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Study of *d*-interior and *d*-closure

Luma S. Abdalbaqi *
Hassan H. Ebrahim †
Department of Mathematics
College of Computer Science & Mathematics
University of Tikrit
Tikrit
Iraq

Ali Al-Fayadh §
Department for Mathematics & Computer Applications
College of Science
University of Al-Nahrain
Baghdad 10072
Iraq

Abstract

A main goal for this article is to generalize the notion open set by introduce an idea d – open set to obtains the d – space which is stronger than classical topological space. Some examples are provided that aim to explain the scientific contribution of the suggested generalization, as well as sundry features of d – open set are presented. The concept of d-closure is introduced. Characterizations and examples for a proposed idea are present and several different properties for d – closure were shown.

Subject Classification: *00A05, 00A06.*

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1. Introduction

Originally coming from questions in analysis and measure [1]–[3] by now topologies permeates mostly in all field for math contains algebra, and combinatorics, and logic and plays a fundamental role in algebraic geometry as known its today. Let $(\mathcal{U}, \mathfrak{J})$ be a topological space and $\mathfrak{D} \subseteq \mathcal{U}$. Then $\mathfrak{D}$ is called

* *E-mail:* **lumahhany1977@tu.edu.iq (Corresponding Author)**
† *E-mail:* **hassan1962pl@tu.edu.iq**
§ *E-mail:* **aalfayadh@yahoo.com**

d −open set if $\mathfrak{D} \cap \mathcal{A} \subseteq int(\mathfrak{D})$ $\forall \mathcal{A} \in \mathfrak{J}$ & $\mathcal{A} \neq \mathcal{U}$. We shall denote the family of d −open set by $dO\mathfrak{J}_{\mathcal{U}}$ and the ordered pair $(\mathcal{U}, dO\mathfrak{J}_{\mathcal{U}})$ is named d −space. While we call a complement of d − open set by d − closed set and the $dC\mathfrak{J}_{\mathcal{U}}$ denote to the family of d −closed set.

An idea for semi-open was studied by Levine [4] in 1963 which is a generalization of an open set. For any topological space $(\mathcal{U}, \mathfrak{J})$, then $dO\mathfrak{J}_{\mathcal{U}}$ is a topology on $\mathcal{U}$. Njastad in 1965 [5] was first introduced α − open set concept as a generalization of open set. There are many authors have been studied some types of classes of sets which are generalizations of σ-algebra and some concepts related to topological spaces for example Ahmed, Ebrahim, and Al-Fayadh, [6] and Sameer and Abdalbaqi [7] and Abdulqader, Esmaeel, and Arif [8].

2. Results

Definition 2.1: Let us assume we have a d −space $(\mathcal{U}, dO\mathfrak{J}_{\mathcal{U}})$ & $v \in \mathcal{A} \subseteq \mathcal{U}$. Then we call v is d −interior point of $\mathcal{A}$ if $\exists\ \mathfrak{D} \in dO\mathfrak{J}_{\mathcal{U}} \ni v \in \mathfrak{D} \subseteq \mathcal{A}$.

Notation 2.2: We shall denote the set of d −interior points of $\mathcal{A}$ by $int_d(\mathcal{A})$.

Theorem 2.3: *In d − space $(\mathcal{U}, dO\mathfrak{J}_{\mathcal{U}})$, we have*

$$int_d(\mathcal{A}) = \cup_{\propto \in I}\left\{\begin{matrix} \mathfrak{D}_\propto : \ \mathfrak{D}_\propto \in dO\mathfrak{J}_{\mathcal{U}} \\ \text{and } \mathfrak{D}_\propto \subseteq \mathcal{A}\ \forall \propto \in I \end{matrix}\right\} \text{whenever } \mathcal{A} \subseteq \mathcal{U}.$$

Proof: Let $v \in int_d(\mathcal{A})$, then $\exists\ \mathfrak{D} \in dO\mathfrak{J}_{\mathcal{U}} \ni v \in \mathfrak{D} \subseteq \mathcal{A}$. So,

$$v \in \cup_{\propto \in I}\left\{\begin{matrix} \mathfrak{D}_\propto : \ \mathfrak{D}_\propto \in dO\mathfrak{J}_{\mathcal{U}} \\ \text{and } \mathfrak{D}_\propto \subseteq \mathcal{A}\ \forall \propto \in I \end{matrix}\right\}.$$

Hence, $int_d(\mathcal{A}) \subseteq \cup_{\propto \in I}\left\{\begin{matrix} \mathfrak{D}_\propto : \ \mathfrak{D}_\propto \in dO\mathfrak{J}_{\mathcal{U}} \\ \text{and } \mathfrak{D}_\propto \subseteq \mathcal{A}\ \forall \propto \in I \end{matrix}\right\}$.

Now, let us assume $v \in \cup_{\propto \in I}\left\{\begin{matrix} \mathfrak{D}_\propto : \ \mathfrak{D}_\propto \in dO\mathfrak{J}_{\mathcal{U}} \\ \text{and } \mathfrak{D}_\propto \subseteq \mathcal{A}\ \forall \propto \in I \end{matrix}\right\}$. Then at least $\exists\ \mathfrak{D}^* \in dO\mathfrak{J}_{\mathcal{U}}$ & $\mathfrak{D}^* \subseteq \mathcal{A} \ni v \in \mathfrak{D}^*$, hence $\in \mathcal{A}$. So, $v \in int_d(\mathcal{A})$. Hence, $int_d(\mathcal{A}) \supseteq \cup_{\propto \in I}\left\{\begin{matrix} \mathfrak{D}_\propto : \ \mathfrak{D}_\propto \in dO\mathfrak{J}_{\mathcal{U}} \\ \text{and } \mathfrak{D}_\propto \subseteq \mathcal{A}\ \forall \propto \in I \end{matrix}\right\}$.

Thus $int_d(\mathcal{A}) = \cup_{\propto \in I}\left\{\begin{matrix} \mathfrak{D}_\propto : \ \mathfrak{D}_\propto \in dO\mathfrak{J}_{\mathcal{U}} \\ \text{and } \mathfrak{D}_\propto \subseteq \mathcal{A}\ \forall \propto \in I \end{matrix}\right\}$.

Theorem 2.4: *In the d −space $(\mathcal{U}, dO\mathfrak{J}_{\mathcal{U}})$, if any subset $\mathcal{A}$ of a universal $\mathcal{U}$, then $int_d(\mathcal{A})$ must be a largest d − open set which included in $\mathcal{A}$.*

Theorem 2.5: *If $(\mathcal{U}, dO\mathfrak{J}_{\mathcal{U}})$ is d −space & $\mathcal{A} \subseteq \mathcal{U}$, then $\mathcal{A} \in dO\mathfrak{J}_{\mathcal{U}}$ iff $int_d(\mathcal{A}) = \mathcal{A}$.*

Theorem 2.6: *If $(\mathcal{U}, dO\mathfrak{J}_{\mathcal{U}})$ is d −space & $\mathcal{A}, \mathcal{B} \subseteq \mathcal{U}$. Then*

1. $int_d(\mathcal{A}) \subseteq int_d(\mathcal{B})$ for $\mathcal{A} \subseteq \mathcal{B}$.
2. $int_d(\mathcal{A} \cap \mathcal{B}) = int_d(\mathcal{A}) \cap int_d(\mathcal{B})$.

3. $int_d\big(int_d(\mathcal{A})\big) = int_d(\mathcal{A})$.
4. $int_d(\emptyset\,) = \emptyset$ and $int_d(\mathcal{U}) = \mathcal{U}$.

Theorem 2.7: *If $\mathcal{A}$ is any subset of a universal $\mathcal{U}$, then $int\,(\mathcal{A}) \subset int_d(\mathcal{A})$.*

Proof: Since $int\,(\mathcal{A})$ is open set & $int\,(\mathcal{A}) \subseteq \mathcal{A}$. Then $int\,(\mathcal{A})$ is d −open set, but $int_d(\mathcal{A})$ is largest d −open set that included in $\mathcal{A}$ by theorem (2.4), so we get $int\,(\mathcal{A}) \subset int_d(\mathcal{A})$.

Remark 2.8: In general $int\,(\mathcal{A}) \neq int_d(\mathcal{A})$.

Example 2.3.9: If $\mathcal{U} = \{d_1, d_2, d_3, d_4\}$ and $\mathfrak{I} = \left\{\begin{matrix} \emptyset, \mathcal{U}, \{d_1\}, \{d_1, d_2\}, \\ \{d_1, d_3\}, \{d_1, d_2, d_3\} \end{matrix}\right\}$

Then $d\mathcal{O}\mathfrak{I}_{\mathcal{U}} = \left\{\begin{matrix} \emptyset, \{d_1\}, \{d_1, d_2\}, \{d_1, d_3\}, \{d_1, d_2, d_3\}, \{d_4\}, \\ \{d_1, d_4\}, \{d_1, d_2, d_4\}, \{d_1, d_3, d_4\}, \mathcal{U} \end{matrix}\right\}$

Now, $int_d(\{d_3, d_4\}) = \{d_4\}$ and $int\,(\{d_3, d_4\}) = \emptyset$, so we note that $int\,(\{d_3, d_4\}) \subset int_d(\{d_3, d_4\})$, but $int_d(\{d_3, d_4\}) \neq int\,(\{d_3, d_4\})$.

This example verifies the equality in Remark (2.8) not hold in general.

Remark 2.10: In the following Theorem we consider the necessary condition of equality to be true in Remark (2.8).

Theorem 2.11: *If the subset $\mathcal{A}$ of a universal set $\mathcal{U}$ is an open set, then $int\,(\mathcal{A}) = int_d(\mathcal{A})$.*

Proof: Let us assume that $\mathcal{A}$ is open set, then $\mathcal{A} = int\,(\mathcal{A})$. Since an open set implies to d −open set, so $\mathcal{A}$ is d −open set. Now, by using theorem (2.5), then we get $\mathcal{A} = int_d(\mathcal{A})$. Hence, $int\,(\mathcal{A}) = int_d(\mathcal{A})$.

Definition 2.12: In the d −space $(\mathcal{U}, d\mathcal{O}\mathfrak{I}_{\mathcal{U}})$ we define the d −closure of arbitrary subset $\mathfrak{D}$ of a universal $\mathcal{U}$ as $cl_d(\mathfrak{D}) = \bigcap_{\propto \in \mathrm{I}} \left\{\begin{matrix} \mathfrak{D}_\propto : \ \mathfrak{D}_\propto \in d\mathcal{C}\mathfrak{I}_{\mathcal{U}} \\ \text{and } \mathfrak{D} \subseteq \mathfrak{D}_\propto \ \forall \propto \in \mathrm{I} \end{matrix}\right\}$.

Theorem 2.13: *$cl_d(\mathfrak{D})$ is the smallest d −closed set including $\mathfrak{D}$.*

Corollary 2.14: *In the d −space $(\mathcal{U}, d\mathcal{O}\mathfrak{I}_{\mathcal{U}})$, subset $\mathfrak{D}$ in universal $\mathcal{U}$ is d −closed iff $\mathfrak{D} = cl_d(\mathfrak{D})$.*

Theorem 2.15: *If $(\mathcal{U}, d\mathcal{O}\mathfrak{I}_{\mathcal{U}})$ is d −space & $\mathfrak{D}_1, \mathfrak{D}_2 \subseteq \mathcal{U}$. Then*

1. $\mathfrak{D}_1 \subseteq \mathfrak{D}_2$, then $cl_d(\mathfrak{D}_1) \subseteq cl_d(\mathfrak{D}_2)$.
2. $cl_d(\mathfrak{D}_1 \cup \mathfrak{D}_2) = cl_d(\mathfrak{D}_1) \cup cl_d(\mathfrak{D}_2)$.
3. $cl_d(\mathfrak{D}_1 \cap \mathfrak{D}_2) \subseteq cl_d(\mathfrak{D}_1) \cap cl_d(\mathfrak{D}_2)$.
4. $cl_d\big(cl_d(\mathfrak{D})\big) = cl_d(\mathfrak{D})$, for any $\mathfrak{D} \subseteq \mathcal{U}$.
5. $cl_d(\emptyset) = \emptyset$ and $cl_d(\mathcal{U}) = \mathcal{U}$.

Proposition 2.16: Every closed implies to d −closed.

Proof: Assume $\mathfrak{D}$ closed, so $\mathfrak{D}^c$ is open set, hence $\mathfrak{D}^c$ is d −open set, therefore $\mathfrak{D}$ is d −closed set.

Proposition 2.17: If $\mathfrak{D} \subseteq \mathcal{U}$, so $cl_d(\mathfrak{D}) \subseteq cl(\mathfrak{D})$.

Proof: Since $cl(\mathfrak{D})$ closed, so $cl(\mathfrak{D})$ d −closed. But $\mathfrak{D} \subseteq cl(\mathfrak{D})$ and $cl_d(\mathfrak{D})$ is smallest d −closed set containing $\mathfrak{D}$, so we have $cl_d(\mathfrak{D}) \subseteq cl(\mathfrak{D})$.

Remark 2.18: If $\mathfrak{D} \subseteq \mathcal{U}$, then it is not necessarily that $cl(\mathfrak{D}) = cl_d(\mathfrak{D})$.

Example 2.19: If $\mathcal{U} = \{d_1, d_2, d_3, d_4\}$ and $\mathfrak{J} = \left\{\begin{matrix} \emptyset, \mathcal{U}, \{d_1\}, \{d_1, d_2\}, \\ \{d_1, d_3\}, \{d_1, d_2, d_3\} \end{matrix}\right\}$

Then $dO\mathfrak{J}_{\mathcal{U}} = \left\{\begin{matrix} \emptyset, \{d_1\}, \{d_1, d_2\}, \{d_1, d_3\}, \{d_1, d_2, d_3\}, \{d_4\}, \\ \{d_1, d_4\}, \{d_1, d_2, d_4\}, \{d_1, d_3, d_4\}, \mathcal{U} \end{matrix}\right\}$

Now, $cl_d(\{d_1\}) = \{d_1, d_2, d_3\}$ and $cl(\{d_1\}) = \mathcal{U}$, so $cl_d(\{d_1\}) \neq cl(\{d_1\})$.This example verifies the equality in Remark (2.18) is not hold.

Remark 2.20: In the following proposition we consider the necessary condition to obtain an equality in Remark (2.18).

Proposition 2.21: If subset $\mathfrak{D}$ in $\mathcal{U}$ is closed set, then $cl(\mathfrak{D}) = cl_d(\mathfrak{D})$.

Proof: Let us assume $\mathfrak{D}$ closed set, then $\mathfrak{D} = cl(\mathfrak{D})$. Since $\mathfrak{D}$ is closed set, then by Proposition (3.16), we get $\mathfrak{D}$ is d −closed set. Hence, $\mathfrak{D} = cl_d(\mathfrak{D})$ by Corollary (2.14). Therefore we are done.

Definition 2.22: Let us assume we have d −space $(\mathcal{U}, dO\mathfrak{J}_{\mathcal{U}})$ and $\mathfrak{D} \subseteq \mathcal{U}$. A point $v \in \mathcal{U}$ is a d −limit point of $\mathfrak{D}$ if $\mathcal{N}$-$\{v\} \cap \mathfrak{D} \neq \emptyset$ $\forall \mathcal{N} \in dO\mathfrak{J}_{\mathcal{U}}$ containing v. We shall denote a set for d −limit point of $\mathfrak{D}$ by $\mathrm{LP}_d(\mathfrak{D})$.

Theorem 2.23: *The subset $\mathfrak{D}$ is d −closed iff $LP_d(\mathfrak{D}) \subseteq \mathfrak{D}$.*

Proof: Assume $\mathfrak{D}$ is d −closed and $v \in \mathrm{LP}_d(\mathfrak{D})$. If $v \notin \mathfrak{D}$, then $v \in \mathfrak{D}^c$, but $\mathfrak{D}^c \in dO\mathfrak{J}_{\mathcal{U}}$, then $\mathfrak{D}^c$-$\{v\} \cap \mathfrak{D} = \emptyset$, which implies that $v \notin \mathrm{LP}_d(\mathfrak{D})$ contradiction. Hence $v \in \mathfrak{D}$ and $\mathrm{LP}_d(\mathfrak{D}) \subseteq \mathfrak{D}$.

Conversely: suppose $\mathrm{LP}_d(\mathfrak{D}) \subseteq \mathfrak{D}$. To prove $\mathfrak{D}$ is d −closed, we must prove $\mathfrak{D}^c \in dO\mathfrak{J}_{\mathcal{U}}$. We us

Now, let $v \in \mathfrak{D}^c$, then $v \notin \mathfrak{D}$, hence $v \notin \mathrm{LP}_d(\mathfrak{D})$, thus $\exists\, \mathcal{N} \in dO\mathfrak{J}_{\mathcal{U}}$ containing v s.t $\mathcal{N}$-$\{v\} \cap \mathfrak{D} = \emptyset$.

Thus $\mathcal{N} \cap \mathfrak{D} = \emptyset$ since $v \notin \mathfrak{D}$. Consequentially, $\mathcal{N} \subseteq \mathfrak{D}^c$. Hence proved.

Theorem 2.24: *In a d −space $(\mathcal{U}, d\mathcal{O}\mathfrak{J}_{\mathcal{U}})$, if $\mathfrak{D} \subseteq \mathcal{U}$, then $LP_d(\mathfrak{D})$ is d −closed set.*

Proof: To prove $\mathrm{LP}_d(\mathfrak{D})$is d −closed set, we must prove $\mathrm{LP}_d(\mathrm{LP}_d(\mathfrak{D})) \subseteq \mathrm{LP}_d(\mathfrak{D})$. Let $\mathscr{v} \in \mathrm{LP}_d(\mathrm{LP}_d(\mathfrak{D}))$.Then $\mathscr{v}$ is a d −limit point of $\mathrm{LP}_d(\mathfrak{D})$. Hence $\mathcal{N}$-$\{\mathscr{v}\} \cap \mathrm{LP}_d(\mathfrak{D}) \neq \emptyset$ $\forall \mathcal{N} \in d\mathcal{O}\mathfrak{J}_{\mathcal{U}}$ containing $\mathscr{v}$, thus $\mathcal{N}$-$\{\mathscr{v}\} \cap \mathfrak{D} \neq \emptyset$ $\forall \mathcal{N} \in d\mathcal{O}\mathfrak{J}_{\mathcal{U}}$ containing $\mathscr{v}$, which implies $\mathscr{v}$ is a d −limit point of $\mathfrak{D}$. So, $\mathscr{v} \in d\mathcal{P}(\mathfrak{D})$. Therefore $\mathrm{LP}_d(\mathfrak{D})$ is d −closed set.

Theorem 2.25: *If $(\mathcal{U}, d\mathcal{O}\mathfrak{J}_{\mathcal{U}})$ is d −space & $\mathfrak{D} \subseteq \mathcal{U}$, then $\mathfrak{D} \cup LP_d(\mathfrak{D})$ is d −closed set.*

Proof: To prove $\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D})$is d −closed set, we must prove $(\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D}))^{c}$ is d −open set. We use Theorem (2.1.11) to prove this claim. Suppose that $\mathscr{v} \in (\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D}))^{c}$, then $\mathscr{v} \notin \mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D})$, thus $\mathscr{v} \notin \mathfrak{D}$ and $\mathscr{v} \notin \mathrm{LP}_d(\mathfrak{D})$This implies that there is $\mathcal{N} \in d\mathcal{O}\mathfrak{J}_{\mathcal{U}}$ containing $\mathscr{v}$ s.t $\mathcal{N}$-$\{\mathscr{v}\} \cap \mathfrak{D} = \emptyset$, but $\mathscr{v} \notin \mathfrak{D}$, then $\mathcal{N} \cap \mathfrak{D} = \emptyset$. To claim $\mathcal{N} \cap \mathrm{LP}_d(\mathfrak{D}) = \emptyset$. Let $z \in \mathcal{N}$. Since $\mathcal{N} \cap \mathfrak{D} = \emptyset$, then $\mathcal{N}$-$\{z\} \cap \mathfrak{D} = \emptyset$, hence $z \notin \mathrm{LP}_d(\mathfrak{D})$. So, we get $\mathcal{N} \cap \mathrm{LP}_d(\mathfrak{D}) = \emptyset$. Now, since $\mathcal{N} \cap [\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D})] = [\mathcal{N} \cap \mathfrak{D}] \cup [\mathcal{N} \cap \mathrm{LP}_d(\mathfrak{D})] = \emptyset$. Then $\mathcal{N} \subseteq [\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D})]^{c}$. Hence, $(\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D}))^{c}$ is the d − neighborhood of its own points. Thus, we have $(\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D}))^{c}$ is d −open set. Hence proved.

Theorem 2.26: *If $(\mathcal{U}, d\mathcal{O}\mathfrak{J}_{\mathcal{U}})$ is d −space & $\mathfrak{D} \subseteq \mathcal{U}$, then $cl_d(\mathfrak{D}) = \mathfrak{D}\cup LP_d(\mathfrak{D})$.*

Proof: By Theorem (2.25), we get $\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D})$ is d −closed set. Since $\mathfrak{D} \subseteq \mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D})$and $cl_d(\mathfrak{D})$ is the smallest d −closed set which includes $\mathfrak{D}$ then we get $cl_d(\mathfrak{D}) \subseteq \mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D})$. To prove $\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D}) \subseteq cl_d(\mathfrak{D})$. Since $\mathfrak{D} \subseteq cl_d(\mathfrak{D})$, then it only remains to prove $\mathrm{LP}_d(\mathfrak{D}) \subseteq cl_d(\mathfrak{D})$, that is we must prove $\mathrm{LP}_d(\mathfrak{D}) \subseteq \bigcap_{\propto\in \mathrm{I}} \{\mathfrak{D}_\propto : \mathfrak{D}_\propto$ is d −closed & $\mathfrak{D} \subseteq \mathfrak{D}_\propto\}$. Let $\mathscr{v} \in \mathrm{LP}_d(\mathfrak{D})$ and $\mathfrak{D}_\propto$ is d −closed & $\mathfrak{D} \subseteq \mathfrak{D}_\propto$. Then $\mathcal{N}$-$\{\mathscr{v}\} \cap \mathfrak{D} \neq \emptyset$ $\forall \mathcal{N} \in d\mathcal{O}\mathfrak{J}_{\mathcal{U}}$ containing $\mathscr{v}$. This mean that there exist at least $m \in \mathcal{N}$-$\{\mathscr{v}\} \cap \mathfrak{D}$ $\forall \mathcal{N} \in d\mathcal{O}_{\mathcal{U}}$ containing $\mathscr{v}$, so $m \in \mathcal{N}$-$\{\mathscr{v}\}$ and $m \in \mathfrak{D}$. Since $\mathfrak{D} \subseteq \mathfrak{D}_\propto$ $\forall \propto\in \mathrm{I}$, then $m \in \mathfrak{D}_\propto$ $\forall\propto\in \mathrm{I}$, hence $m \in \mathcal{N}$-$\{\mathscr{v}\} \cap \mathfrak{D}_\propto$, thus $\mathcal{N}$-$\{\mathscr{v}\} \cap \mathfrak{D}_\propto \neq \emptyset$ $\forall \mathcal{N} \in d\mathcal{O}\mathfrak{J}_{\mathcal{U}}$containing $\mathscr{v}$, $\forall\propto\in \mathrm{I}$. Now, since $\mathscr{v} \in \mathrm{LP}_d(\mathfrak{D}_\propto)$ and $\mathfrak{D}_\propto$ is d −closed set $\forall\propto\in \mathrm{I}$, then by Theorem (2.23), we get $\mathrm{LP}_d(\mathfrak{D}_\propto) \subseteq \mathfrak{D}_\propto$ $\forall\propto\in \mathrm{I}$, hence $\forall\propto\in \mathrm{I}$, $\mathscr{v} \in \mathfrak{D}_\propto \supseteq \mathfrak{D}$. Thus $\mathscr{v} \in \bigcap_{\propto\in \mathrm{I}} \{\mathfrak{D}_\propto : \mathfrak{D}_\propto$ is d −closed that include $\mathfrak{D}\}$, so $\mathscr{v} \in cl_d(\mathfrak{D})$ which implies that $\mathrm{LP}_d(\mathfrak{D}) \subseteq cl_d(\mathfrak{D})$. So, we have $\mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D}) \subseteq cl_d(\mathfrak{D})$. Therefore, $cl_d(\mathfrak{D}) = \mathfrak{D}\cup\mathrm{LP}_d(\mathfrak{D})$.

Conclusion

The main results are obtained are:

1. $int_d(\mathcal{A}) = \bigcup_{\propto\in \mathrm{I}} \left\{ \begin{array}{c} \mathfrak{D}_\propto : \mathfrak{D}_\propto \in d\mathcal{O}\mathfrak{J}_{\mathcal{U}} \\ \text{and } \mathfrak{D}_\propto \subseteq \mathcal{A}\ \forall\propto\in \mathrm{I} \end{array} \right\}$ whenever $\mathcal{A} \subseteq \mathcal{U}$.

2. In the d −space $(\mathcal{U}, d\mathcal{O}\mathfrak{J}_{\mathcal{U}})$, if any subset $\mathcal{A}$ of a universal $\mathcal{U}$, then $int_d(\mathcal{A})$ must be a largest $d-$ open set which included in $\mathcal{A}$.
3. If the subset $\mathcal{A}$ of a universal set $\mathcal{U}$ is open set, then $int\,(\mathcal{A}) = int_d(\mathcal{A})$.
4. Every closed set is d −closed set.
5. If $(\mathcal{U}, d\mathcal{O}\mathfrak{J}_{\mathcal{U}})$ is d −space & $\mathfrak{D} \subseteq \mathcal{U}$, then $\mathfrak{D} \cup \mathrm{LP}_d(\mathfrak{D})$ is d −closed set.

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