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Stabilizability of non-linear dynamic control systems on time scales in Banach spaces

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Abstract

In this article, new method is proposed to study the stabilizability of a nonlinear dynamic control systems $ƫ^{\Delta}(\varkappa)=A(\varkappa)ƫ(\varkappa)+B(\varkappa)u(\varkappa)+g(\varkappa,ƫ(\varkappa),u(\varkappa)),\ \varkappa\in\mathcal{T}$ on a time scale $\mathcal{T}$. Based on using an appropriate Lyapunov function and the derivation of sufficint conditions to be this system, uniformly stabilizable and h – stabilizable. Here, A, $B \in L(X)$. Further, different numerical examples were given as an application.

Subject Classification: *46N10, 47N10, 90C30.*

Keywords: *Time scales, Dynamical control system, Uniformly stabilizable, h – stabilizable.*

1. Introduction

In 1988, Stefan Hilger introduced the theory of dynamic equations on time scales, which combined continuous and discrete analysis into a single theory [7]. Developments in time scale calculus theory make it easier to gain a deeper understanding and improve knowledge of the superfine distinctions between

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continuous and discrete systems, [2, 3]. In recent decades, a large number of researchers have been involved in studying the stability of dynamic equations at time scales, which has relevant applications in mechanics, physics, and economics, see [2, 3, 5, 6, 7, 8], the achieved results of the stability of dynamic equations are derived by estimating the equation's general solutions. In [4, 5, 6] sufficient and necessary conditions for different kind for stability of dynamic equations over time scale was derived. The value and the significance of the achieved results that obtained by Lyapunov functions makes this subject, our considerable interest. Motivated by the researches cited above, this paper is proposed new method to study the stabilizability of a nonlinear dynamic control system on time scales. the stability play in real analysis, see [11] and [12].

2. Basic Concepts

Here , $\mathcal{X}$ represent to Banach Space & $\mathcal{X}^*$ its dual (the space of all continous linear functionals on $\mathcal{X}$), $\mathcal{T}$ is a timescale and $[\varkappa_0, \infty)_{\mathcal{T}} = [\varkappa_0, \infty) \cap \mathcal{T}$.

Definition 2.1: A point $\varkappa \in \mathcal{T}$ named: Riht-scattered if $\sigma(\varkappa) > \varkappa$, Riht-dense if $\varkappa < \varkappa up\mathcal{T}$ and $\sigma(\varkappa) = \varkappa$, & Left-dense if $\varkappa > ing\mathcal{T}$ and $\rho(\varkappa) = \varkappa$. Where $\sigma(\varkappa)$ forward jump operator & $\rho(\varkappa)$ backward jump operator.

We define also $\mathcal{T}^k$ as: if $\mathcal{T}$ have left scatered maximum$m$, so, $\mathcal{T}^k = T - \{m\}$. else $\mathcal{T}^k = \mathcal{T}$. let $\mathfrak{t}: \mathcal{T} \to \mathcal{X}$ & $\exists$ $\mathfrak{t}^{\Delta}(\varkappa)$ satisfy, for $\varepsilon > 0$, $\exists$ a neighborhood U for $\varkappa$, such that

$$\| [\mathfrak{t}(\sigma(\varkappa)) - \mathfrak{t}(\varkappa)] - \mathfrak{t}^{\Delta}(\varkappa)[\sigma(\varkappa) - \mathfrak{r}] \| \leq \varepsilon |\sigma(\varkappa) - \mathfrak{r}|, \ \forall \mathfrak{r} \in U,$$

Then, $\mathfrak{t}$ is saed to be (delta) differentiable at $\varkappa$ & $\mathfrak{t}^{\Delta}(\varkappa)$ named (delta) derivative of $\mathfrak{t}(\varkappa)$. [3]

Theorem 2.2: [2] *Let* $\mathcal{K}: \mathcal{T} \to \mathcal{X}$ *and* $\varkappa \in \mathcal{T}$.

(1) $\mathcal{K}$ differentiable at $\varkappa$, iff $\mathcal{K}$ continuous at $\varkappa$.

(2) Let $\mathcal{K}$ continuous at $\varkappa$, & the graininess function $\mu(\varkappa) = \sigma(\varkappa) - \varkappa > 0$, then

$$\mathcal{K}^{\Delta}(\varkappa) = \frac{\mathcal{K}(\sigma(\varkappa)) - \mathcal{K}(\varkappa)}{\sigma(\varkappa) - \varkappa}$$

(3) Let $\mathcal{K}$ differentiable and$\mu(\varkappa) = 0$. so

$$\mathcal{K}^{\Delta}(\varkappa) = \lim_{\mathfrak{r} \to \varkappa} \frac{\mathcal{K}(\varkappa) - \mathcal{K}(\mathfrak{r})}{\varkappa - \mathfrak{r}}$$

(4) If $\mathcal{K}$ differntiable at, $\varkappa$, so $\mathcal{K}(\sigma(\varkappa)) = \mathcal{K}(\varkappa) + \mu(\varkappa)\mathcal{K}^{\Delta}(\varkappa)$.

Definition 2.3: [2] A map $\mathrm{f}: \mathcal{T} \to \mathcal{X}$ call rd-continuous provided its continuous at all right-dense point in$\mathcal{T}$ & has left-side limts (finit) at each left-dense point, in $\mathcal{T}$. $\mathrm{C_{rd}}$ refers to the set of rd $-$continuous functions, $\mathrm{C_{rd}^1}$ refers to a set for function which is differentiable & whos derivative was rd-continuous.

Definition 2.4: [2] A map $\wp: \mathcal{T} \to \mathbb{R}$ regressive provided $1 + \mu(\varkappa)\wp(\varkappa) \neq 0$, $\forall \varkappa \in \mathcal{T}$. A set for every regressive & C_{rd} map will denotd as $\Re$. $\Re^+ = \{\wp \in \Re: 1 + \mu(\varkappa)\wp(\varkappa) > 0, \forall \varkappa \in \mathcal{T}\}$ set for each positive regressive element for $\Re$.

Definition 2.5: [2] We call $\mathcal{F}: \mathcal{T} \times \mathcal{X} \to \mathcal{X}$:
(i) C_{rd} when $\mathcal{G}$ defined as $\mathcal{G}(\varkappa) = \mathcal{F}(\varkappa, \mathfrak{t}(\varkappa))$ is C_{rd} where $\mathfrak{t} \in \mathcal{X}$ continuous map ,
(ii) $\Re$ at $\varkappa \in \mathcal{T}^k$, where $\mathcal{I} + \mu(\varkappa)\mathcal{F}(\varkappa, .): \mathcal{X} \to \mathcal{X}$ be invertaible mapping, wher $\mathcal{I}$ is the identity functon and $\Re$ on$\mathcal{T}^k$, if it is $\Re$ at each $\varkappa \in \mathcal{T}^k$.

Definition 2.6: The nonlinear dynamic control system (NLDCS):

$$\mathfrak{t}^\Delta(\varkappa) = g[t, \mathfrak{t}(\varkappa), u(\varkappa)],$$

is called stabilizable (Sz) or uniformly stabilizable (USz) If feed back control is present, $u(\varkappa) = K\mathfrak{t}(\varkappa), K \in \mathcal{X}$ s.t. the system $\mathfrak{t}^\Delta(\varkappa) = g[\varkappa, \mathfrak{t}(\varkappa), K\mathfrak{t}(\varkappa))]$ is stable (S) or uniformely stablle (US) respectively.

Different kinds of stability of the special control system

$\mathfrak{t}^\Delta(\varkappa) = A\mathfrak{t}(\varkappa) + Bu(\varkappa)$ were investigated in [1].

3. Main Results

Consider the (NLDCS) on $\mathcal{T}$ of the form

$$\mathfrak{t}^\Delta(\varkappa) = A(\varkappa)\mathfrak{t}(\varkappa) + B(\varkappa)u(\varkappa) + g(\varkappa, \mathfrak{t}(\varkappa), u(\varkappa)), \ \varkappa \in \mathcal{T}, \tag{1}$$

where $A(\varkappa), B(\varkappa) \in L(\mathcal{X})$, and $g(\varkappa, \mathfrak{t}, u): \mathcal{T} \times \mathcal{X} \times \mathcal{X} \longrightarrow \mathcal{X}$ is acontinuous function. When $g(\varkappa, \mathfrak{t}(\varkappa), u(\varkappa)) = 0$, yields the (LDCS)

$$\mathfrak{t}^\Delta(\varkappa) = A(\varkappa)\mathfrak{t}(\varkappa) + B(\varkappa)u(\varkappa), \ \varkappa \in [\varkappa_0, \infty)_\mathcal{T} \tag{2}$$

We now drive adequate conditions for different kinds of these systems' stabilizability (Sz) using the Lyapunov theory.

Theorem 3.1: *If* $\exists$ *a (PD) mapping* $W(\varkappa) \in C_{rd}^1(\mathcal{T}, L(\mathcal{X}))$ *satisfying conditions*
$(c1)$ $W(\varkappa)D(\varkappa) + (I + \mu(\varkappa)D(\varkappa))W^\Delta(\varkappa) \leq 0$, *where* $D(\varkappa) := A(\varkappa) + B(\varkappa)K(\varkappa)$
(c2) $\eta I \leq W(\varkappa) \leq \rho I$, then system (2) is (USz) by the control $u(\varkappa) = K(\varkappa)$ $\mathfrak{t}(\varkappa)$.

Proof: assume $\mathfrak{t}(\varkappa)$ solution for (2) & $L(\varkappa, \mathfrak{t}(\varkappa), \mathfrak{t}^*) = \mathfrak{t}^*(W(\varkappa)\mathfrak{t}(\varkappa))$, $\varkappa \in \mathcal{T}$. $(c2)$ which is implies that condition (ii) of corollary (3.2) in [9]. We get

$$\begin{aligned} L^\Delta(\varkappa, \mathfrak{t}(\varkappa)) &= (\mathfrak{t}^*(W(\varkappa)\mathfrak{t}(\varkappa))^\Delta \\ &= \mathfrak{t}^*(W(\varkappa)\mathfrak{t}(\varkappa)^\Delta + W^\Delta(\varkappa)\mathfrak{t}^\sigma(\varkappa)) \\ &= \mathfrak{t}^*(W(\varkappa)\mathfrak{t}(\varkappa)^\Delta + W^\Delta(\varkappa)\mathfrak{t}(\varkappa) + \mu(\varkappa)W^\Delta(\varkappa)\mathfrak{t}^\Delta(\varkappa)) \end{aligned}$$

$$= \mathfrak{t}^*((W(\varkappa)D(\varkappa) + (I + \mu(\varkappa)D(\varkappa))W^{\Delta}(\varkappa))\mathfrak{t}(\varkappa))$$
$$= \mathfrak{t}^*(Z(\varkappa)\mathfrak{t}(\varkappa))$$

where $Z(\varkappa) = W(\varkappa)D(\varkappa) + (I + \mu(\varkappa)D(\varkappa))W^{\Delta}(\varkappa)$

By (c1) implies $L^{\Delta}(\varkappa, \mathfrak{t}(\varkappa), \mathfrak{t}^*) \leq 0$. so system (2) are (USz).

$$Q(\varkappa) = W(\varkappa)D(\varkappa) + \mu(\varkappa)W^{\Delta}(\varkappa)D(\varkappa).$$

For the remainder, we will assume that $W(\varkappa) \in C^1_{rd}(\mathcal{T}, L(\mathcal{X}))$ is a (PD) mapping that complies with one or more of,

(C1) $\eta_1 \| \mathfrak{t}^* \| \| \mathfrak{t} \| \leq \mathfrak{t}^*(W(\varkappa)\mathfrak{t}) \leq \eta_2 \| \mathfrak{t}^* \| \| \mathfrak{t} \|$,

(C2) $\rho_1 \| \mathfrak{t}^* \| \| \mathfrak{t} \| \leq \mathfrak{t}^*(W^{\Delta}(\varkappa)\mathfrak{t}) \leq \rho_2 \| \mathfrak{t}^* \| \| \mathfrak{t} \|$,

(C3) $\mathfrak{t}^*(Q(\varkappa)\mathfrak{t}) \leq -(\theta\eta_2 + \rho_2 + \rho_2\theta\mu(\varkappa)) \| \mathfrak{t}^* \| \| \mathfrak{t} \|$,

(C4) $\mathfrak{t}^*(Q(\varkappa)\mathfrak{t}) \leq -(\theta\eta_2 + \rho_2 + \rho_2\theta\mu(\varkappa) - \gamma_1 \frac{h^{\Delta}(\varkappa)}{h(\varkappa)}) \| \mathfrak{t}^* \| \| \mathfrak{t} \|$,

$$\text{Where,} \quad \gamma_1 = \begin{cases} \eta_1, & h^{\Delta}(\varkappa) \geq 0 \\ \eta_2, & h^{\Delta}(\varkappa) < 0 \end{cases}$$

Here $\eta_1, \eta_2, \varepsilon \in \mathbb{R}^+$ and $\rho_1, \rho_2 \in \mathbb{R}$.

Theorem 3.2: *Let* $\exists\ \theta > 0$ *such that* $\mathscr{g}$ *satisfies*

$$\| \mathscr{g}(\varkappa, \mathfrak{t}, u) \| \leq \theta \| \mathfrak{t} \|; \varkappa \in \mathcal{T}, \mathfrak{t}, u \in \mathcal{X}. \tag{3}$$

If $\exists$ (PD) mapping $W(\varkappa) \in C^1_{rd}(\mathcal{T}, L(\mathcal{X}))$ satisfying $(C1 - C3)$, then system (1) is (USz) by the control

$$u(\varkappa) = K(\varkappa)\mathfrak{t}(\varkappa). \tag{4}$$

Proof: suppose $\mathfrak{t}(\varkappa)$ solution for, (2) & $L(\varkappa, \mathfrak{t}(\varkappa), \mathfrak{t}^*) = \mathfrak{t}^*(W(\varkappa)\mathfrak{t}(\varkappa)), \varkappa \in \mathcal{T}$. The delta derivative of L is :

$$L^{\Delta}(\varkappa, \mathfrak{t}(\varkappa), \mathfrak{t}^*) = (\mathfrak{t}^*(W(\varkappa)\mathfrak{t}(\varkappa)))^{\Delta}$$
$$= \mathfrak{t}^*(W(\varkappa)\mathfrak{t}(\varkappa)^{\Delta} + W^{\Delta}(\varkappa)\mathfrak{t}(\varkappa) + \mu(\varkappa)W^{\Delta}(\varkappa)\mathfrak{t}^{\Delta}(\varkappa))$$
$$= \mathfrak{t}^*(W(\varkappa)D(\varkappa)\mathfrak{t}(\varkappa) + W(\varkappa)\mathscr{g}(\varkappa, \mathfrak{t}, u) + W^{\Delta}(\varkappa)\mathfrak{t}(\varkappa)$$
$$+ \mu(\varkappa)W^{\Delta}(\varkappa)D(\varkappa)\mathfrak{t}(\varkappa) + \mu(\varkappa)W^{\Delta}(\varkappa)\mathscr{g}(\varkappa, \mathfrak{t}, u))$$

From (3), $(C1)$ and $(C2)$ we have

$$L^{\Delta}(\varkappa, \mathfrak{t}(\varkappa), \mathfrak{t}^*) \leqslant \mathfrak{t}^*\big(W(\varkappa)D(\varkappa)\mathfrak{t}(\varkappa)\big) + \theta\eta_2 \| \mathfrak{t}^* \| \| \mathfrak{t} \| + \rho_2 \| \mathfrak{t}^* \| \| \mathfrak{t}$$
$$\| + \mathfrak{t}^*(\mu(\varkappa)W^{\Delta}(\varkappa)D(\varkappa)\mathfrak{t}(\varkappa)) + \theta\rho_2\mu(\varkappa) \| \mathfrak{t}^* \| \| \mathfrak{t} \|$$
$$\leqslant \mathfrak{t}^*(Q(\varkappa)\mathfrak{t}(\varkappa)) + (\theta\eta_2 + \rho_2 + \theta\rho_2\mu(\varkappa)) \| \mathfrak{t}^* \| \| \mathfrak{t} \|$$

using $(C3)$, $L^{\Delta}(\varkappa, \mathfrak{t}(\varkappa), \mathfrak{t}^*) \leq 0$. so, by corollary (3.2) in [9], system (1) is (USz).

Theorem 3.3: *Let the inequality* (3) *is hold. If* $\exists\ W(\varkappa) \in C^1_{\mathrm{rd}}(\mathcal{T}, L(\mathcal{X}))$ *satisfying* $(C1), (C2)$ *and* $(C4)$*, then system* (1) *is h-(Sz) when* $u(\varkappa)$ *defined as in* (4).

Proof: Let $\varkappa_0 \in \mathcal{T}$, and $\mathfrak{b}(\varkappa) = \mathfrak{b}(\varkappa, \varkappa_0, \mathfrak{b}_0)$ any solution for system (1) correspp-onding of intial value $\mathfrak{b}(\varkappa_0) = \mathfrak{b}_0$. put $L(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*) = \mathfrak{b}^*(W(\varkappa)\mathfrak{b}(\varkappa))$. so,

$$L^\Delta(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*) = \mathfrak{b}^*(W(\varkappa)D(\varkappa)\mathfrak{b}(\varkappa) + W(\varkappa)g(\varkappa, \mathfrak{b}, u) + W^\Delta(\varkappa)\mathfrak{b}(\varkappa) + \mu(\varkappa)W^\Delta(\varkappa)D(\varkappa)\mathfrak{b}(\varkappa) + \mu(\varkappa)W^\Delta(\varkappa)g(\varkappa, \mathfrak{b}, u))$$

By using $(C1), (C2)$ and $(C4)$, we have

$$L^\Delta(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*) \leq \gamma_1 \frac{h^\Delta(\varkappa)}{h(\varkappa)} \parallel \mathfrak{b}^* \parallel \parallel \mathfrak{b} \parallel \leq \begin{cases} \dfrac{\gamma_1}{\eta_1} \dfrac{h^\Delta(\varkappa)}{h(\varkappa)} L(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*), & h^\Delta(\varkappa) \geq 0 \\ \dfrac{\gamma_1}{\eta_2} \dfrac{h^\Delta(\varkappa)}{h(\varkappa)} L(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*), & h^\Delta(\varkappa) < 0 \end{cases}$$

So, $L^\Delta(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*) \leq \frac{h^\Delta(\varkappa)}{h(\varkappa)} L(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*)$. Now, by integration from $\varkappa_0$ to $\varkappa$

$$L(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*) \leq L(\varkappa_0, \mathfrak{b}_0, \mathfrak{b}_0^*) + \int_{\varkappa_0}^{\varkappa} \frac{h^\Delta(\varkappa)}{h(\varkappa)} L(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*)\Delta\varkappa$$

By using Gronwall's Inequality, implies

$$L(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*) \leq L(\varkappa_0, \mathfrak{b}_0, \mathfrak{b}_0^*) e_{h^\Delta(\varkappa)/h(\varkappa)}(\varkappa, \varkappa_0).$$

By $(C1)$ &lemma (2.15) in [5], we get

$$\eta_1 \parallel \mathfrak{b}^* \parallel \parallel \mathfrak{b} \parallel \leq L(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*) \leq \eta_2 \parallel \mathfrak{b}_0^* \parallel \parallel \mathfrak{b}_0 \parallel e_{h^\Delta(\varkappa)/h(\varkappa)}(\varkappa, \varkappa_0)$$
$$\leq \eta_2 \parallel \mathfrak{b}_0^* \parallel \parallel \mathfrak{b}_0 \parallel \frac{h(\varkappa)}{h(\varkappa_0)}.$$

Hence, $\parallel \mathfrak{b}^* \parallel \parallel \mathfrak{b} \parallel \leq \omega \parallel \mathfrak{b}_0^* \parallel \parallel \mathfrak{b}_0 \parallel \frac{h(\varkappa)}{h(\varkappa_0)}$, $\quad \varkappa \geq \varkappa_0$. Whre, $\omega = \eta_2/\eta_1$. Therefore system (1) is h −(Sz).

Theorem 3.4: *Let* (3) *is hold. If* $\exists\ W(\varkappa) \in C^1_{\mathrm{rd}}(\mathcal{T}, L(\mathcal{X}))$ *satisfying* $(C1 - C3)$ *and* $\psi(\varkappa) \in C^1_{\mathrm{rd}}(\mathcal{T}, \mathbb{R}^+)$ *s.t.* $\psi^\Delta(\varkappa) \leq 0, \forall \varkappa \in \mathcal{T}$*, then system* (1) *is* ψ*-(USz) by* (4).

Proof: If $\mathfrak{b}(\varkappa)$ solution for system (1) & $L(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*) = \mathfrak{b}^*(W(\varkappa)\mathfrak{b}(\varkappa))$. Analogously to the demonstration for Theorem 3.2, observe that $L^\Delta(\varkappa, \mathfrak{b}(\varkappa), \mathfrak{b}^*) \leq 0$. Since $\psi^\Delta(\varkappa) \leq 0$, then by Theorem 3.1 in [9, 10], system (1) is ψ-(USz).

4. Numerical Examples

Let the (NLDCS) (1) s.t.

$$A = \begin{pmatrix} y(\varkappa) & -1 \\ 1 & y(\varkappa) \end{pmatrix}, \quad y(\varkappa) = \frac{1}{8}(e_{\ominus 12}(\varkappa, 0) + 8e_{\ominus 8}(\varkappa, 0) + e_{\ominus 4}(\varkappa, 0) - 8)$$

$$B = \begin{pmatrix} e_{\ominus 2}(\varkappa,0) & 0 \\ 0 & e_{\ominus 2}(\varkappa,0) \end{pmatrix} \quad and \quad g(\varkappa,\mathfrak{t},u) = \begin{pmatrix} \frac{1}{10}\mathfrak{t}_1 \cos u_1 \\ \frac{1}{8}\mathfrak{t}_2 \sin u_2 \end{pmatrix}.$$

put$\rho_1 = -1,\ \rho_2 = 0$ &

$$\theta = \frac{1}{8} \,\&\, \gamma = 1 \,\&\, 0 \le \mu(\varkappa) \le 0.25,\ \varkappa \in [0,\infty)_{\mathcal{T}}.$$

$$\varepsilon_1 = 1,\ \varepsilon_2 = \frac{1}{16},\ \varepsilon_3 = \frac{1}{2},\quad \varepsilon_4 = \frac{1}{16},\ \eta_1 = \frac{1}{8},\ \eta_2 = \frac{1}{4},$$

The matrix

$$W = \begin{pmatrix} \frac{1}{8}(1 + e_{\ominus 8}(\varkappa,0)) & 0 \\ 0 & \frac{1}{8}(1 + e_{\ominus 8}(\varkappa,0)) \end{pmatrix}$$

is a (PD) and holds (C1), (C2), and (C4) with $h(\varkappa) = c\varkappa, c > 0$. Therefore this system is h-(Sz)

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