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Compact operator of peridynamic model

Rasha Hamzah Obaid *
Aseel Ameen Harbi †
Zainab Mohammed Najm §
Department of Mathematics
College of Science
University of Al-Qadisiyah
Al-Qadisiyah 54001
Iraq

Abstract

In this work, we examine a one-dimensional compact linear peridynamic model of a spring system through a functional analytical framework.

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1. Introduction

In applications, compact linear operators are crucial [9]. They are essential to the theory of integral equations and a number of other mathematical physics issues [10]. Their characteristics have a strong resemblance to those of the operators in spaces with limited dimensions in analysis, a large number of operators that are linear compact [1]. In this paper, We will examine the idea of Compact linear operator(represented by the peridynamic model [2, 7], on Banach spaces as a map $H \subset (L_2(\Omega,), S_\sigma(\Omega))$ in order for any bonded subset to A

* *ORCID-ID:* **0000-0001-5441-2703**
† *ORCID-ID:* **0009-0007-1987-3425**
§ *ORCID-ID:* **0009-0007-3114-4604**

* *E-mail:* **rasha.hamzah@qu.edu.iq (Corresponding Author)**
† *E-mail:* **aseel.harbi@qu.edu.iq**
§ *E-mail:* **zainab.moha@qu.edu.iq**

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$\subset L_2(\Omega)$ is compact in Additional characterizations of this map H will be expressed about as well [4,5]. The space $S_\sigma(\Omega)$ depends on the kernel function [8], consists of all the functions $u(x) \in L^2(\Omega)$, since the kernel function is sub space of $L^2(\Omega)$; by quotient space [3, 6]. So, we refer [11] and [12-16]. Such that

$$S_\sigma = \{u \in L^2(\Omega) : \sum_k \int_\Omega (1+\phi_\delta)^{1/2} u^*(x).(1+\phi_\delta)^{1/2} u^*(x)dx < \infty\} = \{\sum_k \int_\Omega u^*(x).(1+\phi)u^*(x)dx\}$$

2. Main Results

Definition 2.1: Let $L_2(\Omega)$ and $S_\sigma(\Omega)$ be linear space, $H: L_2(\Omega) \to S_\sigma(\Omega)$ then H is said to be compact if a bounded subset A, then $\overline{H(A)}$ is compact in $S_\sigma(\Omega)$.

Theorem 2.1: *If, $H: L_2(\Omega) \to S_\sigma(\Omega)$, for every bounded sequence $u(x)$ in $L_2(\Omega)$, the sequence $u_n(x)$ convergent in $S_\sigma(\Omega)$, then H is compact.*

Proof: $\|u_n(x) - u(x)\| = \|u(x) - u_n(x)\|$ by properties self adjoint operator and $u(x)$ convergent to $u_n(x)$ and $u(x)$ convergent to $u_n(x)$ $u_n(x) \in L_2(\Omega)$ H is compact.

Definition 2.3: $k(L_2(\Omega), S_\sigma(\Omega)) = \{H: L_2(\Omega) \to S_\sigma(\Omega)\}$ such that H is linear compact.

Definition 2.4: $B(L_2(\Omega), S_\sigma(\Omega)) = \{H: L_2(\Omega) \to S_\sigma(\Omega)\}$ suchthat H is linear bounded.

Lemma 2.5: $k(L_2(\Omega), S_\sigma(\Omega))$ $B(L_2(\Omega), S_\sigma(\Omega))$

Proof: Suppose $H \in k(L_2(\Omega), S_\sigma(\Omega))$ we want to show $H \in B(L_2(\Omega), S_\sigma(\Omega))$ s.t. $\forall\, u \in L_2(\Omega)$ there exist $M > 0, \ni \|H_u\| \leq M\|u\|$, let $u \in L_2(\Omega)$ if $u(x) \in L_2(\Omega)$ if $u = 0$, then by diff.(1) holds trivally. assume $u(x) \neq 0$, $\frac{u(x)}{\|u(x)\|} \in B_u(0,1)$, $H(B_u(0,1))$ is compact since $H \in k(L_2(\Omega), S_\sigma(\Omega))$ so $H(B_u)$ is bounded, there for exist $M > 0$ such that $H(\frac{u(x)}{\|u(x)\|}) \leq M$ implies $\frac{1}{\|x\|} H(u(x)) \leq M$ (by linearity of H) implies $H(u(x)) \leq M\|u_x\|$ implies $H \in B(L_2(\Omega), S_\sigma(\Omega))$ thus $k(L_2(\Omega), S_\sigma(\Omega))$ $B(L_2(\Omega), S_\sigma(\Omega))$.

Theorem 2.6: *Let* $H_1, H_2: L_2(\Omega) \to S_\sigma(\Omega)$ *be compact operator and* $\alpha \in$, *then:*

1) αH_1 *is compact.*
2) $H_1 + H_2$ *is compact.*

Proof:

1. as H_1 is compact, there exist $u_n(x)$ convergent $S_\sigma(\Omega)$ when αthen $\|u_n - u\| \to 0$, $\|\alpha u_n - \alpha u\| \to 0, = \|\alpha(u_n - u)\| \to 0$ $|\alpha|\|u_n - u\| \to 0$, αu_n convergent in $S_\sigma(\Omega)$, then αH_1 is compact.
2. as H_1, H_2 is compact operator then $u_n(x)$ and $v_n(x)$ be bounded and convergent Seq. $u_n(x), v_n(x)\, u_n(x)$ convergent to $u(x)$ in $S_\sigma(\Omega)\, v_n(x)$ convergent to $v(x)$ in $S_\sigma(\Omega)$. $\|u_n(x) - u(x)\| \to 0$ and $\|v_n(x) - v(x)\| \to 0\, \|u_n(x,) - u(x) + v_n(x) - v(x)\| \to 0 \,\|\, u_n(x) + v_n(x) - (u(x) + v(x))\| \to 0$ so $(u_n(x) + v_n(x))$ convergent to $(\, u(x) + v(x))$ in $S_\sigma(\Omega)$ then $H_1 + H_2$ is compact.

Theorem 2.7: *Let* $A: H_1 \to H_2$ *is compact,* $B: H_2 \to H_3$ *is bounded* AB, BA *is compact.*

Proof*:* since *A* is compact then exists $u_n(x)$ convergent to $u(x)$. as *B* is bounded then exists $v_n(x)$ bounded, then $u_n(x)v_n(x)$ are bounded and convergent, $u_n(x)v_n(x)$ is convergent, AB, BA is compact.

Theorem 2.8: *Let* $H: L_2(\Omega) \to S_\sigma(\Omega)$ *then:*

1) H *is compact* $\leftrightarrow$ H^*H *or* HH^* *is compact.*
2) H *is compact* $\leftrightarrow$ H^* *is compact.*

Proof: If *H* is compact, then H^*H or HH^* compact by theorem (2.6). To prove the converse H^*H is compact $\Rightarrow$ $\exists\, u_n v_n$ bounded and convergent set in $L_2(\Omega)$ and $M > 0$ then HH^* has sequence $u_n v_n$ convergent to $uv(x)$ then

$$= \|u_n v_n - uv(x)\| \to 0 = \|(u_n - u)(v_n - v)\| \to 0$$
$$\leq M\|u_n - u\|\|v_n - v\| \text{ hence } \|u_n - u\| \to 0, \|v_n - v\| \to 0.$$

Then *H* is compact.

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