

New result of soft topological space

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Abstract

In this paper, we introduced some new types of separation axioms; sb-separation axioms depending on the concept of boundaries, and also discussed the relationships among sb-separation axioms additionally, all of these axioms have topological properties.

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1. Introduction

Molodtsov first proposed the soft set theory as a new nice mathematical tool for administering uncertainty [3]. The basic ideas behind soft sets have also been studied and investigated by other scholars dealing with operations over soft sets (cf [18, 14, 11, 6]). The researchers in [4] and [5] have used soft sets in diverse fields, including but not limited to measurement theory, probability, operations research, game theory, and function smoothness. Several academics have used soft set theory to examine various structures in mathematics [10]. Researchers Shabir and Naz [16] introduced soft topology as a remarkable development of classical topology defined on initial universe with some

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fixed set of parameters [16]. Sabir Hussain and Bashir Ahmad initiated the discussion of different properties of open, closed, neighborhood, which is defined on the soft set and obtained some interesting results ([17], [9]). In the field of soft topology, numerous basic concepts, such as nice soft axioms, soft connected, etc. (cf [8, 12, 13, 15]). used soft sets to expand (cf [20-27]). The researchers in [2] and [7] introduced new concept of semi theory in soft and soft α -open sets individually in topology defined in soft theory (cf [20, 21]). Salmasi, studied the concept of fuzzy soft bitopological spaces with fuzzy soft pre-open [1]. The manuscript will try to develop the fundamental theories utilizing the soft boundary set to discuss separation axioms on the topology defined in soft theory. This is how the remainder of the manuscript is structured. The first section, briefly recalls the notions of general soft theory. In section Two, we recall how the researcher defined the topology by using the soft theory. In the last section we define the soft boundary T_2 , T_1 , T_0 space, and a few associated properties are discussed. The final section provides a summary of the findings and suggests some areas for further study.

2. Preliminaries

Here, we will state that this section examines some theories and findings relating to soft topological space. For some basic truths about soft set see [3, 18]. Through 2011 Muhammad Shabir and Munazza Naz defined the concept of the soft topological space as follows:

2.1 Soft set

Definition 2.1: [3] In particular, AF is recognized as a soft set (*forshort* AF), where F from A to $P(U)$ be a mapping, U is entire set and A subset of some parameters E .

Definition 2.2 [3] Let AF and BG over an entire U is soft sets, we say AF subset of BG if $A \subseteq B$, $\forall e \in A$, $(e)F$ and $(e)G$ are the same estimates. We are able to write $AF \subseteq BG$.

Definition 2.3 [3] Let AF and BG be a soft, then are equal if $AF \subseteq BG$ and $BG \subseteq AF$.

Definition 2.4: [17] The soft complement of AF is indicated by $(AF)^c$ and is described by $(AF)^c = AF^c$ such that, F^c is a soft mapping provided by $\forall \alpha \in A, (\alpha)F^c = U \setminus (\alpha)F$. The symbol $(F)^c$ is the complement function of F . Evidently the complement of (F^c) is identical to F , $(AF^c)^c = AF$.

Definition 2.5: [3] A soft set $A\mathfrak{F}$ is known as a null, absolute soft set represented as ϕ , respectively $\tilde{A}$ if $\forall e \in A, (e)\mathfrak{F} = \emptyset$, respectively $(e)\mathfrak{F} = U$. Evidently $(\tilde{A})^c = \phi$ and $\phi^c = \tilde{A}$.

Definition 2.6: [3] The union of AF and BG is CH , such that the set

$$C = A \cup B, \quad (\varrho)H = \begin{cases} (\varrho)F & : \varrho \in A - B \\ (\varrho)G & : \varrho \in B - A \\ (\varrho)F \cup (\varrho)G & : \varrho \in A \cap B \end{cases} \quad \forall \varrho \in C. \quad \text{We write}$$

$AF \cup BG = CH$ [2] The linkage CH of AF and BG over U , represented by $AF \cap BG$, is described as $C = A \cap B$ with $(\varrho)H = (\varrho)F \cap (\varrho)G$, $\forall \varrho \in C$.

2.2 Soft Space

In this subsection, we will consider X as an entire set and $E \neq \phi$ be an assortment of standards

Definition 2.7: [3] Give EF be a soft with $x \in X$. That's what we assert $x \in EF$ read as x in EF whenever $x \in (\varrho)F, \forall \varrho \in E$. Observe for any element x with $x \notin EF$, if $x \notin (\varrho)F$, with some $\varrho \in E$.

Definition 2.11: [17] Give (EX, τ) a soft space. The soft interior of EF represented by $(EF)^o$ is the soft union of all soft open sets contained within EF . Thus $(EF)^o$ is the largest set over X which is contained in EF .

Definition 2.12: [17] Suppose (EX, τ) soft space. The closure of EF represented by $\overline{EF}$ is the intersection of all closed of EF . Obviously, $\overline{EF}$ is a smallest closed soft set over X it includes EF .

Definition 2.13: [17] Consider (EX, τ) a soft space. The boundary of EF is represented by $(EF)^b$ and determined by $(EF)^b = \overline{(EF)} \cap \overline{(EF)^c}$. Obviously, $\overline{(EF)}$ is a smallest closed soft set over X containing EF .

Definition 2.19: [19] Let (E_1X, τ) and (E_2Y, τ) be a two soft spaces. The soft mapping $f = (\tilde{f}, \hat{f}): (E_1X, \tau) \rightarrow (E_2Y, \tau)$ is continuous if and only if $f^{-1}(EF) \in \tau_1, \forall EF \in \tau_2$.

Definition 2.20: [19] Let (E_1X, τ) and (E_2Y, τ) be a two soft spaces. The soft mapping $f = (\tilde{f}, \hat{f}): (E_1X, \tau) \rightarrow (E_2Y, \tau)$ is stated to be closed if and only if $f(F)$ is a closed in τ_2 For each closed soft set F in τ_1 .

Definition 2.21: [19] If $f = (\tilde{f}, \hat{f})$ is closed and continuous, then the bijective mapping f is a homeomorphism.

3. Results

Definition 3.1: Consider (EX, τ) a soft space and let x, y two elements in X with $x \neq y$. If there are two open soft sets EU and EV then:

- (EX, τ) is called soft boundary T_0 (for short sbT_0), if $x \tilde{\in} EU \wedge y \tilde{\in} (EU^b)^c$ or $y \tilde{\in} EV \wedge x \tilde{\in} (EV^b)^c$.
- (EX, τ) is called soft boundary T_1 (for short sbT_1), if $x \tilde{\in} EU, y \tilde{\in} (EU^b)^c, y \tilde{\in} EV$ and $x \tilde{\in} (EV^b)^c$.
- (EX, τ) is called soft boundary T_2 (for short sbT_2), if $x \tilde{\in} EU, y \tilde{\in} EV$ and $(EU)^b \tilde{\cap} (EV)^b = \Phi$.

Proposition 3.2: Consider the pair (EX, τ) a soft space then:

- i. Every sbT_2 space is a sbT_0 space.
- ii. Every space sbT_1 is a sbT_0 space.

Proof:

- i. Give (EX, τ) a soft space, Let x, y two elements in X with $x \neq y$. Since (EX, τ) is sbT_2 , then there are disjoint open soft sets EF and EG , where $x \tilde{\in} EF$ with $y \tilde{\in} EG$ and $(EF)^b \tilde{\cap} (EG)^b = \Phi$. Then y in $(EF)^c$ and x in $(EG)^c$. SO, $y \tilde{\in} (EF^b)^c$ and $x \tilde{\in} (EG^b)^c$. Thus, (EX, τ) is a sbT_0 .
- ii. The proof is similar to the proof of (1).

Notes that a sbT_0 not necessary sbT_2, sbT_1 . The next example explains this fact:

Example 3.3: Assume, $X = \{h_1, h_2\}$, $E = \{q'_1, q'_2\}$ and $\tau = \{\Phi, X, E\mathfrak{F}_1, E\mathfrak{F}_2, E\mathfrak{F}_3\}$ be two soft defined on X such that $E\mathfrak{F}_1, E\mathfrak{F}_2$ and $E\mathfrak{F}_3$ defined as follows:

$$\begin{aligned} (q'_1)\mathfrak{F}_1 &= \{h_1, h_2\} (q'_2)\mathfrak{F}_1 = \{h_1\}, \quad (q'_1)\mathfrak{F}_2 = \{h_2\} (q'_2)\mathfrak{F}_2 = \{h_2\}, \\ (q'_1)\mathfrak{F}_3 &= \{h_2\} (q'_2)\mathfrak{F}_3 = \Phi \end{aligned}$$

the space (EX, τ) is a soft with (EX, τ) is a sbT_0 but not a sbT_2 (sbT_1).

Corollary 3.4: Let (EX, τ) be a soft space. We claim that every sbT_2 is sbT_1 .

Proof: Let us assume that (EX, τ) is sbT_2 . To prove that (EX, τ) is sbT_1 . Let (EX, τ) be a soft space with x, y in X and $x \neq y$. Since (EX, τ) is sbT_2 . Next, there is disjoint EF, EG soft open, with $x \tilde{\in} EF, y \tilde{\in} EG$ and $(EF)^b \tilde{\cap} (EG)^b = \Phi$. It follows from this that $y \tilde{\in} (EF)^c$ with $x \tilde{\in} (EG)^c, y \tilde{\in} (EF^b)^c$ and $x \tilde{\in} (EG^b)^c$. Hence the result.

Example 3.5: Assume, $X = \{h_1, h_2\}$, $E = \{q'_1, q'_2\}$ and $\tau = \{\Phi, X, E\mathfrak{F}_1, E\mathfrak{F}_2, E\mathfrak{F}_3, E\mathfrak{F}_4, E\mathfrak{F}_5\}$ where

$$\begin{aligned} (q'_1)\mathfrak{F}_1 &= \Phi (q'_2)\mathfrak{F}_1 = \{h_1\}, \quad (q'_1)\mathfrak{F}_2 = \Phi (q'_2)\mathfrak{F}_2 = \{h_2\}, \\ (q'_1)\mathfrak{F}_3 &= \Phi (q'_2)\mathfrak{F}_3 = X, \quad (q'_1)\mathfrak{F}_4 = X (q'_2)\mathfrak{F}_4 = \{h_1\}, \\ (q'_1)\mathfrak{F}_5 &= X (q'_2)\mathfrak{F}_5 = \{h_2\} \end{aligned}$$

Then (EX, τ) be a soft space and (EX, τ) is a sbT_1 . Now let $h_1, h_2 \in X$ and $h_1 \neq h_2$ but Over, there are non-disjoint soft open sets. We note that (EY, τ) is not a sbT_2 .

Proposition 3.6: Let (EX, τ) be a soft space with $y \neq \phi$ subset of X . If (EX, τ) has a sbT_0 , then (EY, τ_y) has a sbT_0 too.

Proof: Let x, y two elements in X with $x \neq y$. Then we claim there exists EF and EG open in (EX, τ) with $x \tilde{\in} EF \wedge y \tilde{\in} (EF^b)^c$ or $y \tilde{\in} EG \wedge x \tilde{\in} (EG^b)^c$. Now let $x \in Y \Rightarrow x \tilde{\in} Y$. So, $x \tilde{\in} Y \wedge x \tilde{\in} EF \Rightarrow x$ in $(Y \tilde{\cap} EF) = EF_y$ where EF open. Take into consideration $y \tilde{\in} (EF^b)^c \Rightarrow y \tilde{\in} (EF)^c$ Consequently, this implies that for all $\alpha \in E, y \notin (\alpha)F$. So, $y \notin Y \cap (\alpha)F = (\alpha)Y \cap (\alpha)F$. Consequently $y \tilde{\in} (Y \cap EF^b)^c = (EF_y)^c$. Hence $y \tilde{\in} (EF_y^b)^c$. Likewise, it may be demonstrated that if $y \tilde{\in} EG \wedge x \tilde{\in} (EG^b)^c \Rightarrow y \tilde{\in} EG_y \wedge x \tilde{\in} (EG_y^b)^c$. For this reason, (EY, τ_y) will be sbT_0 .

Proposition 3.7: Consider the pair (EX, τ) soft space with $y \neq \phi$ subset of X . We claim, if (EX, τ) is a sbT_1 , then the pair (EY, τ_y) is a sbT_1 .

Proof: The proof is comparable to that of Proposition 3.6.

Remark 3.8: If (EX, τ) is a sbT_2 space with $y \neq \phi$ and subset of X then, (EY, τ_y) need not be a soft bT_2 space. Note that the hereditary property is applied in a general topology by using some of the topological properties and the following example explains that:

Example 3.9: Assume, $X = \{h_1, h_2\}$, $E = \{q'_1, q'_2\}$ and $\tau = \{\Phi, X, EF_1, EF_2\}$ where $(q'_1)F_1 = \{h_1\}$, $(q'_2)F_1 = \{h_1\}$, $(q'_1)F_2 = \{h_2\}$, $(q'_2)F_2 = \{h_2\}$

Then (EX, τ) be a soft space and we note that (EX, τ) is a soft bT_2 . Now we have $Y = \{h_1\}$, then $\tau_Y = \{\Phi, Y\}$ and (EY, τ) does not have a sbT_2 .

Theorem 3.10: *Existence of sbT_2 is a topological property.*

Proof: Let (E_1X, τ_1) and (E_2Y, τ_2) be two soft spaces where $(E_1X, \tau_1) \equiv (E_2Y, \tau_2)$ and (E_1X, τ_1) is a sbT_2 . Since $(E_1X, \tau_1) \equiv (E_2Y, \tau_2)$, then there exists a homeomorphism $f = (\tilde{f}, \hat{f}): (E_1X, \tau_1) \rightarrow (E_2Y, \tau_2)$. Now, let $(a_{e'})_{E_2}, (b_{e'})_{E_2}; (a_{e'})_{E_2} \neq (b_{e'})_{E_2}$. Since f is surjective, then, $\tilde{f}$ and $\hat{f}$ are surjective. So, we have $f^{-1}((a_{e'})_{E_2}), f^{-1}((b_{e'})_{E_2})$ and $f^{-1}((a_{e'})_{E_2}) \neq f^{-1}((b_{e'})_{E_2})$. Since (E_1X, τ_1) is a SbT_2 , then, there exists disjoint soft open sets E_1F and E_1G such that $f^{-1}((a_{e'})_{E_2}) \in E_1F, f^{-1}((b_{e'})_{E_2}) \in E_1G$ and $E_1F \cap E_1G = \tilde{\phi}_{E_1}$. Since f is bijective open mapping, then $(a_{e'})_{E_2} \in f(E_1F), (b_{e'})_{E_2} \in f(E_1G)$ and $f(E_1F)$ and $f(E_1G)$ are soft open sets in Y . Hence the result.

Theorem 3.11: *Topological properties include the property of being sbT_1 .*

Proof: Let (E_1X, τ_1) and (E_2Y, τ_2) be two soft spaces where $(E_1X, \tau_1) \equiv (E_2Y, \tau_2)$ and (E_1X, τ_1) is a sbT_1 . Since $(E_1X, \tau_1) \equiv (E_2Y, \tau_2)$, then there exists a homeomorphism $f = (\tilde{f}, \hat{f}): (E_1X, \tau_1) \rightarrow (E_2Y, \tau_2)$. Now, let $(a_{e'})_{E_2}, (b_{e'})_{E_2}, (a_{e'})_{E_2} \neq (b_{e'})_{E_2}$. Since f is surjective, then $\tilde{f}$ and $\hat{f}$ are surjective. So, we have $f^{-1}((a_{e'})_{E_2}), f^{-1}((b_{e'})_{E_2})$ and $f^{-1}((a_{e'})_{E_2}) \neq f^{-1}((b_{e'})_{E_2})$. Since (X, E_1) is a sbT_1 , then, there exists soft open sets E_1F and E_1G such that $f^{-1}((a_{e'})_{E_2}) \in E_1F,$

$f^{-1}((b_{e'})_{E_2}) \in ((E_1F)^b)^c$ and $f^{-1}((b_{e'})_{E_2}) \in E_1G$, $f^{-1}((a_{e'})_{E_2}) \in ((E_1G)^b)^c$. Since f is bijective open mapping, then $f(E_1F)$ and $f(E_1G)$ are soft open sets in Y and $(a_{e'})_{E_2} \in f(E_1F)$, $(b_{e'})_{E_2} \in ((f(E_1F))^b)^c$ and $(b_{e'})_{E_2} \in f(E_1G)$, $(a_{e'})_{E_2} \in ((f(E_1G))^b)^c$. Hence the result.

Theorem 3.12: *Being sbT_0 is a topological property.*

Proof: From the previous Theorem 3.11 it follows directly that $f(E_1F)$ and $f(E_1G)$ are soft open sets in Y and $(a_{e'})_{E_2} \in f(E_1F)$, $(b_{e'})_{E_2} \in ((f(E_1F))^b)^c$ or $(b_{e'})_{E_2} \in f(E_1G)$, $(a_{e'})_{E_2} \in ((f(E_1G))^b)^c$.

4. Conclusion

This research leads to it achievable to study nice types of soft separation axioms. We construct the basic theories about boundary separation structure in soft space and investigate the properties of these spaces. Additionally, we provide some examples that explain exactly how achieving equality in the relationship between these spaces is not required.

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