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Study on weakly Lambda ideal map

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Abstract

The purpose of this paper is to define a study ideals with the Weakly ideal map and $weakly_0$ ideal map which are defined by Al-Bawi and AL-Swidy in [5] by using the lambda ideal Open set. Some of theorems and properties of these types of map will be studied.

Subject Classification: *32T27, 47L45.*

Keywords: *λI–Open set, λI-local function, λI-den set, λI -Weakly-λI-map, $Weakly_0$-λI-map.*

1. Introduction

The concept of ideality, introduced by physicist Kartowski, stands as one of the most significant ideas in the study of topological spaces. Over time, numerous academics and scientists have expanded its application by linking it to various other topological concepts. In [6], we introduced new types of sets, referred to as λI-open set., by utilizing a novel form of αI-open set. We explored the relationships between open, α-open, αI-open, and λ-open sets, presenting a series of theorems and results related to these concepts. Additionally, we proposed a new type of local function based on ΛI-open set., accompanied by several theorems and properties associated with this formulation. In the same reference [6], we introduced a novel class of dense sets derived from λI-open set., detailing their characteristics and theorems. We further investigated their connections to I-dense, T-dense, and T∗-dense sets. Moreover, we provided a new interpretation of Weakly using λI-open set. and explored various relationships between I-dense, T-dense, and λI-dense sets. We also introduced a new type of Resolvable-space that incorporates ΛI-open set., along with associated theorems, properties, and studies of its relationship with Resolvable-space and I- Resolvable-space sets. This paper focuses on two types of functions, originally defined by Al-Bawi and Al-Swidi in [5], which they termed Weakly0-I-map. These functions were accompanied by a set of results, which we revisit in this work. Specifically, we examine these findings in the context of the ΛI-local function and explore their implications for the concept of ΛI-Resolvable-space lvability. Generalization of algebra used in ideal, see [11] and [12].

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2. Elementary Materials

Definition 2.1: [6] Assume that (H, T, T^{α}) BI-topological-space. & I ideal of H. A subset R of H is named lambda ideal Open set and denoted by λI-Open se tiff every H∈R and every α_I-Open set W so that R ⊆ W, satisfy that h ∈{h∈H:U∩intT^{α} (W) ∉ I, ∀U∈T(H)}. A collection for each λI-Open set symbolize as λI(H). complement for λI-Open named λI-closed [1-4].

Definition 2.2: [6] Assume that (H, T, T^{α}, I) be an ideal BI-topological-space. So H is named P-space if for each T- Open set A of H the subsquent h∈{h∈H: U∩R∉ I}iff H∈R.

Definition 2.3: [5] An ideal topo s. (H, T, I) is named thick if R⊂R^* for each R⊆H. That is each subset of H is ∗-den in itself.

Proposition 2.4: [6] Assume that (H,T,T^{α},I) is ideal BI-topological-space s.t I is codeines ,so all T-Open λI-open set If R λI-Open & R⊑ B, so B is not necessary λI-open set.

Definition 2.5: [6] Assume that (H,T,T^{α},I) is ideal BI-topological-space . An operator $(.)^{*\lambda I}$:P(H) ⟶P(H) is named λI-local function for R over ideal also $O_{\lambda I}$(H) also is define in subsquent for all R⊆H, $R^{*\lambda I}$(I,$O_{\lambda I}$(H))={h∈H:U∩R∉I for each subset U∈$O_{\lambda I}$(H),h∈U}, where is no chance of confusion $R^{*\lambda I}$ (I,$O_{\lambda I}$(H)) symbolize by $R^{*\lambda I}$.

Definition 2.6: [6] Assume that (H,T,T^{α},I) Ideal BI-topological-space & R ⊆ H named λI-den set iff $R^{*\lambda I}$= H

Remark 2.7: [6] Assume that (H,T,T^{α},I) be a hyper connected Ideal BI-topological-space ., so :

1. Assume that Ideal is codes, so each λI- den is Ideal –den set.
2. Assume that Ideal is codes, so each λI-den is T^* -den set.
3. Assume that Ideal is codes, so each λI- den is T- den set.
4. Assume that H is P-space and nowhere space, so each Ideal –den set is λI-den.
5. Assume that H is P.sp, so each T^*-den is λI-den

Definition 2.8 [6]

Assume that (H,T,$T^{*\lambda I}$,I) is ideal BI-topological-space& R⊆H, so R is named Weakly –λI-den if $(R^{*\lambda I})^{*\lambda I}$=H

Definition 2.9: [6] Assume that (H,T,T^{α} ,I) is ideal BI-topological-Space, so ideal is named λI- codes if I ∩$O_{\lambda I}$(H)=∅.

Theory 2.10: [6] *Assume that (H,T,T^{α},I) be an ideal BI-topological-space ., so :-*

1. Assume that Ideal is codes, so each Weakly− λI-den is I- den.
2. Assume that Ideal is codes, so each Weakly− λI-den is T- den.

3. Each Weakly− λI-den is λI-den.
4. Assume that Ideal is λI-codes, so each I- den is Weakly− λI-den.
5. Assume that Ideal is λI-codes, so each λI-den is Weakly− λI-den.

Definition 2.11: [6] Assume that (H,T,T^{α},I) is ideal BI-topological-Space, so H is named λI- Resolvable if H contains two intersection equal ∅ λI-den set and R and B subset for H s.t $R\cup B=H$.

Definition 2.12: [6] A nonempty an ideal BI-topological-space is (H,T,T^{α},I) is named:

1. weakly- λI- Resolvable-Space, where H intersection equal ∅ and unification for two weakly- λI-den.
2. weakly- λI-I–resolvable-space, where H intersection equal ∅ and unification of Two Weakly- λI-den set also I-den set.
3. Weakly- λI-T-Resolvable-space, if H intersection equal ∅ and unification of Two Weakly-λI-den set also T-den set.
4. λI-T-Resolvable-space, if H intersection equal ∅ and unification of Two λI-den

Theory 2.13: [7-10] *Assume that (H,T,T^{α},I) is Ideal BI-Topological-Space with R and B be two subsets in H. if H is $Weakly - \lambda I - Resolvable$ then:*

1. if the ideal is codes, then H weakly-λI-I-resolvable.
2. if Ideal is codes, then H is Weakly-λI-T-resolvable.
3. If ideal is codes, so H is λI-T-Resolvable.
4. If ideal is codes, so H is Weakly- λI-T- Resolvable-Space.
5. H be a λI-T- Resolvable.
6. H be a λI-T- Resolvable.
7. Assume that Ideal be λI- codes, so H be Weakly-λI- Resolvable.
8. Assume that Ideal be a λI- codes, and H be a λI-T- Resolvable, so H must be a Weakly-λI-T- Resolvable. and T- den.

3. Weakly lambda ideal map

Definition 3.1: A function f: $(H,T_1,I_1) \longrightarrow (Y,T_2,I_2)$ for any $R\subseteq H$ is named :

1. Weakly - λI-map iff $f(R^{*\lambda I\ *\lambda I}) \subset (f(R))^{*\lambda I\ *\lambda I}$
2. Weakly_0 - λI-map iff $f(R^{*\lambda I}) \subset (f(R))^{*\lambda I}$.

Example 3.2: Assume that H={d,e,f} , T={H,∅,{a}} also an ideal $I_1=\{\emptyset,\{d\}\}$ also Assume that y={1,2,3} with a topology $T_2=\{Y,\emptyset,\{2,3\}\}$ and an ideal $I_2=\{\emptyset,\{3\}\}$.

Assume that f: $(H,T_H,I_1)\longrightarrow(Y,T_y,I_2)$, so H is not Weakly-λI -map and not $Weakly_0$-λI –map.

Theory 3.3: *A function f: $(H,T_1,I_1)\longrightarrow(Y,T_2,I_2)$ is hold:*

1. Each $Weakly_0$-λI–map is Weakly λI-map.
2. Each $Weakly_0$-λI –map is Weakly λI-map and H is (P-space, thick space) is continuous.

Proof:

1. because f is $Weakly_0$-λI-map so $f(R^{*\lambda I}) \subset (f(R))^{*\lambda I}$, and so $f(R^{*\lambda I})^{*\lambda I} \subset (f(R))^{*\lambda I*\lambda I}$ hence f is Weakly-λI-map.
2. because f is $Weakly_0$-λI-map so $f(R^{*\lambda I}) \subset (f(R))^{*\lambda I}$, because H is (P-space and thick space) so f is continuous.

Theory 3.4: *A function f: $(H,T_1,I_1)\longrightarrow(Y,T_2,I_2)$ is hold:*

If f is open-λI-map and H is P-space and I-codes and bijection map so f(R) is f(I) Open set for each I- open set.

Proof: Since $f(R)\subset f(int(R^*))\subset int\ f(R^*)$, because f is open mapping now because f is $Weakly_0$-λI-map, so $int\ f(R^{*\lambda I}) \subset int\ (f(R))^{*\lambda I}$ since H is P-space and $f(R^*) \subset int\ f(R^{*\lambda I}) \subset int\ (f(R)^{*\lambda I}) \subset int\ f(R^{*\lambda I})$ there for f(R) is f(I)-Open set for each λI–Open set.

Theory 3.5: *Assume that f: $(H,T_1,I)\longrightarrow(Y,T_2,I)$ be a Weakly -λI-map and bijection map if R is Weakly –λI-den in H so f(R) is Weakly f(λI)- den in Y .*

Proof: Assume that R is Weakly-λI –den in H so $R^{*\lambda I\ *\lambda I}= H$, hence $f(R^{*\lambda I\ *\lambda I})\subset(f(R))^{*\lambda I*\lambda I}$,hence $Y\subset f(R)^{*\lambda I*\lambda I}$,because f is Weakly-λI –map and bijection, so f(R) is Weakly-λI -den in Y.

Corollary 3.6: *Assume that f: $(H,T_1,I)\longrightarrow(Y,T_2,f(I))$ be a Weakly -λI-map map bijection map and I is codes .if R is λI-den in H, so f(R) is Weakly f(λI)- den in Y.*

Corollary 3.7: *Assume that f: $(H,T_1,I)\longrightarrow(Y,T_2,f(I))$ be a Weakly -λI-map and bijection map and I is codes and H is P.space.*

If R is T-den in H, so f(R) is Weakly-f(I)-den in Y.

Proof: If we use Theory (2.5) and Theory1.10 (5) with Remark 1.7 (2), (5). The results follows.

Corollary 3.8: *Assume that f: $(H,T_1,I)\longrightarrow(Y,T_1,f(I))$ be a$Weakly_0$ -λI-map, bijection map if R is Weakly-λI-den in H so f(I) is Weakly –f(I)-den in Y.*

Theory 3.9: *Assume that f: $(H,T_1,I)\longrightarrow(Y,T_2,f(I))$ be a$Weakly_0$ -λI-map, bijection if R is λI-den in H so f(R) is f(I)-den in Y.*

Proof: Suppose that R is λI-den in H, so $R^{*\lambda I}$=H, that Y=f(H)=f($R^{*\lambda I}$)⊂$(f(R))^{*\lambda I}$, since f is $Weakly_0$-λI-map & bijection. Y=$(f(R))^{*\lambda I}$. Hence f(R) is f(I)-den of Y.

Corollary 3.10: *Assume that f: $(H,T_1,I)\longrightarrow(Y,T_2,f(I))$ be a $Weakly_0$ -λI-map, bijection, such that H is thick space so each of the below hold:*

1. for R is T^*-den of H, so f(R) is f(I)- den of Y.
2. If R is T-den of H, so f(R) is f(I)-den of Y.

Proof: The prove of (1) and (2) exist by Remark 1.7 and Theory2.9.

Theory 3.11: *Assume that f:$(H,T_1,I)\longrightarrow(Y,T_2,f(I))$ is Weakly-λI-map& bijection. Assume H Weakly-λI- Resolvable-space, then Y must be Weakly-f(I)- resolvable-space.*

Proof: Assume H Weakly-λI –Resolvable-space, so there is R,B sub sets of H s.t R∪B=H & R∩B=∅ , hence R & B are Weakly -λI –den in H and by Theory 2.9 we get each of f(R)&f(B) Weakly f(I)- den in Y, there for Y Weakly f(λI)-Resolvable-space

Corollary 3.12: *Assume that f:$(H,T_1,I)\longrightarrow(Y,T_2,f(I))$ be a Weakly -λI –map & bijection, if H Weakly-λI- Resolvable-space, hence Y be a Weakly f(I)-T_2-Resolvabl-space*

Proof: Clearly that from Theory1.13 and Theory 2.11.

Corollary 3.13: *Assume that f:$(H,T_1,I)\longrightarrow(Y,T_2,f(I))$ be a Weakly-λI-map and λI is codes and bijection map if H is Weakly-λI –T_1-Resolvable-space, thick space , so Y is Weakly-f(I)-Resolvabl-space.*

Proof: Clearly that from Theory 1.13 and Theory 2.11.

Theory 3.14: [11] *Assume that f:$(H,T,I)\longrightarrow(Y,T,f(I))$ be a Weakly-λI –map and bijection map so*

1- If H is Weakly-λI –T_1-Resolvabl-spaceand thick space, so Y is Weakly-f(I)-T_2-Resolvable-space.
2- If H is λI-T_1-Resolvabl-spaceand thick space, so Y is f(I)-T_2-Resolvable-space.

Proof:

1- Assume H weakly-λI –T_1-Resolvabl-space, so ∃ R, B ⊆ H ∋ R∪B=H also R∩B=∅ also R is Weakly-λI –den also B is T-den So by Theory 2.3 we have each $Weakly_0$-λI –map is Weakly-λI –map also by Theory2.5 and

Theory 2.1.23 in [5] we have f(R) Weakly-f(I)-den and f(B) is T-den, so H is Weakly-f(I)-T-Resolvabl-space.

2- Suppose that H is λI –T_1-Resolvabl-space ,so ∃ R, B ⊆ H ∋ R∪B=H and R∩B=∅ also R is Weakly-λI –den also B is I-den So by Theory 2.3 we have each Weakly$_0$-λI –map is Weakly-λI –map also by Theory2.5 we have f(R) Weakly-f(I)-den also f(B) is f(I) -den, so Y is Weakly-f(I)-T_2-Resolvabl-space.

Theory 3.15: *Assume that f:(H,T,I)⟶(Y,T,f(I)) be a Weakly$_0$-λI –I-Resolvabl-space and bijection map*

1- If H be a weakly-λI-I-resolvable-Space, then Y must be weakly-f(I)-T_2-resolvabl-Space.

2- If H is Weakly-λI –I-Resolvable-space, so Y is f(I)-T_2-Resolvabl-space.

Proof: The prove exist by Theorem1.13 and Theory 2.14.

4. Conclusion

In this paper we concluded Weakly -I-map is not necessarily Weakly-λI –map and Weakly$_0$ -I-map is not necessary Weakly$_0$ - λI-map.

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