). If* $j_q\leq j_q^*$, $i_q\geq i_q^*$, $\ell_q\geq\ell_q^*$, *Then*

$$SVNWHWA(\Delta_1,\Delta_2,\ldots,\Delta_n)\leq SVNWHWA(\Delta_1^*,\Delta_2^*,\ldots,\Delta_n^*).$$

4.2 *Weber Weighted Geometric Aggregation Operators*

**Definition 14:** Let $\Delta_q$=$(j_q(x),i_q(x),\ell_q(x))\in$ SVN norms and $q\in\mathbb{N}$. Then, Weber weighted geometric aggregation operators for SVN norms is described as $SVNWWG(\Delta_1,\Delta_2,\ldots,\Delta_n)=\Delta_1^{\delta_1}\Cap\Delta_2^{\delta_2}\Cap\ldots\Cap\Delta_n^{\delta_n}$, the weights $(\delta_1,\delta_2,\ldots,\delta_q)$ of $\Delta_q$ have $\delta_q\geq 0$ & $\sum_{q=1}^{n}\delta_q=1$.

**Theorem 11:** *When* $\Delta_q$=$(j_q(x),i_q(x),\ell_q(x))\in$ *SVN norms and* $q\in\mathbb{N}$ *and the weights* $(\delta_1,\delta_2,\ldots,\delta_q)$ *of* $\Delta_q$ *having* $\delta_q\geq 0$ *and* $\sum_{q=1}^{n}\delta_q=1$. *The SVNWWG aggregation operators are a function from* $\mathcal{V}^n\to\mathcal{V}$ *such that* $SVNWWG(\Delta_1,\Delta_2,\ldots,\Delta_n)=$

$\{max[(\sum_{q=1}^{n}\delta_q(\frac{j_q-1+\beta j_q}{1+\beta})+\sum\prod_{q_1,q_2\in\mathbb{J}_n}\beta\delta_{q_1}(\frac{j_{q_1}-1+\beta j_{q_1}}{1+\beta})\delta_{q_2}(\frac{j_{q_2}-1+\beta j_{q_2}}{1+\beta}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}\beta\delta_{q_1}(\frac{j_{q_1}-1+\beta j_{q_1}}{1+\beta})\ldots\delta_{q_n}(\frac{j_{q_n}-1+\beta j_{q_n}}{1+\beta})),0],$

$min[(\sum_{q=1}^{n}\delta_q(i_q+\beta i_q)+\sum\prod_{q_1,q_2\in\mathbb{J}_n}\beta\delta_{q_1}(i_{q_1}+\beta i_{q_1})\delta_{q_2}(i_{q_2}+\beta i_{q_2}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}\beta\delta_{q_1}(i_{q_1}+\beta i_{q_1})\ldots\delta_{q_n}(i_{q_n}+\beta i_{q_n})),1],$

$min[(\sum_{q=1}^{n}\delta_q(\ell_q+\beta\ell_q)+\sum\prod_{q_1,q_2\in\mathbb{J}_n}\beta\delta_{q_1}(\ell_{q_1}+\beta\ell_{q_1})\delta_{q_2}(\ell_{q_2}+\beta\ell_{q_2}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_n\in\mathbb{J}_n}\beta\delta_{q_1}(\ell_{q_1}+\beta\ell_{q_1})\ldots\delta_{q_n}(\ell_{q_n}+\beta\ell_{q_n})),1]\},$

where $\mathbb{J}_n$=$(1,2,3,\ldots,n)$.

***Proof:*** This proof is given using mathematical induction,

$\Delta_q$= $(j_q(x),i_q(x),\ell_q(x))\in$ SVN norms $\Rightarrow j_q,i_q,\ell_q\in[0,1]$

and $j_q+i_q+\ell_q\leq 3$.

When $n=2$, we obtain $SVNWWG(\Delta_1,\Delta_2)=\Delta_1^{\delta_1}\grave{O}\Delta_2^{\delta_2}$

$= \{max(\delta_1(\frac{j_1-1+\beta j_1}{1+\beta}),0), min(\delta_1(i_1+\beta i_1),1), min(\delta_1(\ell_1+\beta\ell_1),1)\}$

$\Cap \{max(\delta_2(\frac{j_2-1+\beta j_2}{1+\beta}),0), min(\delta_2(i_2+\beta i_2),1), min(\delta_2(\ell_2+\beta\ell_2),1)\}$

$= \{max[(\sum_{q=1}^{2} \delta_q(\frac{j_q-1+\beta j_q}{1+\beta}) + \beta\delta_{q_1}(\frac{j_{q_1}-1+\beta j_{q_1}}{1+\beta})\delta_{q_2}(\frac{j_{q_2}-1+\beta j_{q_2}}{1+\beta})),0],$

$min[(\sum_{q=1}^{2} \delta_q(i_q+\beta i_q) + \beta\delta_{q_1}(i_{q_1}+\beta i_{q_1})\delta_{q_2}(i_{q_2}+\beta i_{q_2})),1],$

$min[(\sum_{q=1}^{2} \delta_q(\ell_q+\beta\ell_q) + \beta\delta_{q_1}(\ell_{q_1}+\beta\ell_{q_1})\delta_{q_2}(\ell_{q_2}+\beta\ell_{q_2})),1]\}.$

Let us assume when $n = k$,

$SVNWWG(\Delta_1, \Delta_2, \ldots, \Delta_k) = \Delta_1^{\delta_1} \Cap \Delta_2^{\delta_2} \Cap \ldots \Cap \Delta_k^{\delta_k}$

$= \{max[(\sum_{q=1}^{k} \delta_q(\frac{j_q-1+\beta j_q}{1+\beta}) + \sum\prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{j_{q_1}-1+\beta j_{q_1}}{1+\beta})\delta_{q_2}(\frac{j_{q_2}-1+\beta j_{q_2}}{1+\beta}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{j_{q_1}-1+\beta j_{q_1}}{1+\beta})\ldots\delta_{q_k}(\frac{j_{q_k}-1+\beta j_{q_k}}{1+\beta})),0],$

$min[(\sum_{q=1}^{k} \delta_q(i_q+\beta i_q) + \sum\prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(i_{q_1}+\beta i_{q_1})\delta_{q_2}(i_{q_2}+\beta i_{q_2}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(i_{q_1}+\beta i_{q_1})\ldots\delta_{q_k}(i_{q_k}+\beta i_{q_k})),1],$

$min[(\sum_{q=1}^{k} \delta_q(\ell_q+\beta\ell_q) + \sum\prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(\ell_{q_1}+\beta\ell_{q_1})\delta_{q_2}(\ell_{q_2}+\beta\ell_{q_2}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(\ell_{q_1}+\beta\ell_{q_1})\ldots\delta_{q_k}(\ell_{q_k}+\beta\ell_{q_k})),1]\}.$

We now need to show that for $n = k + 1$, is true.

$SVNWWG(\Delta_1, \Delta_2, \ldots, \Delta_k, \Delta_{k+1}) =$

$= \{max[(\sum_{q=1}^{k} \delta_q(\frac{j_q-1+\beta j_q}{1+\beta}) + \sum\prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{j_{q_1}-1+\beta j_{q_1}}{1+\beta})\delta_{q_2}$

$(\frac{j_{q_2}-1+\beta j_{q_2}}{1+\beta}))+\ldots+\sum\prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(\frac{j_{q_1}-1+\beta j_{q_1}}{1+\beta})\ldots\delta_{q_k}(\frac{j_{q_k}-1+\beta j_{q_k}}{1+\beta})),$

$0], min[(\sum_{q=1}^{k} \delta_q(i_q+\beta i_q) + \sum\prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(i_{q_1}+\beta i_{q_1})\delta_{q_2}(i_{q_2}+\beta i_{q_2}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(i_{q_1}+\beta i_{q_1})\ldots\delta_{q_k}(i_{q_k}+\beta i_{q_k})),1],$

$min[(\sum_{q=1}^{k} \delta_q(\ell_q+\beta\ell_q) + \sum\prod_{q_1,q_2\in\mathbb{J}_k} \beta\delta_{q_1}(\ell_{q_1}+\beta\ell_{q_1})\delta_{q_2}(\ell_{q_2}+\beta\ell_{q_2}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_k\in\mathbb{J}_k} \beta\delta_{q_1}(\ell_{q_1}+\beta\ell_{q_1})\ldots\delta_{q_k}(\ell_{q_k}+\beta\ell_{q_k})),1]\}$

$\Cap \{max(\delta_{k+1}(\frac{j_{k+1}-1+\beta j_{k+1}}{1+\beta}),0), min(\delta_{k+1}(i_{k+1}+\beta i_{k+1}),1),$

$min(\delta_1(\ell_{k+1}+\beta\ell_{k+1}),1)\} = \{max[(\sum_{q=1}^{k+1} \delta_q(\frac{j_q-1+\beta j_q}{1+\beta}) + \sum\prod_{q_1,q_2\in\mathbb{J}_{k+1}} \beta\delta_{q_1}$

$(\frac{j_{q_1}-1+\beta j_{q_1}}{1+\beta})\delta_{q_2}(\frac{j_{q_2}-1+\beta j_{q_2}}{1+\beta}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_{k+1}\in\mathbb{J}_{k+1}} \beta\delta_{q_1}(\frac{j_{q_1}-1+\beta j_{q_1}}{1+\beta})\ldots\delta_{q_{k+1}}(\frac{j_{q_{k+1}}-1+\beta j_{q_{k+1}}}{1+\beta})),0],$

$min[(\sum_{q=1}^{k+1} \delta_q(i_q+\beta i_q) + \sum\prod_{q_1,q_2\in\mathbb{J}_{k+1}} \beta\delta_{q_1}(i_{q_1}+\beta i_{q_1})\delta_{q_2}(i_{q_2}+\beta i_{q_2}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_{k+1}\in\mathbb{J}_{k+1}} \beta\delta_{q_1}(i_{q_1}+\beta i_{q_1})\ldots\delta_{q_{k+1}}(i_{q_{k+1}}+\beta i_{q_{k+1}})),1],$

$min[(\sum_{q=1}^{k+1} \delta_q(\ell_q+\beta\ell_q) + \sum\prod_{q_1,q_2\in\mathbb{J}_{k+1}} \beta\delta_{q_1}(\ell_{q_1}+\beta\ell_{q_1})\delta_{q_2}(\ell_{q_2}+\beta\ell_{q_2}))$

$+\ldots+\sum\prod_{q_1,q_2,\ldots,q_{k+1}\in\mathbb{J}_{k+1}} \beta\delta_{q_1}(\ell_{q_1}+\beta\ell_{q_1})\ldots\delta_{q_{k+1}}(\ell_{q_{k+1}}+\beta\ell_{q_{k+1}})),1]\}.$

Hence, for any n, the equation holds.

**Theorem 12:** *When* $\Delta_q=(j_q(x), i_q(x), \ell_q(x))$, $\Delta_q^-=(min(j_q(x)), max(i_q(x)), max(\ell_q(x)))$ *and* $\Delta_q^+=(max(j_q(x)), min(i_q(x)), min(\ell_q(x)))$ $\in$ *SVN norm* ($q \in \mathbb{N}$), *Then*

$$\Delta_q^- \leq SVNWWG(\Delta_1, \Delta_2, \ldots, \Delta_n) \leq \Delta_q^+.$$

**Theorem 13:** *When* $\Delta_q=(j_q(x), i_q(x), \ell_q(x))$, $\Delta_q^*=(j_q^*(x), i_q^*(x), \ell_q^*(x))$, $\in$ *SVN norms* ($q \in \mathbb{N}$). *If* $j_q \leq j_q^*$, $i_q \geq i_q^*$, $\ell_q \geq \ell_q^*$, *Then*

$$SVNWWG(\Delta_1, \Delta_2, \ldots, \Delta_n) \leq SVNWWG(\Delta_1^*, \Delta_2^*, \ldots, \Delta_n^*).$$

**Definition 15:** Let $\Delta_q$=$(j_q(x), i_q(x), \ell_q(x))$ $\in$ SVN norms ($q \in \mathbb{N}$). We define Weber ordered weighted geometric aggregation operators for SVN norms as:

$$SVNWOWG(\Delta_1, \Delta_2, \ldots, \Delta_n) = \Delta_{\Im(1)}^{\delta_1} \grave{O} \Delta_{\Im(2)}^{\delta_2} \grave{O} \ldots \grave{O} \Delta_{\Im(n)}^{\delta_n}$$

where $x(q)$ represents the order and $(\Im(1), \Im(2), \ldots, \Im(n))$ is a permutation of (1,2,3,...,n), with the condition $\Delta_{\Im(q-1)} \geq \Delta_{\Im(q)}$ for all q. Also, the weights $(\delta_1, \delta_2, \ldots, \delta_q)$ of $\Delta_q$ having $\delta_q \geq 0$ and $\sum_{q=1}^{n} \delta_q = 1$.

**Theorem 14:** *Let* $\Delta_q=(j_q(x), i_q(x), \ell_q(x))$ $\in$ *SVN norms* ($q \in \mathbb{N}$) *and the weights* $(\delta_1, \delta_2, \ldots, \delta_q)$ *of* $\Delta_q$ *having* $\delta_q \geq 0$ *and* $\sum_{q=1}^{n} \delta_q = 1$. *Then, the SVNWOWG aggregation operators is a function from* $\mathcal{V}^n \to \mathcal{V}$ *such that* $SVNWOWA(\Delta_1, \Delta_2, \ldots, \Delta_n) =$

$$\{max[(\textstyle\sum_{q=1}^{n} \delta_q(\frac{j_{x(q)}-1+\beta j_{x(q)}}{1+\beta}) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta \delta_{q_1}(\frac{j_{x(q_1)}-1+\beta j_{x(q_1)}}{1+\beta})$$
$$\delta_{q_2}(\frac{j_{x(q_2)}-1+\beta j_{x(q_2)}}{1+\beta}))$$
$$+\ldots+\textstyle\sum \prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta \delta_{q_1}(\frac{j_{x(q_1)}-1+\beta j_{x(q_1)}}{1+\beta})\ldots \delta_{q_n}(\frac{j_{x(q_n)}-1+\beta j_{x(q_n)}}{1+\beta})),0],$$
$$min[(\textstyle\sum_{q=1}^{n} \delta_q(i_{x(q)} + \beta i_{x(q)}) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta \delta_{x(q_1)}(i_{x(q_1)} + \beta i_{x(q_1)})$$
$$\delta_{q_2}(i_{x(q_2)} + \beta i_{x(q_2)}))$$
$$+\ldots+\textstyle\sum \prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta \delta_{x(q_1)}(i_{x(q_1)} + \beta i_{x(q_1)})\ldots \delta_{q_n}(i_{x(q_n)} + \beta i_{x(q_n)})),1],$$
$$min[(\textstyle\sum_{q=1}^{n} \delta_q(\ell_{x(q)} + \beta \ell_{x(q)}) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta \delta_{q_1}(\ell_{x(q_1)} + \beta \ell_{x(q_1)})$$
$$\delta_{q_2}(\ell_{x(q_2)} + \beta \ell_{x(q_2)}))$$
$$+\ldots+\textstyle\sum \prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta \delta_{q_1}(\ell_{x(q_1)} + \beta \ell_{x(q_1)})\ldots \delta_{q_n}(\ell_{x(q_n)} + \beta \ell_{x(q_n)})),1]\},$$

where $\mathbb{J}_n$=$(1,2,3,\ldots,n)$.

**Theorem 15:** *Let* $\Delta_q=(j_q(x), i_q(x), \ell_q(x))$, $\Delta_q^-=(min(j_q(x)), max(i_q(x)), max(\ell_q(x)))$ *and* $\Delta_q^+=(max(j_q(x)), min(i_q(x)), min(\ell_q(x)))$ $\in$ *SVN norm* ($q \in \mathbb{N}$), *Then*

$$\Delta_q^- \leq SVNWOWG(\Delta_1, \Delta_2, \ldots, \Delta_n) \leq \Delta_q^+.$$

**Theorem 16:** *When* $\Delta_q = (j_q(x), i_q(x), \ell_q(x))$, $\Delta_q^* = (j_q^*(x), i_q^*(x), \ell_q^*(x))$, $\in$ *SVN norms* ($q \in \mathbb{N}$). *If* $j_q \leq j_q^*$, $i_q \geq i_q^*$, $\ell_q \geq \ell_q^*$, *Then*

$$SVNWOWG(\Delta_1, \Delta_2, \ldots, \Delta_n) \leq SVNWOWG(\Delta_1^*, \Delta_2^*, \ldots, \Delta_n^*).$$

**Definition 16:** Let $\Delta_q$=$(j_q(x), i_q(x), \ell_q(x))$ $\in$ SVN norms ($q \in \mathbb{N}$). Weber hybrid weighted geometric aggregation operators for SVN norms,

$$SVNWHWG(\Delta_1, \Delta_2, \ldots, \Delta_n) = (\widetilde{\Delta}_{x(1)})^{\kappa_1} ⋒ (\widetilde{\Delta}_{x(2)})^{\kappa_2} ⋒ \ldots ⋒ (\widetilde{\Delta}_{x(n)})^{\kappa_n}$$

where the weights $(\delta_1, \delta_2, \ldots, \delta_q)$ associated with $\Delta_q$ satisfy $\delta_q \geq 0$ and $\sum_{q=1}^{n} \delta_q = 1$, where the $k$-th highest weighted value is denoted as $\widetilde{\Delta}x(q)$ (with $\widetilde{\Delta}x(q) = n\delta_q\Delta_{x(q)}$ for $q = 1,2, \ldots, n$), determined accordingly to the total order $(x(1), x(2), \ldots, x(n))$.

**Theorem 17:** *Let* $\Delta_q = (j_q(x), i_q(x), \ell_q(x))$ $\in$ *SVN norms* ($q \in \mathbb{N}$) *and the weights* $(\delta_1, \delta_2, \ldots, \delta_q)$ *of* $\Delta_q$ *having* $\delta_q \geq 0$ *and* $\sum_{q=1}^{n} \delta_q = 1$. *The SVNWHWG aggregation operators are a function from* $\mathcal{V}^n \to \mathcal{V}$ *with associated weights* $(\kappa_1, \kappa_2, \ldots, \kappa_q)$ *of* $\Delta_q$ *having* $\kappa_q \geq 0$ *and* $\sum_{q=1}^{n} \kappa_q = 1$, *and we have* $SVNWHWG(\Delta_1, \Delta_2, \ldots, \Delta_n) =$

$$\{max[(\textstyle\sum_{q=1}^{n} \kappa_q(\frac{\tilde{j}_q - 1 + \beta\tilde{j}_q}{1+\beta}) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\kappa_{q_1}(\frac{\tilde{j}_{q_1} - 1 + \beta\tilde{j}_{q_1}}{1+\beta})\kappa_{q_2}(\frac{\tilde{j}_{q_2} - 1 + \beta.\tilde{j}_{q_2}}{1+\beta}))$$
$$+ \ldots + \textstyle\sum \prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\kappa_{q_1}(\frac{\tilde{j}_{q_1} - 1 + \beta\tilde{j}_{q_1}}{1+\beta}) \ldots \kappa_{q_n}(\frac{\tilde{j}_{q_n} - 1 + \beta\tilde{j}_{q_n}}{1+\beta})),0],$$
$$min[(\textstyle\sum_{q=1}^{n} \kappa_q(\tilde{\imath}_q + \beta\tilde{\imath}_q) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\kappa_{q_1}(\tilde{\imath}_{q_1} + \beta\tilde{\imath}_{q_1})\kappa_{q_2}(\tilde{\imath}_{q_2} + \beta\tilde{\imath}_{q_2}))$$
$$+ \ldots + \textstyle\sum \prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\kappa_{q_1}(\tilde{\imath}_{q_1} + \beta\tilde{\imath}_{q_1}) \ldots \kappa_{q_n}(\tilde{\imath}_{q_n} + \beta\tilde{\imath}_{q_n})),1],$$
$$min[(\textstyle\sum_{q=1}^{n} \kappa_q(\tilde{\ell}_q + \beta\tilde{\ell}_q) + \sum \prod_{q_1,q_2 \in \mathbb{J}_n} \beta\kappa_{q_1}(\tilde{\ell}_{q_1} + \beta\tilde{\ell}_{q_1})\kappa_{q_2}(\tilde{\ell}_{q_2} + \beta\tilde{\ell}_{q_2}))$$
$$+ \ldots + \textstyle\sum \prod_{q_1,q_2,\ldots,q_n \in \mathbb{J}_n} \beta\kappa_{q_1}(\tilde{\ell}_{q_1} + \beta\tilde{\ell}_{q_1}) \ldots \kappa_{q_n}(\tilde{\ell}_{q_n} + \beta\tilde{\ell}_{q_n})),1]\},$$

where $\mathbb{J}_n$=$(1,2,3, \ldots, n)$.

**Theorem 18:** *Let* $\Delta_q = (j_q(x), i_q(x), \ell_q(x))$, $\Delta_q^- = (min(j_q(x)), max(i_q(x)), max(\ell_q(x)))$ *and* $\Delta_q^+ = (max(j_q(x)), min(i_q(x)), min(\ell_q(x)))$ $\in$ *SVN norm* ($q \in \mathbb{N}$), *Then*

$$\Delta_q^- \leq SVNWHWG(\Delta_1, \Delta_2, \ldots, \Delta_n) \leq \Delta_q^+.$$

**Theorem 19:** *When* $\Delta_q = (j_q(x), i_q(x), \ell_q(x))$, $\Delta_q^* = (j_q^*(x), i_q^*(x), \ell_q^*(x))$, $\in$ *SVN norms* ($q \in \mathbb{N}$). *If* $j_q \leq j_q^*$, $i_q \geq i_q^*$, $\ell_q \geq \ell_q^*$, *Then*

$$SVNWHWG(\Delta_1, \Delta_2, \ldots, \Delta_n) \leq SVNWHWG(\Delta_1^*, \Delta_2^*, \ldots, \Delta_n^*).$$

## 5. Multicriteria Decision Making using SVN data

5.1 *Algorithm*

In this part, Using SVN data, we present a layout for resolving the problem of making decisions with multiple aspects. Consider a decision making problem with multiple attributes $\{\kappa_1, \kappa_2, \ldots, \kappa_m\}$ whose weight vectors are $\{\rho_1, \rho_2, \ldots, \rho_m\}$ where $\rho_a \in [0,1]$ and $\sum_{a=1}^{m} \rho_a = 1$, having alternatives $\{\zeta_1, \zeta_2, \ldots, \zeta_m\}$. Let $\{A_1, A_2, \ldots, A_{\breve{e}}\}$ be the set of decision makers whose weight vectors are $(\varpi_1, \varpi_2, \ldots, \varpi_{\breve{e}})$ where $\varpi_s \in [0,1]$ and $\sum_{s=1}^{\breve{e}} \varpi_s = 1$. The generalized form of decision matrix by a decision maker for a SVN set is :

$$\begin{bmatrix} (j_{11}^{\breve{e}}, i_{11}^{\breve{e}}, \ell_{11}^{\breve{e}}) & (j_{12}^{\breve{e}}, i_{12}^{\breve{e}}, \ell_{12}^{\breve{e}}) & \cdots & (j_{1m}^{\breve{e}}, i_{1m}^{\breve{e}}, \ell_{1m}^{\breve{e}}) \\ (j_{21}^{\breve{e}}, i_{21}^{\breve{e}}, \ell_{21}^{\breve{e}}) & (j_{22}^{\breve{e}}, i_{22}^{\breve{e}}, \ell_{22}^{\breve{e}}) & \cdots & (j_{2m}^{\breve{e}}, i_{2m}^{\breve{e}}, \ell_{2m}^{\breve{e}}) \\ (j_{31}^{\breve{e}}, i_{31}^{\breve{e}}, \ell_{31}^{\breve{e}}) & (j_{32}^{\breve{e}}, i_{32}^{\breve{e}}, \ell_{32}^{\breve{e}}) & \cdots & (j_{3m}^{\breve{e}}, i_{3m}^{\breve{e}}, \ell_{3m}^{\breve{e}}) \\ \vdots & \vdots & \vdots & \vdots \\ (j_{m1}^{\breve{e}}, i_{m1}^{\breve{e}}, \ell_{m1}^{\breve{e}}) & (j_{m2}^{\breve{e}}, i_{m2}^{\breve{e}}, \ell_{m2}^{\breve{e}}) & \cdots & (j_{mm}^{\breve{e}}, i_{mm}^{\breve{e}}, \ell_{mm}^{\breve{e}}) \end{bmatrix}$$

Where the truth $j(x) \in [0,1]$, the indeterminacy $i(x) \in [0,1]$, and falsity $\ell(x) \in [0,1]$. Also,

$$0 \le j(x) + i(x) + \ell(x) \le 3, \quad \forall x \in X.$$

**Step 1:** Subject to expert estimation, construct the SVN set decision matrix corresponding to each decision maker $A_j$ where $j = (1,2,\ldots,\breve{e})$.

**Step 2:**The aggregated matrix is constructed by combining each decision matrices derived from the aggregation operators. The resulting decision matrix is expressed as follows:

$$\begin{bmatrix} (j_{11}, i_{11}, \ell_{11}) & (j_{12}, i_{12}, \ell_{12}) & \cdots & (j_{1m}, i_{1m}, \ell_{1m}) \\ (j_{21}, i_{21}, \ell_{21}) & (j_{22}, i_{22}, \ell_{22}) & \cdots & (j_{2m}, i_{2m}, \ell_{2m}) \\ (j_{31}, i_{31}, \ell_{31}) & (j_{32}, i_{32}, \ell_{32}) & \cdots & (j_{3m}, i_{3m}, \ell_{3m}) \\ \vdots & \vdots & \vdots & \vdots \\ (j_{m1}, i_{m1}, \ell_{m1}) & (j_{m2}, i_{m2}, \ell_{m2}) & \cdots & (j_{mm}, i_{mm}, \ell_{mm}) \end{bmatrix}$$

**Step 3:** Apply the aggregation operators SVNWWA, SVNWOWA, SVNWHWA, SVNWOWG, SVNWOWG, SVNWHWG to obtain the total aggregate preference values for the alternatives $\zeta_m$ where $\rho = (\rho_1, \rho_2, \ldots, \rho_m)$ is the weight vectors of the attributes.

**Step 4:** For the hybrid operator we take [9] $\rho =$ (0.112, 0.236, 0.304, 0.236, 0.112).

**Step 5:** Calculate the score value for the alternatives $\zeta_m$. Where the score value $\widehat{\mathbb{S}}$ is defined as $\widehat{\mathbb{S}} = j(x) - i(x) - \ell(x)$.

**Step 6:** Sort the score values in descending order and select the option with the highest value as the optimal choice.

5.2 *Numerical Example*

The above algorithm can be implemented in many aspects of decision making problems. Let us consider, the problem to select the suitable modern manufacturing system available in the market. We have 5 alternate choices: $\zeta_1$, $\zeta_2$, $\zeta_3$, $\zeta_4$, $\zeta_5$. We take some criteria into account because each alternate has its own advantages and drawbacks. The attributes are: $\kappa_1$- Operational cost, $\kappa_2$- feasibility, $\kappa_3$- time requirement, $\kappa_4$- risk of failure, $\kappa_5$- availabilty of resources. As per the requirement we characterize the weight vector $\rho_m (m = 1,2,3,4,5)$ as (0.13, 0.30, 0.17, 0.25, 0.15). We use SVN information to identify the best manufacturing system.

**Step 1:** The information about expert evaluation is given in Table .

**Step 2:** Since there is only one decision-maker here, the aggregated matrix is not required.

**Step 3:** The weight vector is $\rho = (\rho_1 = 0.13, \rho_2 = 0.30, \rho_3 = 0.17, \rho_4 = 0.25, \rho_5 = 0.15)$.

**Step 4:** We make use of the propsed WAO to evaluate the collective performances of the alternatives. That is shown in Table 2 and 3**.**

**Step 5:** The score value as shown in Table 4.

**Step 6:** The scores are sorted in descending order, with highest value determining optimal choice, as illustrated in Table 5. Consequently, $\zeta_4$ is identified as the best alternative.

**Table 1**
**Expert evaluation information**

| | $\kappa_1$ | $\kappa_2$ | $\kappa_3$ | $\kappa_4$ | $\kappa_5$ |
|---|---|---|---|---|---|
| $\zeta_1$ | [0.4,0.2,0.3] | [0.2,0.1,0.3] | [0.1,0.1,0.5] | [0.4,0.1,0.2] | [0.2,0.2,0.3] |
| $\zeta_2$ | [0.6,0.1,0.2] | [0.4,0.3,0.1] | [0.4,0.2,0.1] | [0.6,0.2,0.2] | [0.6,0.1,0.1] |
| $\zeta_3$ | [0.4,0.2,0.3] | [0.4,0.1,0.2] | [0.5,0.1,0.1] | [0.5,0.1,0.3] | [0.5,0.3,0.2] |
| $\zeta_4$ | [0.6,0.2,0.1] | [0.4,0.1,0.1] | [0.3,0.4,0.1] | [0.6,0.1,0.1] | [0.3,0.4,0.3] |
| $\zeta_5$ | [0.3,0.1,0.2] | [0.3,0.2,0.5] | [0.3,0.1,0.4] | [0.4,0.1,0.1] | [0.3,0.1,0.3] |

**Table 2**
**Weber weighted averaging**

| | SVNWWA | SVNWOWA | SVNWHWA |
|---|---|---|---|
| $\zeta_1$ | [0.2660, 0, 0] | [0.2710, 0, 0] | [0.2396, 0, 0] |
| $\zeta_2$ | [0.4510, 0, 0] | [0.4490, 0, 0] | [0.4069, 0, 0] |
| $\zeta_3$ | [0.4630, 0, 0] | [0.4630, 0, 0] | [0.4539, 0, 0] |
| $\zeta_4$ | [0.5160, 0, 0] | [0.5260, 0, 0] | [0.5152, 0, 0] |
| $\zeta_5$ | [0.3280, 0, 0] | [0.3150, 0, 0] | [0.3161, 0, 0] |

**Table 3**
**Weber weighted geometric**

| | SVNWWG | SVNWOWG | SVNWHWG |
|---|---|---|---|
| $\zeta_1$ | [0, 0.1300, 0.3120] | [0, 0.1500, 0.3150] | [0, 0.1575, 0.3196] |
| $\zeta_2$ | [0, 0.2200, 0.1300] | [0, 0.2390, 0.1330] | [0, 0.2588, 0.1270] |
| $\zeta_3$ | [0, 0.1450, 0.2230] | [0, 0.1620, 0.2350] | [0, 0.1427, 0.2191] |
| $\zeta_4$ | [0, 0.1920, 0.1430] | [0, 0.1720, 0.1480] | [0, 0.1703, 0.1397] |
| $\zeta_5$ | [0, 0.1220, 0.2930] | [0, 0.1150, 0.2940] | [0, 0.1211, 0.3168] |

**Table 4**
**Score Values**

| | $\widehat{\mathbb{S}}(\zeta_1)$ | $\widehat{\mathbb{S}}(\zeta_2)$ | $\widehat{\mathbb{S}}(\zeta_3)$ | $\widehat{\mathbb{S}}(\zeta_4)$ | $\widehat{\mathbb{S}}(\zeta_5)$ |
|---|---|---|---|---|---|
| SVNWWA | 0.266 | 0.451 | 0.463 | 0.516 | 0.328 |
| SVNWOWA | 0.271 | 0.449 | 0.463 | 0.526 | 0.315 |
| SVNWHWA | 0.239 | 0.406 | 0.453 | 0.515 | 0.316 |
| SVNWWG | -0.442 | -0.350 | -0.368 | -0.335 | -0.415 |
| SVNWOWG | -0.465 | -0.372 | -0.397 | -0.320 | -0.409 |
| SVNWHWG | -0.477 | -0.385 | -0.361 | -0.310 | -0.437 |

**Table 5**
**Ranking**

| Existing Operators | Score Ranking | Optimal Alternative |
|---|---|---|
| SVNWWA | $\widehat{\mathbb{S}}(\zeta_4) > \widehat{\mathbb{S}}(\zeta_3) > \widehat{\mathbb{S}}(\zeta_2) > \widehat{\mathbb{S}}(\zeta_5) > \widehat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNWOWA | $\widehat{\mathbb{S}}(\zeta_4) > \widehat{\mathbb{S}}(\zeta_3) > \widehat{\mathbb{S}}(\zeta_2) > \widehat{\mathbb{S}}(\zeta_5) > \widehat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNWHWA | $\widehat{\mathbb{S}}(\zeta_4) > \widehat{\mathbb{S}}(\zeta_3) > \widehat{\mathbb{S}}(\zeta_2) > \widehat{\mathbb{S}}(\zeta_5) > \widehat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNWWG | $\widehat{\mathbb{S}}(\zeta_4) > \widehat{\mathbb{S}}(\zeta_3) > \widehat{\mathbb{S}}(\zeta_2) > \widehat{\mathbb{S}}(\zeta_5) > \widehat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNWOWG | $\widehat{\mathbb{S}}(\zeta_4) > \widehat{\mathbb{S}}(\zeta_3) > \widehat{\mathbb{S}}(\zeta_2) > \widehat{\mathbb{S}}(\zeta_5) > \widehat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNWHWG | $\widehat{\mathbb{S}}(\zeta_4) > \widehat{\mathbb{S}}(\zeta_3) > \widehat{\mathbb{S}}(\zeta_2) > \widehat{\mathbb{S}}(\zeta_5) > \widehat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |

**Table 6**
**Existing geometric aggregated SVN Yager information**

| | SVNYWG | SVNYOWG | SVNYHWG |
|---|---|---|---|
| $\zeta_1$ | [4.9373, 0.0300, 0.0723] | [4.9389, 0.0258, 0.0686] | [4.9930, 0.0399, 0.0639] |
| $\zeta_2$ | [4.9693, 0.0593, 0.0313] | [4.9716, 0.0475, 0.0313] | [4.9978, 0.0647, 0.0276] |
| $\zeta_3$ | [4.9789, 0.0353, 0.0414] | [4.9782, 0.0361, 0.0425] | [4.9991, 0.0353, 0.0391] |
| $\zeta_4$ | [4.9818, 0.0350, 0.0269] | [4.9826, 0.0404, 0.0292] | [4.9996, 0.0340, 0.0332] |
| $\zeta_5$ | [4.9593, 0.0236, 0.0656] | [4.9605, 0.0260, 0.0653] | [4.9972, 0.0290, 0.0752] |

**Table 7**
**Existing average aggregated SVN Yager information**

| | SVNYWA | SVNYOWA | SVNYHWA |
|---|---|---|---|
| $\zeta_1$ | [0.0476, 4.9184, 4.9628] | [0.0486, 4.9116, 4.9599] | [0.0680, 4.9901, 4.9971] |
| $\zeta_2$ | [0.0837, 4.9482, 4.9172] | [0.0870, 4.9342, 4.9172] | [0.0841, 4.9945, 4.9816] |
| $\zeta_3$ | [0.0965, 4.9237, 4.9330] | [0.0952, 4.9244, 4.9355] | [0.1032, 4.9912, 4.9923] |
| $\zeta_4$ | [0.1058, 4.9247, 4.9134] | [0.1077, 4.9320, 4.9171] | [0.0978, 4.9920, 4.9813] |
| $\zeta_5$ | [0.0630, 4.9080, 4.9555] | [0.0660, 4.9119, 4.9532] | [0.0709, 4.9809, 4.9950] |

**Table 8**
**Ranking using existing operators**

| Existing Operators | Score Ranking | Optimal Alternative |
|---|---|---|
| SVNYWA [36] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNYOWA [36] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNYHWA [36] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNYWG [36] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNYOWG [36] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNYHWG [36] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNWA [39] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNOWA [39] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| NWA [40] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNFWA [38] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| SVNHWA [37] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| L-SVNWA [35] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| L-SVNOWA [35] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| ST-SVNWA [34] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| ST-SVNWG [34] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| ST-SVNOWA [34] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |
| ST-SVNOWG [34] | $\hat{\mathbb{S}}(\zeta_4) > \hat{\mathbb{S}}(\zeta_3) > \hat{\mathbb{S}}(\zeta_2) > \hat{\mathbb{S}}(\zeta_5) > \hat{\mathbb{S}}(\zeta_1)$ | $\zeta_4$ |

5.3 Comparison Analysis

To establish the efficiency of the suggested aggregation operators, the comparison with the Yager [36] aggregation operators are employed. The weighted aggregation and geometric aggregation for already existing operators are calculated and shown in Table 7 and 6. Therefore from Tables 8 and 5, we can conclude that their rankings are identical. Hence under SVN environment, Weber-norm based aggregation operator is more adaptable and effective at determing the optimal alternative.

## 6. Conclusion

Neutrosophic set theory becomes extremely significant in decision-making to deal with uncertainties. The neutrosophic set's aggregation operators provide an organized approach for handling uncertainties. This paper presents a comprehensive examination of the Weber operational principles applied to NS, with the definition of aggregation operators derived from these established principles. We also provide a systematic algorithm for handling decision-making scenarios. The numerical example and comparison study illustrate the practicality and efficiency of our suggested methodology. For future research, we suggest extending these operators to Neutrosophic soft sets, Neutrosophic cubic sets, etc., The proposed framework can be employed in ambiguous environment such as risk estimation, decision-making theory, and other uncertain domains.

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