Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 28 (2025), No. 5, pp. 2031–2043
DOI : 10.47974/JIM-2265

A simple mathematical model for assessing water quality in a closed-system shrimp farm

Kaboon Thongtha †
Department of Mathematics
Faculty of Science
Mahanakorn University of Technology
Bangkok 10530
Thailand

Nopparat Pochai *
Department of Mathematics
School of Science
King Mongkut's Institute of Technology Ladkrabang
Bangkok 10520
Thailand

Abstract

The problem of wastewater from shrimp farming affects the environment, both in terms of wastewater discharge and soil deterioration. Wastewater management is also quite expensive for the production costs of shrimp farmers. Therefore, the approach to using shrimp farming technology in closed-system farms is proposed, which reduces wastewater discharge into the environment and reduces the cost of wastewater treatment for farmers. This research presents a simple mathematical model for assessing water quality in such closed-system shrimp farms. The method for determining various parameters for determining the mathematical model is presented. The model solution is estimated by the Runge-Kutta method of the fourth order. This research simulates the situation to compare the different parameter values in each situation, which affect the level of water quality in closed-system shrimp farms at different times. The research found that the initial water quality, the rate of chemical reaction of pollutants, the rate of pollution formation, the rate of pollution decomposition, the rate of decrease in pollution concentration due to water circulation between the farm and the water treatment pond, and time all affect water quality. The results from the calculation can help closed-system shrimp farmers know the trend of

† *E-mail:* **kaboon.t@gmail.com**
* *E-mail:* **nopparat.po@kmitl.ac.th (Corresponding Author)**

pollution concentration changes in closed-system shrimp farms in order to find ways to develop techniques for improving water quality.

Subject Classification: *65L05.*

Keywords: *Closed-system shrimp farm, Mathematical model, Water quality, Pollution, Fourth-order Runge-Kutta method.*

1. Introduction

Wastewater in shrimp ponds caused by biological materials produced in the ponds is frequently the cause of issues in open-system shrimp farming. As a result, wastewater that is discharged into natural water sources may pollute water resources and have an adverse effect on the ecosystem and other living beings [1]. By decreasing the outflow from farms to natural water sources, closed-system shrimp farming practices and water recycling techniques for recirculating farm water [2] enhance water quality. The efficiency of shrimp production is increased, and the risk of shrimp sickness is eliminated when water is used efficiently and to its fullest potential. This allows farmers to produce high-quality shrimp that are safe from the usage of chemicals or antibiotics. Some studies use a mathematical model to quantify water quality [3] in enclosed drainage systems for shrimp farming and water flow that impacts water quality [4, 5, 6, 7, 8, 9]. By providing treatment and recycling techniques [10] to enhance water quality, the water circulation on the farm and the flow rate from the well drainage were used to measure the water quality. A fourth-order Rung Kutta approach is used to estimate the solution of the governing equation [11], [12], [13], which is practical for computations. In order to compare the water quality produced by taking biological oxygen demand (BOD) [14] into consideration as an index for evaluating water quality, it simulates a scenario in which water in a well is pumped at varying flow rates while the wastewater treatment plant is operating at different intervals. In [15], numerical simulations of water quality in a couple of ponds of shrimp farming are proposed.

They plan to use closed-system shrimp farming technology, which involves circulating and cleaning water, to protect water quality while also maximizing efficiency and benefit. In this study, a mathematical model will be written as an ordinary differential equation using a fourth-order Runge-Kutta approximation.

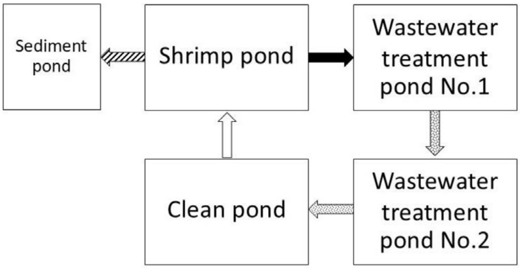

Figure 1

A closed-system shrimp farm diagram

2. A simple model for assessing water quality in a closed-system shrimp farm

In order to maintain water quality and maximize efficiency and benefit, they plan on using closed-system shrimp farming technology, which involves circulating and treating water. The BOD value of the pond water will be determined by the water quality assessment. For farming, a BOD value of 1-6 mg/L is suitable. Water becomes unfit for farming (regarded as effluent) if the BOD value exceeds 10 mg/L. A closed-system shrimp farm and their process diagram are shown in Figures 1.

In this research, a simple mathematical model of two pounds between a shrimp pond and a wastewater treatment pond is proposed. The proposed mathematical model is used to describe the water quality of both ponds when the water is circulated. In the simulation, water will be transferred between the two ponds at a certain time. The relation between two ponds can be described as shown in Figure 2.

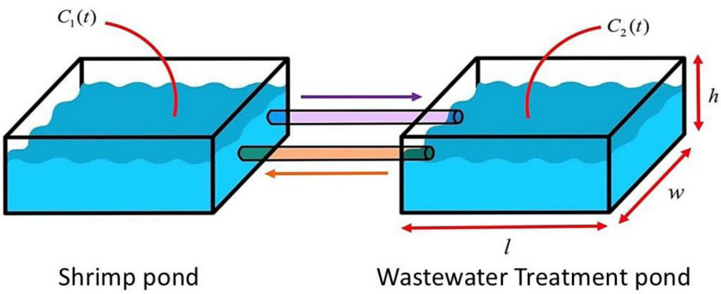

Figure 2

The relation between shrimp and wastewater treatment ponds

3. A simple mathematical model for assessing water quality in a closed-system shrimp farm d

3.1 *A water quality model of the shrimp pond*

Water quality of the shrimp pond can be described by a zero-dimensional water quality model,

$$V_1 \frac{dC_1}{dt} = K_1 C_1 - Q_1 + S_1 - PC_1 \quad (1)$$

where C_1 is the pollution concentration in shrimp pond, V_1 is the volume of water in shrimp pond, K_1 is the reaction rate of toxic substances in shrimp pond, Q_1 is the decomposition rate of toxic substances in shrimp pond, S_1 is the pollution emission rate from the source of shrimp pond and P is the pollution reduction rate due to water pumping.

3.2 *A water quality model of the wastewater treatment pond*

Water quality of the shrimp pond can be described by a zero-dimensional water quality model,

$$V_2 \frac{dC_2}{dt} = K_2 C_2 - Q_2 + KPC_1 \quad (2)$$

where C_2 is the concentration of pollutants in the treatment pond, V_2 is the volume of water in the treatment pond, K_2 is the reaction rate of toxic substances in the treatment pond, Q_2 is the decomposition rate of toxic substances in the treatment pond, C_1 is the concentration of pollutants in the shrimp pond pumped into the treatment pond, and K is the rate of increase of pollutants due to pumping water in.

4. Mathematical simulation

This section will demonstrate the differences in pollutant concentration levels in shrimp farms under different circumstances. A commonly used Runge-Kutta method to find the solution of Eqs. (1) and (2) is the fourth-order Runge-Kutta method.

4.1 *A water quality of the shrimp pond when the rate of toxic substances varies*

Parameters are set as shown in Table 1. The concentration level of pollutants in the shrimp pond when the rate of toxic substances, K_1, varies over a period of 3 days, with the concentration values displayed every 6 hours, is shown in Table 2 and Figure 3.

Table 1

Parameter setting of the scenario 1.

Parameters	Setting value
K_1	0.1,0.3, 0.5, 0.7
Q_1	1
S_1	1
P	0.5
V_1	625
$C_1(0)$	5

Table 2

The concentration level of pollutants in the shrimp pond of the scenario 1.

t (hr)	$C_1(t)(K_1=0.1)$	$C_1(t)$ $(K_1=0.3)$	$C_1(t)$ $(K_1=0.5)$	$C_1(t)$ $(K_1=0.7)$
0	5	5	5	5
6	3.9711	4.4559	5	5.6105
12	3.1539	3.9711	5	6.2955
18	2.5049	3.5390	5	7.0642
24	1.9895	3.1539	5	7.9267
30	1.5801	2.8107	5	8.8945
36	1.2549	2.5049	5	9.9805
42	0.9967	2.2323	5	11.1991
48	0.7916	1.9894	5	12.5665
54	0.6287	1.7729	5	14.1009
60	0.4993	1.5800	5	15.8226
66	0.3966	1.4081	5	17.7545
72	0.3150	1.2549	5	19.9222

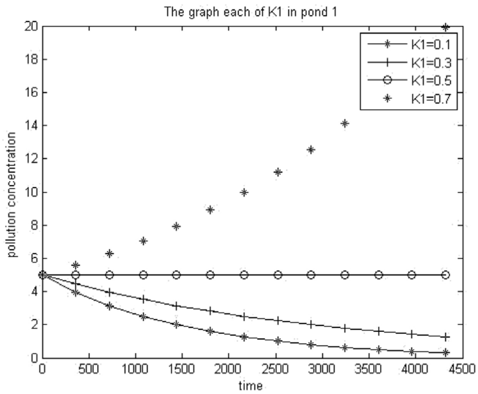

Figure 3

The concentration level of pollutants in the shrimp pond of the scenario 1.

4.2 *A water quality of the shrimp pond when the pumping rate varies*

Parameters are set as shown in Table 3. The concentration level of pollutants in the shrimp pond when the pollution reduction rates due to pumping, P, varies over a period of 3 days, with the concentration values displayed every 6 hours, is shown in Table 4 and Figure 4.

Table 3

Parameter setting of the scenario 2

Parameters	Setting value
K_1	0.5
Q_1	1
S_1	1
P	0.4, 0.5, 0.6, 0.7
V_1	625
$C_1(0)$	3

Table 4

The concentration level of pollutants in the shrimp pond of the scenario 2.

t (hr)	C_1(t)(P=0.4)	C_1(t) (P=0.5)	C_1(t) (P=0.6)	$C_1(t)$ (P=0.7)
0	5	5	5	5
6	5.2965	5	4.7201	4.4559

Contd...

12	5.6105	5	4.4559	3.9711
18	5.9431	5	4.2065	3.5390
24	6.2955	5	3.9711	3.1539
30	6.6688	5	3.7488	2.8107
36	7.0642	5	3.5390	2.5049
42	7.4830	5	3.3409	2.2323
48	7.9267	5	3.1539	1.9894
54	8.3967	5	2.9774	1.7729
60	8.8945	5	2.8107	1.5800
66	9.4219	5	2.6534	1.4081
72	9.9805	5	2.5049	1.2549

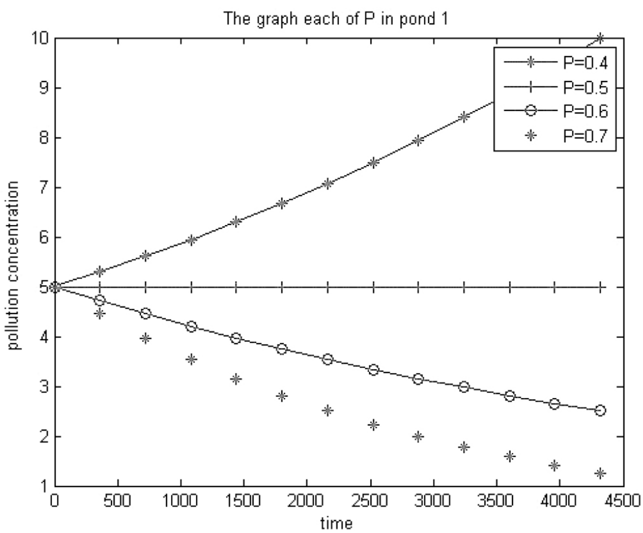

Figure 4

The concentration level of pollutants in the shrimp pond of the scenario 2.

4.3 *A water quality of the treatment pond when the decomposition rate of toxic substances varies*

Parameters are set as shown in Table 5. The concentration level of pollutants in the treatment pond when the pollution reduction rates due to decomposition rate of toxic substances, Q_2, varies over a period of 3 days, with the concentration values displayed every 6 hours, is shown in Table 6 and Figure 5.

Table 5

Parameter setting of the scenario 3.

Parameters	Setting value
K_2	0.005
Q_2	0.01, 0.07, 0.14, 0.28
K	0.05
P	0.45
V_2	312.5
$C_1(0)$	3
Shrimp pond parameters	$Q_1 = 5$, $K_1 = 0.4$, $S_1 = 4.5$, $P = 0.45$, $V_1 = 625$

Table 6

The concentration level of pollutants in the treatment pond of the scenario 3.

t (hr)	$C_1(t)(Q_2=0.01)$	$C_1(t)$ $(Q_2=0.07)$	$C_1(t)$ $(Q_2=0.14)$	$C_1(t)$ $(Q_2=0.28)$
0	3	3	3	3
6	3.1358	3.0664	2.9856	2.8238
12	3.2612	3.1222	2.9600	2.6355
18	3.3767	3.1675	2.9235	2.4354
24	3.4823	3.2026	2.8763	2.2237
30	3.5784	3.2278	2.8187	2.0006
36	3.6652	3.2432	2.7509	1.7663
42	3.7429	3.2492	2.6732	1.5212
48	3.8118	3.2459	2.5857	1.2652
54	3.8720	3.2335	2.4886	0.9988
60	3.9237	3.2122	2.3822	0.7220
66	3.9672	3.1823	2.2666	0.4351
72	4.0027	3.1439	2.1420	0.1382

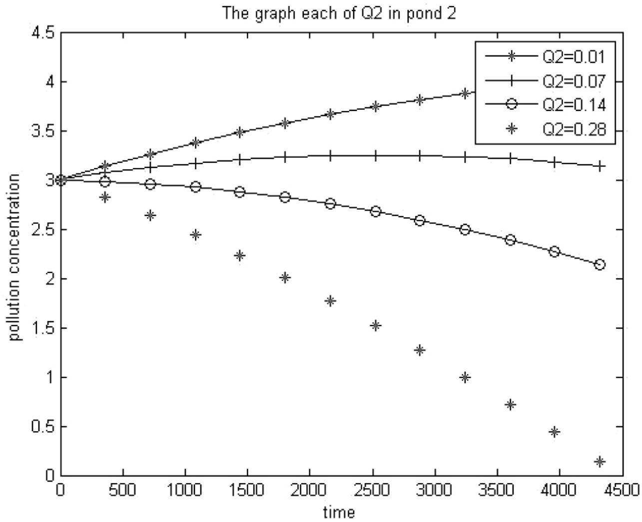

Figure 5

The concentration level of pollutants in the treatment pond of the scenario 3.

4.4 *A water quality of the treatment pond when the rate of pumping water in varies*

Parameters are set as shown in Table 7. The concentration level of pollutants in the treatment pond when the rate of pumping water in, K, varies over a period of 3 days, with the concentration values displayed every 6 hours, is shown in Table 8 and Figure 6.

Table 7

Parameter setting of the scenario 4.

Parameters	Setting value
K_2	0.01
Q_2	0.5
K	0.25, 0.3, 0.5, 0.75
P	0.4
V_2	312.5
$C_1(0)$	3
Shrimp pond parameters	$Q_1 = 5$, $K_1 = 0.4$, $S_1 = 4.5$, $P = 0.4$, $V_1 = 625$

Table 8

The concentration level of pollutants in the treatment pond of the scenario 4.

t (hr)	$C_1(t)(K=0.25)$	$C_1(t)$ $(K=0.3)$	$C_1(t)$ $(K=0.5)$	$C_1(t)$ $(K=0.75)$
0	3	3	3	3
6	3.0348	3.1506	3.6141	4.1934
12	3.0366	3.2630	4.1686	5.3006
18	3.0050	3.3365	4.6627	6.3204
24	2.9397	3.3709	5.0958	7.2520
30	2.8403	3.3657	5.4673	8.0942
36	2.7064	3.3203	5.7763	8.8461
42	2.5375	3.2344	6.0221	9.5067
48	2.3333	3.1074	6.2040	10.0747
54	2.0934	2.9390	6.3213	10.5492
60	1.8173	2.7285	6.3732	10.9292
66	1.5047	2.4755	6.3590	11.2134
72	1.1551	2.1796	6.2779	11.4008

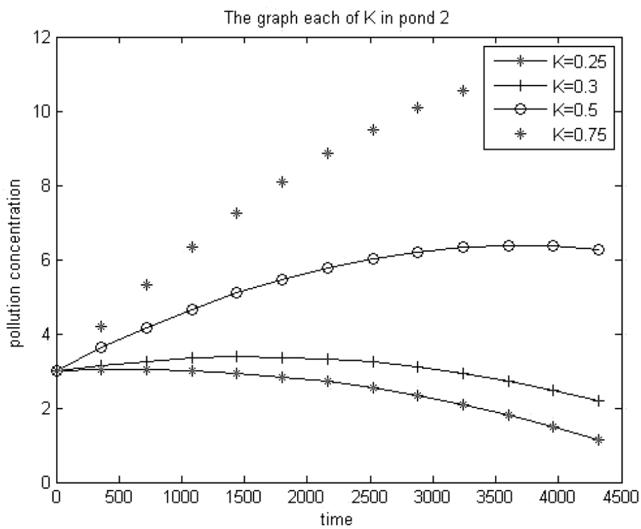

Figure 6

The concentration level of pollutants in the treatment pond of the scenario 4.

5. Discussion

As the results of scenarios 1-4, we can conclude the water quality due to several parameters as shown in Table 9.

Table 9

The concentration level of pollutants trend in the shrimp pond.

Parameter setting		Better water quality of the shrimp pond	Poor water quality of the shrimp pond
The reaction rate of toxic substances in shrimp pond	Low K_1	√	
	High K_1		√
The pollution reduction rate due to water pumping	Low P		√
	High P	√	
The decomposition rate of toxic substances in the treatment pond	Low Q_2		√
	High Q_2	√	
The rate of increase of pollutants due to pumping water in	Low K		√
	High K	√	

6. Conclusion

This study proposes a simple mathematical technique for monitoring water quality in closed-system shrimp farms. The procedure for determining various parameters for the mathematical model is described. The model solution is calculated using the fourth-order Runge-Kutta method. This study reproduces the situation to compare the various parameter values that affect the level of water quality in closed-system shrimp farms at various points in time. The study discovered that water quality is influenced by the initial water quality, the rate of chemical reaction of pollutants, the rate of pollution formation, the rate of pollution decomposition, the rate of decrease in pollution concentration due to water circulation between the farm and the water treatment pond, and time. The calculation results can help closed-system shrimp farmers understand the trend of pollutant concentration changes in closed-system shrimp farms, allowing them to create strategies to improve water quality.

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Received February, 2025