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Characterizations of IIS-Compactness space

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Abstract

The idea of the present study is based on the fundamental idea of defining a new set, which we called interior isolated set and the old topological concept of isolated point. Therefore, the topological properties of this concept and its most important relationships were examined. Moreover, we built new types of sets and spaces as follows: IIS*-set, IIS-open and $\mathfrak{T}_{IIS}$-space. The most important algebraic properties of these sets were also studied. In addition, several new definitions were presented for compact based on interior isolated

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set, which was called IIS-compact spaces IIS-O-compact. The researcher also discussed their most important properties and relationships with each other.

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1. Introduction and Preliminaries

Derived set is considered one of the essential concepts in building general topology. Mckingsey and Tarski [7] were the first to identify this concept in 1944. If A is a subset of the topological space $(\mathfrak{X}, \mathfrak{T})$, the set of all limit points of set A is given the symbol $\mathbf{D}(\mathbf{A})$. It should be remembered that A subset A of space $(\mathfrak{X}, \mathfrak{T})$ is named dense if $\overline{\mathbf{A}} \subseteq \mathfrak{X}$ [8-10]. Later, Munker and Sharma [8,12-17]) proved that for any subset A of space $(\mathfrak{X}, \mathfrak{T})$, cl $(\mathbf{A}) = \mathbf{A} \cup \mathbf{D}(\mathbf{A})$, int $(\mathbf{A}) = \{\boldsymbol{x} : \boldsymbol{x} \in \mathbf{A}, \boldsymbol{x} \notin \mathbf{D}(\mathbf{A})\}$. Hewitt [1-4] established that a space $(\mathfrak{X}, \mathfrak{T})$ is said to be resolvable if there exists two disjoint dense subsets $\mathbf{A}$ and $\boldsymbol{B}$ of $\mathfrak{X}$ such that $\mathbf{A} \cup \boldsymbol{B} = \mathfrak{X}$. Mckingsey and Tarski [7] defined isolated points as a point $\boldsymbol{x} \in \mathfrak{X}$ of a space is $(\mathfrak{X}, \mathfrak{T})$ is named isolated point of a subset $\boldsymbol{B}$ of $\mathfrak{X}$ if $\boldsymbol{x} \in \mathbf{B}$ and $\boldsymbol{x} \notin \mathbf{D}(\mathbf{B})$.The set of all isolated points of B is denoted by $\boldsymbol{is}(\mathbf{B})$.

Among the crucial mathematical concepts in space topology is ideal. It was first defined by Kuratwaski [6]. Jankovic and Hamlett [5] proposed building a new topology from space topology $(X, \mathfrak{T})$ induced by any $\boldsymbol{\mathcal{J}}$ ideal on X such that: The operator $(\,)^*: \mathbf{p}(\mathfrak{X}) \to \mathbf{p}(\mathfrak{X})$ is said to be local function of $\mathbf{A}$ with respect to $\mathfrak{T}$ and $\boldsymbol{\mathcal{J}}$ defined as $\mathbf{A}^*(\boldsymbol{\mathcal{J}}, \mathfrak{T}) = \{\mathbf{x} : \mathbf{A} \cap \mathbf{U} \notin \boldsymbol{\mathcal{J}}$ for every open $\mathbf{U}$ of $\mathbf{x}$ $\}$such that $\boldsymbol{\beta}(\boldsymbol{\mathcal{J}}, \mathrm{T}) = \{\mathrm{V} - \mathbf{A} : \mathrm{V} \in \mathfrak{T}, \mathbf{A} \in \boldsymbol{\mathcal{J}}\}$ is the basis for this new topology $\mathfrak{T}^*(\boldsymbol{\mathcal{J}})$ on $\mathfrak{X}$ such that $\mathfrak{T}^*(\boldsymbol{\mathcal{J}})$ is strictly finer with $\mathfrak{T}$. Natkaniec [11] studied another type of operator on the $(\mathfrak{X}, \mathfrak{T}, \boldsymbol{\mathcal{J}})$ ideal space and called it Ψ-operator, which is defined as follows: The operator $\boldsymbol{\Psi} : \mathbf{P}(\mathfrak{X}) \to \mathfrak{T}, \forall \mathbf{A} \subseteq \mathfrak{X}, \boldsymbol{\Psi}(\boldsymbol{\mathcal{J}}, \mathfrak{T})(\mathbf{A}) = \{\mathbf{x} : \exists\, \mathbf{V} \in \mathfrak{T}, \mathbf{x} \in \mathbf{V}, \ni\, \mathbf{V} - \mathbf{A} \in \boldsymbol{\mathcal{J}}\}$. Many researchers have presented studies about ideal, including (1,2,3,9,10). The current paper consists of two sections. Section one introduces a new definition, which is interior isolated set. Its most essential properties and relationships were studied, and several new sets and spaces were defined based on it. Finally, section two presents several new definitions of compact based on interior isolated set, and its most important properties and relationships were discussed.

2. Some types of interior isolated sets

In this section, new definitions were introduced. These definitions are interior isolated set, IIS*-set and IIS-open,. Their most essential properties and relations were also examined

Definition 2.1: Let $(\mathfrak{X}, \mathfrak{T})$ be space. A point $x \in \mathfrak{X}$ is called an interior isolated point of $B \subseteq \mathfrak{X}$ iff $\exists\, V \in \mathfrak{T}(x) \ni V - \{x\} \subseteq \mathrm{B}$ and IIS(B) is the set of all interior isolated points of a set B.

In light of the above definition, it is concluded that: If $x \in \mathfrak{X} - D(\mathfrak{X} - \mathrm{B})$ iff $x \notin D(\mathfrak{X} - \mathrm{B})$ iff $\exists\, V \in \mathfrak{T}(x) \ni (V - \{x\}) \cap (\mathfrak{X} - \mathrm{B}) = \emptyset$ iff $\in \mathrm{IIS}(\mathrm{B})$, so $\mathrm{IIS}(\mathrm{B}) = \mathfrak{X} - D(\mathfrak{X} - \mathrm{B})$, for any $B \subseteq \mathfrak{X}$.So $int(A) \subseteq \mathrm{IIS}(A)\ \forall A \subseteq \mathfrak{X}$.There is relationship between IIS(B) and B^*, $\forall\, \mathrm{B} \subseteq \mathfrak{X}$ embodicd in the following: $\mathrm{IIS}_{\mathfrak{T}}(B) \subseteq \mathrm{IIS}_{\mathfrak{T}^*}(B)$ and$\psi(B) \subseteq \mathrm{IIS}_{\mathfrak{T}^*}(B)$,for any ideal $\mathcal{J}$ on $\mathfrak{X}$,. It is evident that $\mathrm{IIS}\,(\mathfrak{X}) = \mathfrak{X}$ and $\mathrm{IIS}(\emptyset) = \mathfrak{X} - \mathrm{D}(\mathfrak{X})$.

Theorem 2.2: *Let* $(\mathfrak{X}, \mathfrak{T})$ *be space and* $A, B \subseteq \mathfrak{X}$.*Then:*

(i) $int(\mathrm{B}) = \mathrm{B} \cap \mathrm{IIS}(\mathrm{B})$.

(ii) $\mathrm{IIS}(\mathrm{B}) \subseteq \mathrm{IIS}(\mathrm{IIS}(\mathrm{B}))$.

(iii) $\mathrm{IIS}(\mathrm{A} \cap B) = \mathrm{IIS}(\mathrm{A}) \cap \mathrm{IIS}(B)$

(iv) $\mathrm{IIS}(\mathfrak{X} - D(A)) = \mathfrak{X} - \mathrm{D}(\mathrm{D}(\mathrm{A}))$

(v) If $\mathrm{A} \in \mathfrak{T}$, then $\mathrm{A} \subseteq \mathrm{IIS}(\mathrm{A})$.

(vi) $D(\mathrm{IIS}(\mathfrak{X} - A)) = \mathfrak{X} - \mathrm{IIS}(\mathrm{D}(\mathrm{A}))$.

(vii) $D(\mathfrak{X} - \mathrm{IIS}(A)) = \mathfrak{X} - \mathrm{IIS}(\mathrm{IIS}(\mathrm{A}))$.

(viii) For any index Δ, $\bigcup_{\lambda\in\Delta} \mathrm{IIS}(A_\lambda) \subseteq \mathrm{IIS}(\bigcup_{\lambda\in\Delta}(A_\lambda))$.

Proof: (i) Assume that $x \in int\,(\mathrm{A})$ iff $\exists\, V \in \mathfrak{T}(x) \ni x \in V \subseteq A$ iff $V \cap (\mathfrak{X} - \mathrm{A}) = \emptyset$ and $x \notin (\mathfrak{X} - \mathrm{A})$ iff $(V - \{x\}) \cap (\mathfrak{X} - \mathrm{A}) = \emptyset$ iff $x \in \mathrm{A} \cap \mathrm{IIS}(A)$.

(iii) Since $\mathrm{IIS}(\mathrm{A} \cap B) = \mathfrak{X} - D(\mathfrak{X} - (\mathrm{A} \cap B)) = (\mathfrak{X} - D(\mathfrak{X} - \mathrm{A})) \cap (\mathfrak{X} - d(\mathfrak{X} - B)) = \mathrm{IIS}(\mathrm{A}) \cap \mathrm{IIS}(\mathrm{B})$.

(viii) Since $\mathrm{IIS}(\bigcup_{\lambda\in\Delta}(A_\lambda)) = \mathfrak{X} - D(\mathfrak{X} - \bigcup_{\lambda\in\Delta}(A_\lambda)) = \mathfrak{X} - D(\bigcap_{\lambda\in\Delta}(\mathfrak{X} - A_\lambda))$, and $D(\bigcap_{\lambda\in\Delta}(\mathfrak{X} - A_\lambda)) \subseteq \bigcap_{\lambda\in\Delta} D(\mathfrak{X} - A_\lambda)$, then $\bigcup_{\lambda\in\Delta} \mathrm{IIS}(A_\lambda) \subseteq \mathrm{IIS}(\bigcup_{\lambda\in\Delta}(A_\lambda))$.

Remark 2.3: Let $(\mathfrak{X}, \mathfrak{T})$ be space and $\mathrm{A} \subseteq \mathfrak{X}$, then:

(i) $is(\mathfrak{X} - \mathrm{A}) \subseteq \mathrm{IIS}(\mathrm{A})$.

(ii) $D(\mathfrak{X}) = \mathfrak{X}$ iff $\forall x \in \mathfrak{X}, \{x\} \notin \mathfrak{T}$.

(iii) $\mathrm{IIS}(\emptyset) = \emptyset$ iff $D(\mathfrak{X}) = \mathfrak{X}$.

(iv) $\mathrm{A} \cap \mathrm{IIS}(\mathrm{A}) = \mathrm{A} - \mathrm{D}(\mathfrak{X} - \mathrm{A})$.

(v) If $\mathrm{A} \subseteq B$, then $\mathrm{IIS}(\mathrm{A}) \subseteq \mathrm{IIS}(B)$

Definition 2.4: Let $(\mathfrak{X}, \mathfrak{T})$ be space and $\mathcal{B} \subseteq \mathfrak{X}$,then:

(i) $\mathcal{B}$ is said to be perfect if $\mathcal{B} = \mathrm{D}(\mathcal{B})$.

(ii) $\mathcal{B}$ is said to be IIS^*-set iff $\mathcal{B} = \mathrm{IIS}(\mathcal{B})$.

According to the definition: The complement of IIS^*-set is perfect and every IIS^*-set is open.

Definition 2.5: A function $h: (\mathfrak{X}, \mathfrak{T}) \to (\Upsilon, \mathrm{T})$ be function. Then:

(i) h is called IIS-continuous if $\forall\, \mathcal{A} \subseteq \Upsilon$, $\mathrm{IIS}(h^{-1}(\mathcal{A}) \subseteq h^{-1}(\mathrm{IIS}(\mathcal{A}))$.

(ii) h is called IIS-irresolute if $\forall\, \mathcal{A} \subseteq \Upsilon$, $h^{-1}(\mathrm{IIS}(\mathcal{A}) \subseteq \mathrm{IIS}(h^{-1}(\mathcal{A}))$.

Definition 2.6: A subset K of space $(\mathfrak{X}, \mathfrak{T})$ is named IIS-open (IIS-O) iff $\exists\, B \subset \mathfrak{X}, B \in \mathfrak{T}$ such that $\mathrm{K} \subseteq \mathrm{IIS}(B)$ and $\mathfrak{T}^{\mathrm{IIS}}$ is the set of all IIS-open sets. The family of all IIS-open sets is topology on $\mathfrak{X}$ and its finer than $\mathfrak{T}$.

The sets $\emptyset$ and $\mathfrak{X}$ are IIS-open sets. Also, the intersection of two IIS-open sets is IIS-open set, and the union of any family of IIS-open sets is IIS-open sets. It is noted that every open set is IIS-open, but not in general, as the opposite is not true if suppose that $\mathfrak{X} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$ and $\mathfrak{T} = \{\emptyset, \mathfrak{X}, \{\mathfrak{a}\}, \{\mathfrak{b}, \mathfrak{a}\}\}$ be topological space on $\mathfrak{X}$, then $\mathfrak{T}^{\mathrm{IIS}} = \mathrm{P}(\mathfrak{X})$.

Corollary 2.7: *Let $f: (\mathfrak{X}, \mathfrak{T}) \to (\Upsilon, \mathrm{T})$ be continuous and IIS-irresolute function and $\mathcal{B} \subseteq \mathfrak{X}$. If $\mathcal{B}$ is IIS-open set in* Υ, *then $f^{-1}(\mathcal{B})$ is IIS-open set in $\mathfrak{X}$.*

Proof: The proof is evident.

3. Certain types of interior isolated compact space.

In this section, new definitions were given. These definitions are IIS-compact space and IIS-O-compact space IIS. Most of their topological properties of these spaces were also examined.

Definition 3.1: A space $(\mathfrak{X}, \mathfrak{T})$ is called IIS-compact if each open cover $\{\mathcal{H}_\lambda : \lambda \in \Lambda\}$ has finite subsets Λ_0 of Λ such that $\mathfrak{X} = \bigcup_{\lambda \in \Lambda_0} \mathrm{IIS}\,(\mathcal{H}_\lambda)$. In other word $\mathfrak{X}$ is IIS-compact if every open cover $\{\mathcal{H}_\lambda : \lambda \in \Lambda\}$, there exists a finite subsets Λ_0 of Λ with if $\bigcap_{\lambda \in \Lambda_0} D(\mathfrak{X} - \mathcal{H}_\lambda) = \emptyset$.

It is easy to determine that every compact space is IIS-compact, but the opposite is not necessarily true. The following example shows that:

Example 3.2: Each discrete space on infinite set is IIS-compact.

Proposition 3.3: Every perfect subset of IIS-compact space is IIS-compact.

Proof: Let $A = D(A)$,then $\mathfrak{X} - A = \mathfrak{X} - \mathrm{D}(A)$, A is closed set,. For any $\{\mathcal{H}_\lambda : \lambda \in \Delta\}$ be open cover of A, thus $\bigcup_{\lambda \in \Delta} \mathcal{H}_\lambda \cup (\mathfrak{X} - A)$ is open cover of $\mathfrak{X}$, then there exist a finite subsets Δ_0 of Δ such that $\mathfrak{X} = \bigcup_{\lambda \in \Delta_0} \mathrm{IIS}(\mathcal{H}_\lambda) \cup \mathrm{IIS}(\mathfrak{X} - A) = \bigcup_{\lambda \in \Delta_0} \mathrm{IIS}(\mathcal{H}_\lambda) \cup (\mathfrak{X} - A)$ thus $A \subseteq \bigcup_{\lambda \in \Delta_0} \mathrm{IIS}(\mathcal{H}_\lambda))$, Therefore A is IIS-compact set.

Definition 3.4: A family $G = \{G_\lambda : \lambda \in \Delta\}$ of subsets of $\mathfrak{X}$ has the DFIP iff there exists a finite subsets Δ_0 of Δ with $\bigcap_{\lambda \in \Delta_0} D(G_\lambda) \neq \emptyset$.

Theorem 3.5: A space $(\mathfrak{X}, \mathfrak{T})$ is IIS-compact space iff for all family of closed subsets of $\mathfrak{X}$ with $DFIP$ has anon empty intersection.

Proof: $\Longrightarrow$) Assume that $F = \{F_\beta : \beta \in \Omega\}$be family of closed subsets of $\mathfrak{X}$ with $DFIP$,if possible $\bigcap_{\beta\in\Omega} F_\beta = \emptyset$, then $\mathfrak{X} = \bigcup_{\beta\in\Omega}(\mathfrak{X} - F)$, there exists a finite subsets Ω_0 of Ω with $\mathfrak{X} = \bigcup_{\beta\in \Omega_0} \mathrm{IIS}\,(\mathfrak{X} - F_\beta) = \bigcup_{\beta\in \Omega_0} (\mathfrak{X} - \mathrm{D}(F_\beta))$, thus $\bigcap_{\beta\in \Omega_0} \mathrm{D}(F_\beta) = \emptyset$ which is contradiction. Hence $\bigcap_{\beta\in\Omega} F_\beta \neq \emptyset$.

$\Longleftarrow$) Let $\{G_\alpha : \alpha \in \Lambda\}$ be any open cover of $\mathfrak{X}$, then $\mathfrak{X} = \bigcup_{\alpha\in\Lambda} G_\alpha$, there exists a finite subsets Λ_0 of Λ with $\emptyset = \bigcap_{\alpha\in \Lambda_0} \mathrm{D}\,(\mathfrak{X} - G_\alpha) = \bigcap_{\alpha\in \Lambda_0} (\mathfrak{X} - \mathrm{IIS}(G_\alpha))$ iff $\mathfrak{X} = \bigcup_{\alpha\in \Lambda_0} \mathrm{IIS}(G_\alpha)$.Therefore $(\mathfrak{X}, \mathfrak{T})$ is IIS-compact space.

Proposition 3.6: If $\mathbb{h}: (\mathfrak{X}, \mathfrak{T}) \to (\mathrm{Y}, \mathrm{T})$ is onto, continuous and IIS-continuous function such that $(\mathfrak{X}, \mathfrak{T})$ is IIS-compact space, then (Y,T) is IIS-compact space.

Proof: Let $\{\mathbb{G}_\delta : \delta \in \Lambda\}$ is open cover of Y, then $\{\mathbb{h}^{-1}(\mathbb{G}_\delta): \delta \in \Lambda\}$ is open cover of $\mathfrak{X}$, there is finite subsets Λ_0 of Λ such that $\mathfrak{X} = \bigcup_{\delta\in \Lambda_0} \mathrm{IIS}\,(\mathbb{h}^{-1}(\mathbb{G}_\delta))$, given $\mathbb{h}$ is IIS-continuous function, thus $\mathrm{Y} = \bigcup_{\delta\in \Lambda_0} \mathrm{IIS}(\mathbb{G}_\delta))$.Therefor Y is IIS-compact space.

Proposition 3.7: If $\mathcal{B}$ is closed subset of topology space $(\mathfrak{X}, \mathfrak{T})$ and $\mathcal{K}$ is IIS-compact subset of $\mathfrak{X}$ such that $\mathcal{B}$ is dense in $\mathfrak{X}$, then $\mathcal{B} \cap \mathcal{K}$ is IIS-compact set in $\mathfrak{X}$.

Proof: Assume that $\{\mathcal{H}_\lambda : \lambda \in \Delta\}$ be open cover of $\mathcal{B} \cap \mathcal{K}$,then $(\mathcal{B} \cap \mathcal{K}) \cup (\mathfrak{X} - \mathcal{B}) \subseteq \bigcup_{\lambda\in\Delta} \mathcal{H}_\lambda \cup (\mathfrak{X} - \mathcal{B})$. So $\{\mathcal{H}_\lambda : \lambda \in \Delta\} \cup (\mathfrak{X} - \mathcal{B})$ is open cover of $\mathcal{K}$. Since $\mathcal{K}$ is IIS-compactset in $\mathfrak{X}$, then there exists a finite subsets Δ_0 of Δ with $\mathcal{K} \subseteq \bigcup_{\lambda\in \Delta_0} \mathrm{IIS}\,(\mathcal{H}_\lambda \cup (\mathfrak{X} - \mathcal{B}))$. Since $\mathcal{B}$ is dense in $\mathfrak{X}$, then $\mathcal{K} \subseteq \bigcup_{\lambda\in \Delta_0} \mathrm{IIS}(\mathcal{H}_\lambda) \cup (\mathfrak{X} - \mathcal{B}))$, thus $\mathcal{K} \cap \mathcal{B} \subseteq \bigcup_{i=1}^{n} \mathrm{IIS}(\mathcal{H}_{\lambda_i})$.Hence $\mathcal{B} \cap \mathcal{K}$ is IIS-compact set in $\mathfrak{X}$.

Proposition 3.8: If $\mathfrak{D}$ and $\mathfrak{K}$ are IIS-compact sets in topology space $(\mathfrak{X}, \mathfrak{T})$, then $\mathfrak{D} \cup \mathfrak{K}$ is IIS-compact.

Proof: The proof is evident.

Definition 3.9: A space $(\mathfrak{X}, \mathfrak{T})$ is named IIS-O-compact if each IIS-open cover $\{\mathbb{G}_\eta : \eta \in \Lambda\}$ has finite subsets $\Lambda_0$of Λ with $\mathfrak{X} = \bigcup_{\lambda\in\Lambda_0} \mathrm{IIS}\ (\mathbb{G}_\eta)$.In other word $\mathfrak{X}$ is IIS-O-compact if ever IIS-open cover $\{\mathbb{G}_\eta : \eta \in \Lambda\}$, there exists a finite subsets $\Lambda_0$of Λ with $\bigcap_{i=1}^{n} \mathrm{D}\left(\mathfrak{X} - \mathbb{G}_\eta\right) = \emptyset$.

Example 3.10: Every discrete space is IIS-O--compact.

It is clear that every IIS-O-compact is IIS-compact, but the opposite is not necessarily true, for example if $(\mathfrak{X}, \mathfrak{T})$ is indiscrete space on infinite set is IIS-compact but not IIS-O-compact..

Proposition 3.11: Let $\mathbb{h}$: $(\mathfrak{X}, \mathfrak{T}) \rightarrow (Y, T)$ is continuous,onto, IIS-irresolute, IIS-ontinuous function.If $(\mathfrak{X}, \mathfrak{T})$ is IIS-O-compact space, then (Y, T) is IIS-O-compact space.

Proof: Similar proof to the proposition (3.6), by Corollary (2.7)

4. Conclusion and future work

Based on the definition of the isolated point and Theorem 2.2 it is noticed that IIS(A) lies between int(A) and cl(A). It can be discussed in ideal topologies and their relations to the mathematical concepts with common properties between them. We have drawn attention to a new definition of continuity and compactness using interior isolated point through which we can involve gem and ω-open sets to study these two concepts together to obtain results with satisfactory effects.

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