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A semi-analytical method to solve the Fitzhugh-Nagumo equation

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Abstract

In this paper, a solution of the FNE is approximated via the residual power series method, RPSM. To test the effectiveness of the RPSM, its approximation is compared with those obtained from the finite difference method, FDM, the Galerkin finite element method, GFEM, and the exact solution of the equation. These results illustrate the RPSM's effectiveness in approximating FNE solutions.

Subject Classification (2010): *65M60, 65M06, 35G20, 35G25.*

Keywords: *Residual power series method, FitzHugh-Nagumo equations, Finite difference, Galerkin finite element method, Numerical solutions.*

1. Introduction

Theoretical and experimental investigations have been conducted for a hundred years to understand the rich electrical activity of excitable cells - neurons and myocytes. The Hodgkin-Huxley model has perhaps been one of the most seminal developments in modern biophysics and shed much light on the membrane activity in the squid giant axon [14]. This model fully revolutionized our understanding of cellular electrophysiology and served as a stimulant for providing a variety of detailed models to capture a broader range of neuronal events. Efforts have been made to

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reduce the HH model to its bare essentials and to provide simplified models, analytic solutions of which are more tractable. Only simplified models can indicate how such complex neural networks of hundreds of mutually interacting cells are to be understood and reproduced.

These models have also found applications beyond neuroscience. In cardiac electrophysiology, they are fundamental for understanding the electrical activity of cardiac cells and the underlying mechanisms of cardiac arrhythmias. Furthermore, they enable studies of complex spatiotemporal patterns such as traveling and spiral waves [9, 22] that are very important to understanding the organization of electrical activity in both neural and cardiac tissues. Investigations of biological rhythms, especially circadian rhythms, have wide ramifications in understanding mechanisms at work in many physiological processes.

It can be demonstrated that the Hodgkin-Huxley model can be reduced to give the FitzHugh-Nagumo equation,[12],[19]:

$$\frac{\partial v}{\partial \tau}=\frac{\partial^2 v}{\partial y^2}-v(1-v)(a-v), \tag{1}$$

where a is a constant. FNE is considered a paradigm nonlinear reaction-diffusion equation. It arises in a wide range of fields like circuit theory, biology, and in particular in population genetics [5]. The FNE has also been used to study the dynamical aspects of Rayleigh Bénard convection near the bifurcation point in binary fluid mixtures [23].

In the last couple of decades, nonlinear partial differential equations have been solved using semi-analytical approaches such as the Variational Iteration Method (VIM), Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), He-Laplace Decomposition Method (HLDM), Variational Iteration Adomian Decomposition Method (VIADM), and Modified Sumudu Transform Adomian Decomposition Method (MSTADM) among others as documented in [4, 25, 13, 26, 11, 20, 7, 18, 27, 2, 3, 16, 21, 28, 15, 1, 29]. Using analytical and numerical methods, different techniques have been widely adopted to solve the FitzHugh-Nagumo equation. For instance, Ali gives the numerical solution of the FNE by the generalized finite element method, presenting comparisons with the exact solution and errors at different t-values. In Soliman, VIM and ADM were applied to solve FNE in a particular form, and the obtained results were compared with the exact values presented via numerical simulations. Nourazar solved FNE using the HPM method. He proved the validity and efficiency of the applied method by introducing two nonhomogeneous nonlinear differential equations and computing the relative error for several values of and. Soori [26] applied VIM to solve two case studies of the FNE. On the other hand, Wei [29] studied a fractional FitzHugh-Nagumo equation (FFFNE) using the He-Laplace decomposition method to approximate its solution.

Starting from the time of the appearance of the RPSM that was first introduced by Arqub in [6], most of the researchers and scientists put this method within the circle of the most reliable, effective, and efficient methods in solving nonlinear differential equations. The idea of this method relies on obtaining truncated series solutions for initial value problems by calculating the power series coefficients. One can refer to [10, 17, 24, 8] to get more information about RPSM. In this paper, we apply RPSM to obtain the analytical solution for the undisturbed and undiscretized (FNE). The main aim of RPSM is to describe the FNE in the form of a power series centered at the start point τ_0 such that the solution will take the following form:

$$v(y,\tau)=\sum_{k=0}^{\infty}\phi_k(y)(\tau-\tau_0)^k.$$

The n^{th} truncated solution, represented by

$$v_n(y,\tau)=\phi_0(y)+\sum_{k=1}^{n}\phi_k(y)(\tau-\tau_0)^k,$$

can be used to approximate the true solution $v(y,\tau)$, where $\phi_0(y)$ = $v_0(y,\tau)$ = $v(y,0)$. Additionally, the n^{th} residual function, denoted by $Res_n(y,\tau)$, is an infinitely differentiable function at $\tau=0$, and thus,

$$Res_\infty(y,\tau)=\lim_{n\to\infty}Res_n(y,\tau)=0.$$

Furthermore,

$$\frac{\partial^s}{\partial\tau^s},Res_\infty(y,0)=\frac{\partial^s}{\partial\tau^s},Res_n(y,0)=0,$$

for $s=1,2,3,\cdots,n$. This paper is organized as follows: in section (2), the (RPSM) is introduced and applied to the (FNE). Section (3) demonstrates the (RPSM) with examples and presents several numerical solutions and simulations for the (FNE). The paper concludes in section (4).

2. The RPSM algorithm to solve FitzHugh-Nagumo equation

This section describes the (RPSM) algorithm used to find series solutions for the (FNE) subject to a specific initial condition. To do this, we considered the following initial value problem (IVP):

$$\frac{\partial v}{\partial \tau}=\frac{\partial^2 v}{\partial y^2}-v(1-v)(a-v), \tag{2}$$

subject to the initial condition $v(y,0)=\phi(y)$. We assumed that the solution would take the following form:

$$v(y,\tau)=\sum_{k=0}^{\infty}\phi_k(y)\tau^k, \tag{3}$$

where $\phi_0(y)=v(y,0)=\phi(y)$. Then we can approximate the solution $v(y,\tau)$ of (2) by the n^{th}-truncated series

$$v_n(y,\tau)=\sum_{k=0}^{n}\phi_k(y)\tau^k. \tag{4}$$

Next, the residual function is defined as follows:

$$Res(y,\tau)=\frac{\partial v}{\partial \tau}-\frac{\partial^2 v}{\partial y^2}+v(1-v)(a-v)$$

and the truncated residual function is expressed as

$$Res_n(y,\tau)=\frac{\partial v_n}{\partial \tau}-\frac{\partial^2 v_n}{\partial y^2}+v_n(1-v_n)(a-v_n),$$

It is clear that $Res(y,\tau)=0$ and as shown in [6], $\lim_{n\to\infty} Res_n(y,\tau) = Res(y,\tau)$ for all x in the domain and $\tau\geq 0$. Moreover, $\frac{\partial^s}{\partial\tau^s}Res(y,\tau) = \frac{\partial^s}{\partial\tau^s}Res_n(y,\tau) = 0$, at $\tau=0$ for each $s=0,1,2,\dots,n$. Now, to obtain the first approximate solution $v_1(y,\tau) = \phi_0(y) + \phi_1(y)\tau$, we calculate

$$\begin{aligned}Res_1(y,\tau)&=\frac{\partial v_1}{\partial \tau}-\frac{\partial^2 v_1}{\partial y^2}+v_1(1-v_1)(a-v_1)\\&=\phi_1(y)-\phi_0''(y)-\tau\phi_1''(y)\\&\quad+(\phi_0(y)+\phi_1(y)\tau)(1-\phi_0(y)-\phi_1(y)\tau)(a-\phi_0(y)-\phi_1(y)\tau).\end{aligned}$$

Upon substituting $\tau=0$ and by using the fact that $Res(y,0) = Res_1(y,0) = 0$, we can solve for $\phi_1(y)$.

For the second approximated solution $v_2(y,\tau) = \phi_0(y) + \phi_1(y)\tau + \phi_2(y)\tau^2$, we calculate

$$Res_2(y,\tau)=\frac{\partial v_2}{\partial \tau}-\frac{\partial^2 v_2}{\partial y^2}+v_2(1-v_2)(a-v_2)$$

$$= \phi_1(y) + 2\tau\phi_2(y) - \phi_0''(y) - \tau\phi_1''(y) - \tau^2\phi_2''(y)$$
$$+(\phi_0(y) + \phi_1(y)\tau + \phi_2(y)\tau^2)(1 - \phi_0(y) - \phi_1(y)\tau - \phi_2(y)\tau^2)$$
$$.(a - \phi_0(y) - \phi_1(y)\tau - \phi_2(y)\tau^2).$$

Thus, by using the fact that $\frac{\partial}{\partial\tau} Res_2(y,\tau) = 0$ at $\tau = 0$, we can solve for $\phi_2(y)$. Repeating this procedure, we can find the n^{th}-approximated solution $v_n(y,\tau)$. The following algorithm describes this process.

Table 1

RPSM algorithm for FNE

RPSM algorithm for solving the FitzHugh-Nagumo equation	
$v_0 = \phi_0(y)$;	
for $k = 1, 2, \ldots, n$ do	
$\quad v_k = v_{k-1} + \phi_k(y)\tau^k$	
$\quad Res_k = \frac{\partial v_k}{\partial \tau} - \frac{\partial^2 v_k}{\partial y^2} + v_k(1 - v_k)(a - v_k)$	Compute the residue
$\quad eq_k = \left(\frac{\partial^{k-1}}{\partial \tau^{k-1}} Res_k\right)_{\tau=0}$	Use the residue condition
$\quad$ Solve $(eq_k = 0, \phi_k)$	
end for	
return v_n	The n^{th}-approximation

The convergence of the (RPSM) is demonstrated in the following theorem:

Theorem 2.1: [6] *Assume that $v(y,\tau)$ represents the precise solution of the initial value problem 1. In this scenario, the approximate solutions derived through the residual power series method correspond to the Taylor series expansion of $v(y,\tau)$.*

Corollary 2.2: [6] The residual power series method will obtain the exact solution if $v(y,\tau)$ is a polynomial.

3. Numerical discussion and examples

In this section, the (RPSM) is applied to solve several cases of the (FNE) with different initial conditions.

Example 3.1: Consider the IVP

$$\frac{\partial v}{\partial \tau}=\frac{\partial^2 v}{\partial y^2}-v(1-v)^2, \tag{5}$$

with the initial condition

$$\phi_0(y)=v(y,0)=1-\frac{\exp\left(-\frac{y}{\sqrt{2}}\right)}{2+\exp\left(-\frac{y}{\sqrt{2}}\right)},$$

Applying the (RPSM) algorithm, we have $v_1(y,\tau)=\phi_0(y)+\phi_1(y)\tau$, which implies that

$$\begin{aligned}Res_1(y,\tau)&=\frac{\partial v_1}{\partial \tau}-\frac{\partial^2 v_1}{\partial y^2}+v_1(1-v_1)^2\\&=\phi_1(y)-\phi_0''(y)-\tau\phi_1''(y)\\&\quad+(\phi_0(y)+\phi_1(y)\tau)(1-\phi_0(y)-\phi_1(y)\tau)^2,\end{aligned}$$

substituting $\tau=0$,

$$Res_1(y,0)=\phi_1(y)-\phi_0''(y)+\phi_0(y)(1-\phi_0(y))^2=0.$$

Solving for $\phi_1(y)$ we get

$$\phi_1(y)=\frac{\exp\left(-\frac{y}{\sqrt{2}}\right)}{\left(2+\exp\left(-\frac{y}{\sqrt{2}}\right)\right)^2}.$$

Now, $v_2(y,\tau)=\phi_0(y)+\phi_1(y)\tau+\phi_2(y)\tau^2$ which implies

$$\begin{aligned}Res_2(y,\tau)&=\frac{\partial v_2}{\partial \tau}-\frac{\partial^2 v_2}{\partial y^2}+v_2(1-v_2)^2\\&=\phi_1(y)+2\tau\phi_2(y)-\phi_0''(y)-\tau\phi_1''(y)-\tau^2\phi_2''(y)\\&\quad+(\phi_0(y)+\phi_1(y)\tau+\phi_2(y)\tau^2)(1-\phi_0(y)-\phi_1(y)\tau-\phi_2(y)\tau^2)^2.\end{aligned}$$

Which implies that

$$\begin{aligned}\frac{\partial}{\partial \tau}Res_2(y,\tau)&=2\phi_2(y)-\phi_1''(y)-2\tau\phi_2''(y)\\&\quad+2(\phi_0(y)+\phi_1(y)\tau+\phi_2(y)\tau^2)\\&\quad.(1-\phi_0(y)-\phi_1(y)\tau-\phi_2(y)\tau^2)(\phi_1(y)-2\tau\phi_2(y))\end{aligned}$$

$$.(\phi_1(y)+2\tau\phi_2(y))(1-\phi_0(y)-\phi_1(y)\tau-\phi_2(y)\tau^2)^2,$$

using the fact that $\frac{\partial}{\partial\tau}Res_2(y,\tau)=0$ at $\tau=0$ we get

$$2\phi_2(y)-\phi_1''(y)+2\phi_0(y)\phi_1^2(y)(1-\phi_0(y))^3=0,$$

solving for $\phi_2(y)$ we get

$$\phi_2(y)=\frac{\exp\left(-\frac{y}{\sqrt{2}}\right)\left(\exp\left(-\frac{y}{\sqrt{2}}\right)-2\right)}{4\left(2+\exp\left(-\frac{y}{\sqrt{2}}\right)\right)^3}.$$

Similarly, $v_3(y,\tau)=\phi_0(y)+\phi_1(y)\tau+\phi_2(y)\tau^2+\phi_3(y)\tau^3$ and

$$\begin{aligned}\frac{\partial^2}{\partial\tau^2}Res_3(y,\tau)=2\phi_2(y)-\phi_1''(y)-2\tau\phi_2''(y)\\ +2(\phi_0(y)+\phi_1(y)\tau+\phi_2(y)\tau^2)\\ .(1-\phi_0(y)-\phi_1(y)\tau-\phi_2(y)\tau^2)(\phi_1(y)-2\tau\phi_2(y))\\ .(\phi_1(y)+2\tau\phi_2(y))(1-\phi_0(y)-\phi_1(y)\tau-\phi_2(y)\tau^2)^2,\end{aligned}$$

using the fact that $\frac{\partial^2}{\partial\tau^2}Res_3(y,\tau)=0$ at $\tau=0$ we get,

$$\phi_3(y)=\frac{\exp\left(-\frac{y}{\sqrt{2}}\right)\left(\left(\exp\left(-\frac{y}{\sqrt{2}}\right)\right)^2-8\exp\left(-\frac{y}{\sqrt{2}}\right)+4\right)}{24\left(2+\exp\left(-\frac{y}{\sqrt{2}}\right)\right)^4}.$$

Now we compare the numerical solution obtained by the (RPSM) with the one obtained by the finite difference method (FDM) in [10] using the following domains $-10<y<10$ and $0<\tau<10$.

Table 2

Numeric and analytic solutions of (5) when $\Delta y=0.001$ and $\tau=0.001$ using $v_3(y,\tau)$.

y	(RPSM) Relative Error	(FDM) Relative Error
0	4.823×10^{-16}	1.389×10^{-8}
0.001	4.824×10^{-16}	2.357×10^{-4}
0.002	4.824×10^{-16}	4.712×10^{-4}
0.003	4.824×10^{-16}	7.066×10^{-4}
0.004	4.825×10^{-16}	9.418×10^{-4}
0.005	4.825×10^{-16}	1.177×10^{-3}
0.006	4.826×10^{-16}	1.412×10^{-3}

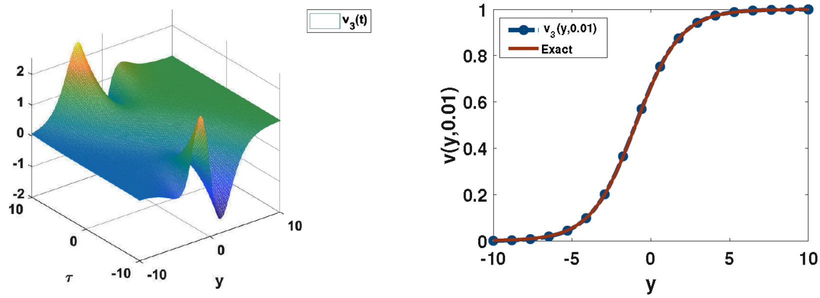

Figure 1

The 3D plot of $v_3(y,\tau)$ (Left) and the plot of the third approximation $v_3(y,\tau)$ and the exact solution $v(y,\tau)$ of the equation (5) at $\tau = 0.01$ (Right).

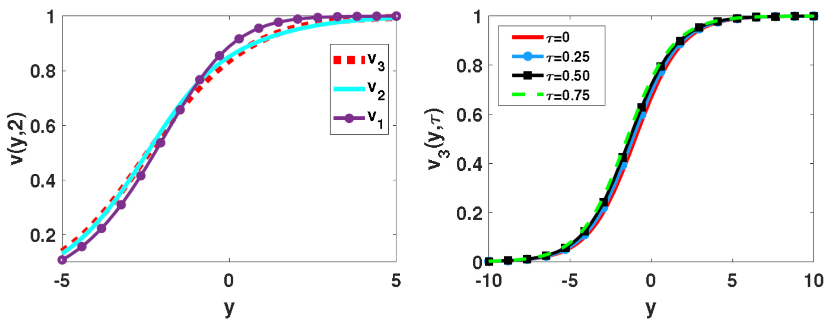

Figure 2

The plot of the first three approximations of the solution of equation (5) for $\tau = 2$ (Left) and the plot of third approximation $v_3(y,\tau)$ of the solution of equation (5) for several τ values (Right).

It is clear from Table 2, that the approximation obtained using the (RPSM) is several magnitudes better than the ones obtained using the (FDM) in [30].

Example 3.2: Consider the IVP

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial y^2} - v(1-v)(1-v), \tag{6}$$

with the initial condition

$$\phi_0(y) = v(y,0) = 1 - \frac{1}{2}\left[1 - \coth\left(\frac{y}{2\sqrt{2}} + \frac{\pi}{4}\right)\right].$$

Applying the (RPSM) algorithm we have $v_1(y,\tau) = \phi_0(y) + \phi_1(y)\tau$, which implies

$$\begin{aligned} Res_1(y,\tau) &= \frac{\partial v_1}{\partial \tau} - \frac{\partial^2 v_1}{\partial y^2} + v_1(1-v_1)^2 \\ &= \phi_1(y) - \phi_0''(y) - \tau\phi_1''(y) \\ &\quad + (\phi_0(y) + \phi_1(y)\tau)(1 - \phi_0(y) - \phi_1(y)\tau)^2, \end{aligned}$$

substituting $\tau = 0$,

$$Res_1(y,0) = \phi_1(y) - \phi_0''(y) + \phi_0(y)(1-\phi_0(y))^2 = 0,$$

solving for $\phi_1(y)$ we get

$$\phi_1(y) = \left(-\frac{1}{8} + \frac{1}{8}\coth^2\left(\frac{y}{2\sqrt{2}} + \frac{\pi}{4}\right)\right).$$

Table 3

Numeric and analytic solutions of (6) when $\Delta y = 0.2$ and $\tau = 0.05$ using $v_3(y,\tau)$.

y	**(RPSM) Relative Error**	**(GFEM) Relative Error**
–1	7.890×10^{-7}	2.982×10^{-6}
–0.8	3.718×10^{-7}	1.036×10^{-6}
–0.6	1.928×10^{-7}	1.9850×10^{-6}
–0.4	1.084×10^{-7}	4.5095×10^{-6}
–0.2	6.420×10^{-8}	2.375×10^{-7}
0	4.030×10^{-8}	3.0055×10^{-6}
0.2	2.620×10^{-8}	3.8725×10^{-6}
0.4	1.790×10^{-8}	4.0740×10^{-6}
0.6	1.240×10^{-8}	3.7225×10^{-6}
0.8	8.90×10^{-9}	6.255×10^{-7}
1	6.00×10^{-9}	1.8575×10^{-6}

Continuing with the (RPSM) algorithm (1) we find

$$\phi_2(y) = 5.945210450\times10^{-13} + \frac{1}{32}\coth\left(\frac{y}{2\sqrt{2}}+\frac{\pi}{4}\right) - \frac{1}{32}\coth^3\left(\frac{y}{2\sqrt{2}}+\frac{\pi}{4}\right),$$

$$\phi_3(y) = -0.01041666667\,\coth^2\left(\frac{y}{2\sqrt{2}}+\frac{\pi}{4}\right) + 0.002604166667$$
$$+\frac{1}{128}\coth^4\left(\frac{y}{2\sqrt{2}}+\frac{\pi}{4}\right) - 1.0\times10^{-12}\coth\left(\frac{y}{2\sqrt{2}}+\frac{\pi}{4}\right).$$

We now compare the numerical solution obtained by the (RPSM) with the one received by the (GFEM) in [4].

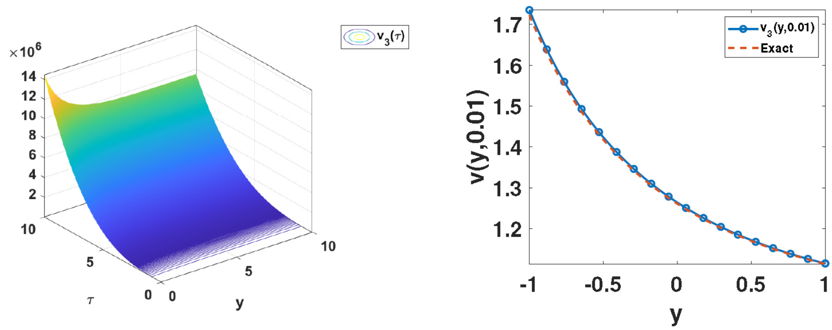

Figure 3

The 3D plot of $v_3(y,\tau)$ (Left) and the graphs of $v_3(y,\tau)$ and the exact solution $v(y,\tau)$ of (6) at $\tau = 0.01$ (Right).

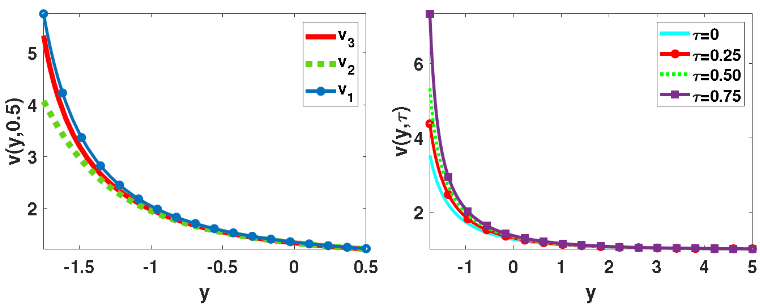

Figure 4

The plot of the first three approximations of the solution of equation (6) for $\tau = 0.5$ (Left) and the plot of third approximation $v_3(y,\tau)$ of the solution of equation (6) for several τ values (Right).

Example 3.3: Consider the IVP

$$\frac{\partial v}{\partial \tau}=\frac{\partial^2 v}{\partial y^2}-0.5v(1-v)(0.2-v), \tag{7}$$

with the initial condition

$$\phi_0(y)=v(y,0)=\frac{1}{2}-\frac{\tanh\left(\frac{y}{4}\right)}{2}.$$

This IVP has been studied in [2] by constructing four versions of NSFD schemes. We will apply the (RPSM) algorithm to find a numerical solution and compare it with the result found in [2]. Applying the (RPSM) algorithm, we have

$$\phi_1(y)=\frac{3}{80}-\frac{3\tanh\left(\frac{y}{4}\right)^2}{80},$$

$$\phi_2(y)=\frac{9\tanh\left(\frac{y}{4}\right)}{3200}-\frac{9\tanh\left(\frac{y}{4}\right)^3}{3200},$$

$$\phi_3(y)=-\frac{27\tanh\left(\frac{y}{4}\right)^4}{128000}+\frac{9\tanh\left(\frac{y}{4}\right)^2}{32000}-\frac{9}{128000}.$$

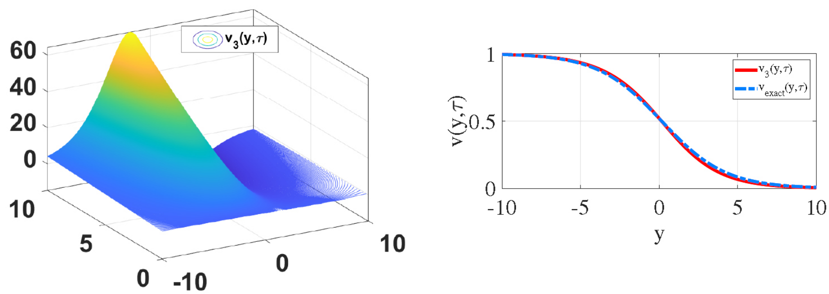

Figure 5

The 3D plot of $v_3(y,\tau)$ (Left) and the graphs of $v_3(y,\tau)$ and the exact solution $v(y,\tau)$ of (7) at $\tau=0.01$ (Right).

Example 3.4: Consider the IVP

$$\frac{\partial v}{\partial \tau}=\frac{\partial^2 v}{\partial y^2}-v(1-v)(0.2-v), \tag{8}$$

with the initial condition

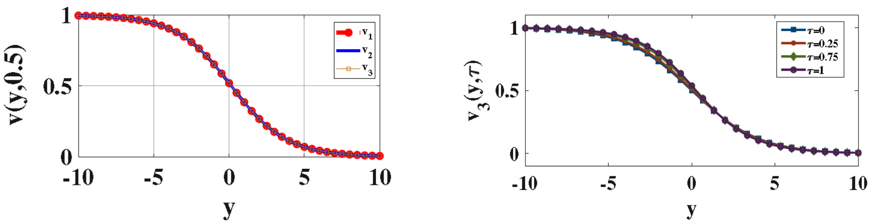

Figure 6

The plot of the first three approximations of the solution of equation (7) for $\tau = 0.5$ (Left) and the plot of third approximation $v_3(y,\tau)$ of the solution of equation (7) for several t values (Right).

$$\phi_0(y) = v(y,0) = \frac{1}{2} - \frac{\tanh\left(\frac{\sqrt{2}y}{4}\right)}{2}.$$

This IVP has been studied in [2] by constructing four versions of NSFD schemes. We will apply the (RPSM) algorithm to find a numerical solution and compare it with the result found in [2].

Applying the (RPSM) algorithm, we have

$$\phi_1(y) = \frac{3}{40} - \frac{3\tanh\left(\frac{\sqrt{2}y}{4}\right)^2}{40},$$

$$\phi_2(y) = \frac{9\tanh\left(\frac{\sqrt{2}y}{4}\right)}{800} - \frac{9\tanh\left(\frac{\sqrt{2}y}{4}\right)^3}{800},$$

$$\phi_3(y) = -\frac{27\tanh\left(\frac{\sqrt{2}y}{4}\right)^4}{16000} + \frac{9\tanh\left(\frac{\sqrt{2}y}{4}\right)^2}{4000} - \frac{9}{16000}.$$

The following table 4 shows the relative error when using the (RPSM) algorithm on the IVP from 7 for various values of τ.

Table 4

Absolute error of (RPSM) of equation (8) when $\tau = 0.005$ using $v_3(y,\tau)$.

y	RPSM	Exact Solution	RPSM Absolute Error
−10	0.9999999998	0.9999999998	0
−8	0.999999981	0.9999999809	1.000000×10^{-10}
−6	0.9999983833	0.9999983752	8.100000×10^{-09}
−4	0.9998625708	0.9998618823	6.885000×10^{-07}

Contd...

–2	0.988447886	0.9883919779	5.590810×10^{-05}
0	0.50037547	0.5003749999	4.701000×10^{-07}
2	0.0115864222	0.0116424924	5.607020×10^{-05}
4	0.000137842	0.0001385327	6.907000×10^{-07}
6	0.0000016215	0.0000016296	8.100000×10^{-09}
8	0.0000000191	0.0000000192	1.000000×10^{-10}
10	0.0000000002	0.0000000002	0

4. Conclusion

The (RPSM) was introduced in this paper, and its application in obtaining approximate solutions to the (FNE) was demonstrated using different parameters and initial conditions. Four examples were presented to showcase the efficiency of (RPSM) in solving such problems with minimal computations.

In Example 3.1, Equation (1) was solved by using parameter $a = 1$ and the same initial condition as in [30]. The outcomes are depicted in Figures (1) and (2), and the relative error is computed in Table (2), which is then compared with the relative error computed in [30].

Example 3.2 involves solving equation (1) with parameter $a = 1$ and the same initial condition used in [4]. The results are presented in Figures (3) and (4), and the relative error is computed in Table (3), which is then compared to the relative error computed in [4].

The examples mentioned above illustrate that the RPSM exhibits higher accuracy when compared to the other two methods used in the references [4] and [30].

Furthermore, Examples 3.3 and 3.4 involve the solving equation (1) with parameter $a = 0.2$ and the same initial condition as in [2]. The outcomes are presented in Figures (5) and (6), and Table (4) displays the corresponding calculations. It is worth mentioning that all computations in these examples were carried out using MAPLE.

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Competing interests

The authors declare no disclosures of relevant financial or non-financial interests.

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