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Some results about (Micro)topological spaces

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Abstract

This work introduces and investigates the properties for (Micro)-Ri Spaces (i = O, 1) and (Micro)-Ti Spaces (i = O, 1, 2), as well as identify relationships between existing spaces.

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1. Introduction

Thivagar and Richard [1] introduced (Nano) topology in 2013. Many researchers have presented new results about (Nano)topology, including the author of this research [2] we also used [3]. (Nano)topology has a maximum of 5 (Nano)open sets and a minimum of 3, such as U and φ. If we want to include more open sets, we can apply Levine's straightforward extension idea. This topology is referred to (Micro) topology (µicTopology). All (Nano) topologies are (Micro)topologies Chandrasekar [4] introduced and studied a notion for (Micro)topology, which was define as (micro)closed, (micro)open, (micro) interior, and (micro) closure. Ibrahim [5] and Sathishmohan et al. [6] introduce a

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new concepts of (Micro)separation axioms. In this research we present new findings on the concepts mentioned in the sources, this essay seeks to present and examine additional properties for (Micro)-Ri Spaces (i = O,1) and (Micro)-Ti Spaces (i = O,1,2), it was not mentioned or addressed in [5] as well as identify relationships between existing spaces.

2. Preliminary

Definition 2.1: [4] Here τR(X) is (Nano) topological $\mu_{ic}R(X)= \{\mathcal{N}\cup(\mathcal{N}_0 \cap\mu_{ic})\}$

$\mathcal{N}$, $\mathcal{N}_0$ ∈ τR(X) and named it μ_{ic}Topology of τR(X) by μ_{ic} where $\mu_{ic} \notin$ τR(X).

Definition 2.2: [4] μ_{ic}-cl(S) is the symbol for (Micro) closure for S, which is described as μ_{ic}-cl(S) = ∩{G:G is (Micro) closed and S⊆ G}. μ_{ic}-int(S) represents (Micro) interior for S, which is defined as μ_{ic}-int(S) = ∪{G:G is (Micro) open and S⊇ G}.

Lemma 2.3: [7] *In a μ_{ic}Topological space U, if G_i: $i \in I$ family for (micro)open set. so, $\cup G_i \in \mu_{ic}$-O(U).*

Remark 2.4: [5] The combination of two weaker axioms is the definition of each (Micro) separation axiom μ_{ic}-T_k space = μ_{ic} - R_{k-1} space and μ_{ic}-T_{k-1} space = μ_{ic}-R_{k-1} space and μ_{ic}-T_0 space

Definition 2.5: [5] Let Ube a μ_{ic}Topological. So U is:

(1) (Micro)T_0space(μ_{ic}-T_0 space) if ∀ (r,s) and $r\neq s : r, s$ in U, ∃ one (Micro) open Ҡ ∋ either $r \in$ Ҡ , $s \notin$ Ҡ or $r \notin$ Ҡ and $s \in$ Ҡ.

(2) (Micro)T_1space(μ_{ic}-T_1 space) if ∀ (r,s) and $r\neq s$ $(s.t)$ r, s in U, ∃ two (Micro) open sets $Ҡ_1$ & $Ҡ_2$ ∋ $r \in Ҡ_1$ but $s \notin Ҡ_1$ and $s \in Ҡ_2$ but $r \notin Ҡ_2$

(3) (Micro)T_2space(μ_{ic}-T_2space) if ∀ (r, s) and $r\neq s : r, s$ in U, ∃ two distinct (Micro) open sets $Ҡ_1$ & $Ҡ_2$ which contain r & s correspondingly.

Definition 2.6: [5] A μ_{ic} Topological space is called a (Micro) R_0- space (μ_{ic}-R_0 space) if ∀ (Micro) open set Ҡ and $r \in$Ҡ then μ_{ic}-$cl\{r\}\subseteq$ Ҡ.

Definition 2.7: [5] A μ_{ic}Topological space is called a (Micro) R_1- space (μ_{ic}-R_1space) if ∀ (r,s) and $r\neq s : r$, s in U with μ_{ic}-$cl\{r\} \neq \mu_{ic}$-$cl\{s\}$, ∃ disjoint (Micro) open sets $Ҡ_1$, $Ҡ_2$ ∋ μ_{ic}-$cl\{r\}\subseteq Ҡ_1$ and μ_{ic}-$cl\{s\}\subseteq Ҡ_2$.

Definition 2.8: [8] Let Ube a μ_{ic}Topological space and Ꞙ ⊂U. Next, the (Micro) kernel of Ꞙ indicated by μ_{ic}-ke(Ꞙ) is refers to the set μ_{ic}-ker(Ꞙ) = ∩{ꝉ∈ μ_{ic}R (X): Ꞙ ⊆ ꝉ}.

3. Some new results on the (Micro)-R_i Spaces (i = O, 1)

Theorem 3.1: *A μ_{ic} Topological space is a μ_{ic}-R_0 space iff ∀ F (Micro) closed set and $r \in F$ then μ_{ic}-{r} ⊆ F.*

Proof': Let a μ_{ic}Topological space be a μ_{ic}-R_0 space and Ḟ be a (Micro) closed set and $r \in$Ḟ. Then $\forall\ s \notin$ Ḟ implies $s \in$ Ḟ^c is (Micro) open set, then μ_{ic}-$cl\{s\} \subseteq$ Ḟ^c , so $r \notin \mu_{ic}$-$cl\{s\}$, so $s \notin \mu_{ic}$-$\{r\}$. Thus μ_{ic}-$ker\{r\} \subseteq$ Ḟ. Contrariwise, $\forall$ Ḟ (Micro) closed set and $r \in$ Ḟ then μ_{ic}-$ker\{r\} \subseteq$ Ḟ and let Ҡ $\in U$, $r \in$ Ҡ then $\forall\ s \notin$ Ҡ implies $s \in$ Ҡ^c is a (Micro)closed set implies μ_{ic}-$ker\{s\} \subseteq$ Ҡ^c . Therefore $r \notin \mu_{ic}$-$cl\{s\}$ and $s \notin \mu_{ic}$- $cl\{r\}$, so μ_{ic}-$cl\{r\} \subseteq$Ҡ, thus $U$is a μ_{ic}-R_0 space.

Theorem 3.2: *A μ_{ic}Topological space is μ_{ic}-R_1space iff $\forall\ r \neq s \in U$ with μ_{ic}-ker{r}$\neq \mu_{ic}$-ker{s} then $\exists$(Micro)closed sets Ҡ$_1$, Ҡ$_2$ $\ni$ μ_{ic}-ker{r}$\subseteq$ Ҡ$_1$, μ_{ic}-ker{r} $\cap$Ҡ$_2$ = $\emptyset$ and μ_{ic}-ker{s}$\subseteq$ Ҡ$_2$, μ_{ic}-ker{s}$\cap$Ҡ$_1$ = $\emptyset$ and Ҡ$_1$$\cup$Ҡ$_2$ = U*

Proof': Let a μ_{ic}Topological space be an μ_{ic}-R_1 space. Then $\forall\, r \neq s \in U$ with μ_{ic}- $ker\ \{r\} \neq \mu_{ic}$- $ker\ \{s\}$. Since every μ_{ic}-R_1 *space is a* μ_{ic}-Ro *space*, μ_{ic}-$cl\{r\} \neq \mu_{ic}$-$cl\{s\}$, then $\exists$ (Micro) open sets G_1, G_2 $\ni$ μ_{ic}-$cl\{r\} \subseteq G_1$ *and* μ_{ic}-$cl\{s\} \subseteq G_2$ *and* $G_1 \cap G_2 = \emptyset$ [Since $U$is μ_{ic}-R_1 space], then G_1^c *and* G_2^c are (Micro) closed sets $\ni G_1^c \cup G_2^c = U$. Put Ҡ$_1$ = G_1^c *and* Ҡ$_2$ = G_2^c Thus x $\in G_1 \subseteq$ Ҡ$_2$ *and* $s \in G_2 \subseteq$ Ҡ$_1$ so that μ_{ic}- $ker\ \{r\} \subseteq G_1 \subseteq$ Ҡ$_2$ *and* μ_{ic}-$ker\{s\} \subseteq G_2 \subseteq$ Ҡ$_1$. Contrariwise, Let $\forall\ r \neq s \in U$ with μ_{ic}-$ker\{r\} \neq \mu_{ic}$-$ker\{s\}$, $\exists$ (Micro) closed sets Ҡ$_1$, Ҡ$_2$ such that μ_{ic}-$ker\{r\} \subseteq$Ҡ$_1$, μ_{ic}-$ker\{r\} \cap$Ҡ$_2$ = $\emptyset$ *and* μ_{ic}-$ker\{s\} \subseteq$Ҡ$_2$, μ_{ic}-$ker\{s\} \cap$Ҡ$_1$ = $\emptyset$ *and* Ҡ$_1 \cup$Ҡ$_2$ = U, then Ҡ$_1^c$ *and* Ҡ$_2^c$ are (Micro)open sets $\ni$ Ҡ$_1^c \cap$ Ҡ$_2^c$ = $\emptyset$. Put Ҡ$_1^c$ = G_2 *and* Ҡ$_2^c$ = G_1. Thus μ_{ic}-$ker\{r\} \subseteq G_1$ *and* μ_{ic}-$ker\{s\} \subseteq G_2$ and $G_1 \cap G_2 = \emptyset$, so that $r \in G_1$ *and* $s \in G_2$ implies $r \notin \mu_{ic}$-$cl\{s\}$ *and* $s \notin \mu_{ic}$-$cl\{r\}$, then μ_{ic}-$cl\{r\} \subseteq G_1$ *and* μ_{ic}-$cl\{s\} \subseteq G_2$.Thus U is a μ_{ic}-R_1 space.

Corollary 3.3: μ_{ic}Topological space is a μ_{ic}-R_1 space iff $\forall\, r \neq s \in U$ with μ_{ic}-$cl\{r\} \neq \mu_{ic}$-$cl\{s\}$ then $\exists$disjoint (Micro)open sets Ҡ$_1$, Ҡ$_2$ such that μ_{ic}-cl(μ_{ic}-$k$er $\{r\}$) $\subseteq$ Ҡ$_1$ *and* μ_{ic}-cl(μ_{ic}-$k$er$\{s\}$) $\subseteq$ Ҡ$_2$.

Proof': Let U be a μ_{ic}-R_1space and let $r \neq s \in U$ with μ_{ic}-$cl\{x\} \neq \mu_{ic}$-$cl\{y\}$ then $\exists$ disjoint (Micro)open sets Ҡ$_1$, Ҡ$_2$ such that μ_{ic}-$cl\{r\} \subseteq$ Ҡ$_1$ *and* μ_{ic}-$cl\{s\} \subseteq$ Ҡ$_2$. Also $U$is μ_{ic}-R_0 space, implies $\forall\, r \in U$, then μ_{ic}-$cl\{r\}$ = μ_{ic}-$k$er$\{r\}$, but μ_{ic}-$cl\{r\}$= μ_{ic}-cl(μ_{ic}-$cl\{r\}$) = μ_{ic}-cl(μ_{ic}-$k$er$\{r\}$).Thus μ_{ic}-cl(μ_{ic}-$k$er$\{r\}$) $\subseteq$ Ҡ$_1$ and μ_{ic}-cl(μ_{ic}-$ker\{s\}$)$\subseteq$Ҡ$_2$. Contrariwise, Let $\forall\ r \neq s \in U$ with μ_{ic}-$cl\{r\} \neq \mu_{ic}$-$cl\{s\}$ then $\exists$ disjoint (Micro)open sets Ҡ$_1$, Ҡ$_2$ such that μ_{ic}-cl(μ_{ic}-$ker\{r\}$) $\subseteq$ Ҡ$_1$ *and* (μ_{ic}-$k$er$\{s\}$)$\subseteq$ Ҡ$_2$. Since $\{r\} \subseteq \mu_{ic}$-$k$er$\{r\}$ *then* μ_{ic}-$cl\{r\} \subseteq \mu_{ic}$-$cl$($\mu_{ic}$-$ker\{r\}$), $\forall\, r \in$Ҡ$_1$. So, we get μ_{ic}-$cl\{r\} \subseteq$Ҡ$_1$ *and* μ_{ic}-$cl\{s\} \subseteq$Ҡ$_2$. Therefore μ_{ic}- $\{r\} \neq \mu_{ic}$-$cl\{s\}$.

4. Some new results on the (Micro)-T_i Spaces (i = O, 1, 2)

Theorem 4.1: *If Ҡ$_1$ is μ_{ic}-T_0 space and Ҡ$_2$ $\subseteq$Ҡ$_1$ then Ҡ$_2$ is also μ_{ic}-T_0 space.*

Proof: Assume Ҡ$_1$ be μ_{ic}-T_0 space. To demonstrate that Ҡ$_2$ is μ_{ic}-T_0 space. Select distinct points. r, s from Ҡ$_2$ and $r \neq s$, since Ҡ$_2$ $\subseteq$ Ҡ$_1$, we have r, $s \in$ Ҡ$_1$.

But Ꝁ$_1$ is μ_{ic}-T$_0$ space, so ∃ (Micro) open Ꞙ ∋ $r \in$ Ꞙ & $s \notin$ Ꞙ. So Ꝁ$_2$∩Ꞙ (micro) open in Ꝁ$_2$ ∋ $r \in$ Ꝁ$_2$∩ Ꞙ & $s \notin$ Ꝁ$_2$∩Ꞙ. As a result, Ꝁ$_2$ is also a μ_{ic}-T$_0$ space.

Theorem 4.2: *A μ_{ic}Topological space is a μ_{ic}-T$_0$ space if and only if either $s \notin \mu_{ic}$-{r} or $r \notin \mu_{ic}$-ker{s}, ∀ $r \neq s \in U$.*

Proof': Let $U$is a μ_{ic}-T$_0$ space then ∀ $r \neq s \in U$, ∃ a (Micro) open set Ꞙ $s.t\ r \in$ Ꞙ, $s \notin$ Ꞙ $or\ r \notin$ Ꞙ, &$s \in$ Ꞙ. Thus, either $r \in$ Ꞙ,& $s \notin$ Ꞙ implies $s \notin \mu_{ic}$-ker$\{r\}$ or $r \notin$ Ꞙ, $s \in$ Ꞙ demonstrates $r \notin \mu_{ic}$-$ker\{s\}$. Contrariwise, Let either $s \notin \mu_{ic}$-ker$\{r\}$ or $r \notin \mu_{ic}$-$ker\{s\}$, ∀ $r \neq s \in U$. Then ∃ a (Micro)open set Ꞙ such that $r \in$ Ꞙ,& $s \notin$ Ꞙ $or\ r \notin$ Ꞙ,& $s \in$ Ꞙ, as result U μ_{ic}-T$_0$ space.

Theorem 4.3: *If Ꝁ$_1$ is μ_{ic}-T$_1$ space and Ꝁ$_2$ ⊆Ꝁ$_1$ then Ꝁ$_2$ is also μ_{ic}-T$_1$ space.*

Proof': Let Ꝁ$_1$ be μ_{ic}-T$_1$ space. To demonstrate that Ꝁ$_2$ is μ_{ic}-T$_1$space, let's Ꝁ$_2$ ⊆Ꝁ$_1$. Let's $r \neq s$ in Ꝁ$_2$, since Ꝁ$_2$ ⊆ Ꝁ$_1$, we possess r, s ⊆ Ꝁ$_1$. However, Ꝁ$_1$ is μ_{ic}-T$_1$space, Consequently, a (Micro)open sets Q_1,Q_2 ∋ $r \in Q_1$, $s \notin Q_1$, $s \in Q_2$, $r \notin Q_2$. Let N$_1$= Ꝁ$_2$∩Q_1 and N$_2$= Ꝁ$_2$∩Q_2. Then N$_1$ and N$_2$ are (Micro)open sets $r \in$ N$_1$ but $s \notin$ N$_1$, as a result Ꝁ$_2$ is μ_{ic}-T$_1$ space.

Theorem 4.4: *A μ_{ic}Topological space is a μ_{ic}-T$_1$ space iff ∀ $r \neq s \in U$. $s \notin \mu_{ic}$-ker{r} and $r \notin \mu_{ic}$- ker{s}*

Proof': Let U is a μ_{ic}-T$_1$ space then ∀ $r \neq s \in U$, ∃ a (Micro)open sets Q_1, Q_2 ∋ $r \in Q_1$, $s \notin Q_1$ or $s \in Q_2$, $r \notin Q_2$. Implies $s \notin \mu_{ic}$-ker$\{r\}$ and $r \notin \mu_{ic}$-$ker\{s\}$. Contrariwise, Let $s \notin \mu_{ic}$-ker$\{r\}$ and $r \notin \mu_{ic}$-$ker\{s\}$, ∀ $r \neq s \in U$. Then ∃ a (Micro) open sets Q_1, Q_2 such that $r \in Q_1$, $s \notin Q_1$ and $s \in Q_2$, $r \notin Q_2$. As a result, U is a μ_{ic}-T$_1$ space.

Theorem 4.5: *A μ_{ic}Topological space is a μ_{ic}-T$_1$ space iff ∀ $r \in U$ then μ_{ic}- ker {r}={r}.*

Proof': Let U is a μ_{ic}-T$_1$ space and let μ_{ic}-ker$\{r\} \neq \{r\}$, then μ_{ic}-$ker\{r\}$ includes a different point, say s, from r. So $s \in \mu_{ic}$-ker$\{r\}$, hence U is not a μ_{ic}-T$_1$ space. This is in inconsistency. Thus μ_{ic}-ker$\{r\}$=$\{r\}$. Contrariwise, Let μ_{ic}-ker$\{r\}$= $\{r\}$, ∀ $r \in U$ and let $U$is not a μ_{ic}-T$_1$ space. Then $s \in \mu_{ic}$-ker$\{r\}$, implies μ_{ic}-$ker\{r\} \neq \{r\}$, This is in inconsistency. As a result, U is a μ_{ic}-T$_1$ space.

Theorem 4.6: *A μ_{ic}Topological space is a μ_{ic}-T$_1$ space iff ∀ $r \neq s \in U$ implies μ_{ic}-ker{r} ∩ μ_{ic}-ker{s} = ∅.*

Proof': Let a μ_{ic}Topological space Ube a μ_{ic}-T$_1$ space. Then μ_{ic}-$ker\{r\}$ = $\{r\}$ and μ_{ic}-$ker\{s\}$= s. Thus μ_{ic}-$ker\{r\}$ ∩ μ_{ic}- $ker\{s\}$ = ∅. Contrariwise, Let ∀ $r \neq s \in$ U implies μ_{ic}-$ker\{r\}$∩ μ_{ic}-$ker\{s\}$ = ∅ and let $U$is not μ_{ic}-T$_1$ space Then ∀ $r \neq s \in U$ implies $s \in \mu_{ic}$-$ke\{r\}$ or $r \in \mu_{ic}$-$ker\{s\}$, The μ_{ic}-$ker\{r\}$ ∩ μ_{ic}-$ker\{s\} \neq$ ∅ . This is in inconsistency. As a result, U is a μ_{ic}-T$_1$ space.

Theorem 4.7: *Every μ_{ic}-T_2 space is μ_{ic}-T_1 space.*

Proof': Let U is μ_{ic}- T_2 space. Let $r \neq s \in U$. So, there are two (Micro) open sets $Ꮶ_1, Ꮶ_2$, ∋ $Ꮶ_1 \cap Ꮶ_2 = \emptyset$,$r \in Ꮶ_1$ but $s \notin Ꮶ_1$ and $s \in Ꮶ_2$ but $r \notin Ꮶ_2$ So, $Ꮶ_1$ and $Ꮶ_2$ are (Micro) open sets ∋ either $r \in Ꮶ_1$ but $s \notin Ꮶ_2$ or $s \in Ꮶ_2$ but $r \notin Ꮶ_1$. So U is μ_{ic}- T_1 space.

Here's an example demonstrating that the converse of 4.8 is not true in overall.

Example 4.8: Let $U = \{ƚ_1, ƚ_2, ƚ_3, ƚ_4, ƚ_5\}$, $U/R = \{\{ƚ_1\}, \{ƚ_2, ƚ_3, ƚ_4\}, \{ƚ_5\}\}$ and $X = \{ƚ_1, ƚ_2\} \subset U$. Then $\tau R(X) = \{U, \emptyset, \{ƚ_1\}, \{ƚ_1, ƚ_2, ƚ_3, ƚ_4\}, \{ƚ_2, ƚ_3, ƚ_4\}\}$. Then $\mu_{ic} = \{ƚ_5\}$. Then $\mu_{ic}(X) = \{U, \emptyset, \{ƚ_1\}, \{ƚ_5\}, \{ƚ_1, ƚ_5\}, \{ƚ_1, ƚ_2, ƚ_3, ƚ_4\}, \{ƚ_2, ƚ_3, ƚ_4\}, \{ƚ_2, ƚ_3, ƚ_4, ƚ_5\}\}$ Then U is μ_{ic}-T_1 space and not μ_{ic}-T_2 space

Theorem 4.9: *If $Ꮶ_1$ is μ_{ic}-T_2 space and $Ꮶ_2 \subseteq Ꮶ_1$ then $Ꮶ_2$ is also μ_{ic}-T_2 space.*

Proof': Let $Ꮶ_1$ be μ_{ic}- T_2 space. To demonstrate that $Ꮶ_2$ is μ_{ic}-T_2space, suppose $Ꮶ_2 \subset Ꮶ_1$, $r \neq s \in Ꮶ_2$. Since $Ꮶ_2 \subseteq Ꮶ_1$, we currently possess $r, s \in Ꮶ_1$. But $Ꮶ_1$ is μ_{ic}- T_2 space, so ∃ two (Micro)open sets Q_1 and Q_2 ∋ $Q_1 \cap Q_2 = \emptyset$, $r \in Q_1$, $s \in Q_2$. Thus, we have $Q_1 \cap Ꮶ_2$, $Q_2 \cap Ꮶ_2$ are (Micro)open sets in $Ꮶ_2$ ∋ $(Q_1 \cap Ꮶ_2) \cap (Q_2 \cap Ꮶ_2) = (Q_1 \cap Q_2) \cap Ꮶ_2 = \emptyset \cap Ꮶ_2 = \emptyset$. Also, $r \in Q_1 \cap Ꮶ_2$ and $s \in Q_2 \cap Ꮶ_2$, hence $Ꮶ_2$ is μ_{ic}-T_2 space.

Theorem 4.10: *A μ_{ic}-R_1space is μ_{ic}-T_2space iff at least one of these conditions applies:*(1) *∀ $r \in U$, μ_{ic}-$ker\{r\} = \{r\}$, (2) ∀ $r \neq s \in U$, μ_{ic}-$ker\{r\} \neq \mu_{ic}$- $ker\{s\} \Rightarrow \mu_{ic}$-$ker\{r\} \cap \mu_{ic}$-$ker\{s\} = \emptyset$, (3) ∀ $r \neq s \in U$, either $r \notin \mu_{ic}$-$ker\{s\}$ or $y \notin \mu_{ic}$-$ker\{r\}$, (4) ∀ $r \neq s \in U$, then $r \notin \mu_{ic}$-$ker\{s\}$ and $y \notin \mu_{ic}$-$ker\{r\}$.*

Proof : (1) Let Ube a μ_{ic}-T_2 space. Then $U$is a μ_{ic}-T_1 space and μ_{ic}-R_1 space. Hence, μ_{ic}-$ker\{r\} = \{r\}$ ∀ $r \in U$. Contrariwise, let ∀ $r \in U$, μ_{ic}-$ker\{r\} = \{r\}$, then, $U$is a μ_{ic}-T_1 space, also $U$is a μ_{ic}-R_1space, hence U is a μ_{ic}-T_2 space.

Proof: (2) Let U be a μ_{ic}-T_2 space. Then $U$is a μ_{ic}-T_1space.Hence μ_{ic}-$ker\{r\} \cap \{s\} = \emptyset$, ∀ $r \neq s \in U$. Contrariwise, Assume that ∀ $r \neq s \in$U, μ_{ic}-$ker\{r\} \neq \mu_{ic}$-$ker\{s\}$ implies μ_{ic}-$ker\{r\} \cap \mu_{ic}$-$ker\{s\} = \emptyset$, so the μ_{ic}Topological space is a μ_{ic}-T_1space, also $U$is a μ_{ic}- R_1 space. Hence, U is a μ_{ic}-T_2 space.

Proof: (3) Let $U$is a μ_{ic}-T_2 space .Then U is a μ_{ic}-T_0 space, hence $r \notin \mu_{ic}$-$ker\{s\}$ *or* $s \notin \mu_{ic}$-$ker\{r\}$ ∀ $r \neq s \in U$. Contrariwise, Assume that ∀ $r \neq s \in U$, *either* $r \notin \mu_{ic}$-$ker\{s\}$ *or* $s \notin \mu_{ic}$-$ker\{r\}$ ∀ $r \neq s \in U$.So, U is a μ_{ic}-T_0 space also $U$is μ_{ic}-R_1space, thus $U$is a μ_{ic}-T_2 space.

Proof: (4) Let Ube a μ_{ic}-T_2 space. Then U is a μ_{ic}-T_1 space and a μ_{ic}-R_1space, hence, $r \notin \mu_{ic}$-$ker\{s\}$ and $s \notin \mu_{ic}$-$ker\{r\}$. Contrariwise, Let ∀ $r \neq s \in U$ then $r \notin \mu_{ic}$-$ker\{s\}$ and $s \notin \mu_{ic}$-$ker\{r\}$. Then, $U$is a μ_{ic}-T_1space, also U is μ_{ic}-R_1space. Hence, U is a μ_{ic}-T_2space.

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