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Proposing an effective numerical algorithm based on analysis to solve two-dimensional Friedholm equations

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Abstract

In order to solve 2-Dimention Fredholm integral formulas, a brand-new and effective numerical approach based on interpolating scaled functions is devised. FIE simplifies to the set for algebraic equation which solved to approximate the solution using functional matrices of integral for combining scaling variables. A convergence method is examined, and certain numerical tests support the precision and applicability of the approach. We contrast the proposed strategy with others to demonstrate its effectiveness.

Subject Classification: *Primary 65Dxx, Secondary 65Rxx.*

Keywords: *Dimention friedholm integral equations, Scaled functions, The functional matrices of integral, Legendre equations.*

1. Introduction

The integral equations were the subject of numerous numerical studies. One of them is a powerful numerical technique constructed using block-pulse values [1]. For the purpose of resolving this kind of integral equation, Banifatemi, Razzaghi, and Yousefi, [2] employed a neural network strategy. An effective approach for solving the FIE system is being developed, and it uses a sparse mathematical model based on multiwavelets [3]. Many numerical approaches are described and examined in [4] in order to solve the computational solution of the FIE. We direct readers to a few similar titles, the development and analysis of numerical methods to address two-dimensional FIE

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has been the subject of numerous articles. The integral median value theorem is used to create a novel method in [5] that resolves the issue. To solve 2D-FEI, the Wang and Wang [6] suggested an iterative technique based on the quadrature formula. In [7], a numerical technique based on the creation of dimensional triangle function orthogonal is used. Problem (1) was solved by Karamete and Sezer [8] using the Haar wavelet approach. The method's main benefit is that, in contrast to previous approaches, it's not using numerical integral. [9] proposes an Euler-type approach to address consideration issues.

For example [5, 6, 10], numerous numerical methods have been suggested for solving limited IEs. The authors of [7] introduced a method based on for solving MV-F-IEs using Chebyshev polynomials and the moving least squares method. To obtain a precise numerical answer to MV-F-IEs, the authors in [8] used the Legender dispersion technique. Whilst, to computationally solve the MV-F-IEs, multiquadric basis functions with radial bases have been examined. To explore two - dimensional MV-F-IEs, authors recently introduced and developed the three-dimensional functions approach in [11]. To get the numerical answers of MV-F-IEs, in [12, 13] use the Taylor & Lagrange interpolation technique. Additionally, different numerical approaches for MVF-IEs were researched.

In the past forty years, spectral techniques [14-17] were widely applied in variety for fields. In the past, the few scopes, including straightforward geometric region &periodic boundary circumstances, has employed spectral approaches based on Fourier expansions. They are now being used as potent strategies to handle a variety of issues, including optimum control, variational challenges, integral, integro-differential formulas, fractional differential equation, & nonlinear as well as linear differential equations. Where compared to other numerical techniques, a spectrum techniques have a premium quality based on completeness&exponential average for convergence. A core component for all spectral approaches is formulation for issue solution as the finite series for many function. A coefficients of variation will then be selected to minimize the total error as much as possible. While the spectral clustering technique's computational solution will be used to almost fulfill the differential equations. but, at a selection points, the residuals can be zero. Due to its evident benefits, A family strategy is effectively used in the wide variety for scientific &engineering fields. Spectral aggregation techniques are promising options for solving fractional differential equations as their global character fits well with the nonlocal definition of fractional operators.

In order to resolve the multivariate Fredholm integral equations, Mirzaee [11] suggested numerical approach depend on Legendary scaling function. A precise method for resolving the 2D Fredholm integral issue utilizing cosine and trigonometrical functions is proposed in [12]. The issue is resolved in [13] using an iterative patchy least-squares approach. A number of equations can be solved using multiwavelets as an intriguing starting point [15–17]. Wavelet characteristics including orthogonality, fading moments, and compacted support also apply to multiwavelets. They high smoothing & high approximation order

along with short support, in contrast to wavelets that are and biorthogonal wavelets that are [18]. In contrast to biorthogonal wavelets [19-25].

2. Scaling The interpolation of Functions

Let J ∈ Z and ∪ 0. The subintervals X J,b [xb, xb+1] are given by the point xb (b/(2J)), with B 0,..., 2J 1. We then have a look at the homogeneous finite discretizations [20] bBXJ,b. We introduce the subdomain Vr J as space for piece-wise quadratic bases with thedegree r lower the redundancy parameter for k R 0, 1, r1

$$V_J^r \coloneqq \text{Span}\left\{\phi_{j.b}^k \coloneqq \mathcal{D}_{2^j}\mathcal{T}_b\phi^k . b \in \mathcal{B}_j . k \in \mathcal{R}\right\} \subset L^2(\Omega); r \geq 0 \tag{1}$$

T, D stand for the workers for translations & reduction, in turn, and k k kR stands for the Hacia [4] proposed primal interpolating scales bases. A roots for a Legendre equation with degree r are nodes k k kR, and the combining scale bases are specified as:

$$\phi^k(x) = \begin{cases} \sqrt{\frac{2}{\omega_k}} L_k(2x-1) & x \in \Omega \\ 0 & \text{o.w,} \end{cases} \tag{2}$$

Where the Gauss-Legendre summation weights at the k and k R are the Lagrangian interpolating polynomials, respectively [1, 4]. With regard to the L2-inner product, these bases create orthonormal basis on. The spaces Vr j jZ+ 0 dimension N 2Jr &manifestly nested, pursuant to the description of the space Vr J.

$$V_0^r \subset V_1^r \subset \cdots \subset V_J^r \subset \cdots \subset L^2(\Omega) \tag{3}$$

Each function with the form p L2() can be expressed.

$$p = \sum_{b\in\mathcal{B}_J} \sum_{k\in\mathcal{R}} p_{J.b}^k \phi_{J.b}^k \tag{4}$$

where Pr J denotes the straight-line projection of L2() onto space Vr Jin order to determine the coefficients for pk J,b the fact that, k J,b determine. We will compute the integrals for X J; b f(x)k J,b(x)dx. To avoid problematic integrals, this r-point G-Legendre algorithm is used. by appropriately selecting the weights and nodes for k R [2]. Via

$$p^k{}_{t.b} \approx 2^{-\frac{J}{2}} \sqrt{\frac{\omega_k}{2}} p\left(2^{-J}\left(\frac{\tau_k+1}{2}+b\right)\right). k \in \mathcal{R}. b \in \mathcal{B}_J \tag{5}$$

Investigated is the convergence analysis of the projected Pr J(p) for the r-times constantly differentiable function p Cr [1, 2].

$$\left\|\mathcal{P}_J^r(p) - p\right\| \leq 2^{-Jr} \frac{2}{4^r r!} \sup_{x\in[0.1]} \left|p^{(r)}(x)\right|. \tag{6}$$

We direct readers to [21] for further information and the complete verification of this estimate. Pr J(p) ends to p &rate for converging of O(2 Jr), according to *us.

Assume the vector product r J [r,J,0,..., r,J,2J1]T with r,J,b [0 J,b,..., r J,b 1] contains scaling operations & referred to as multiscaling value. The vector P,

which contains the entries Pbr+k+1 pk J,b, can be used to rewrite estimate (5) as follows:

$$\mathcal{P}_j^r(p) = P^t \theta_j^r \tag{7}$$

Each of the components of these base constructions can be used to mimic an equation of greater dimensions, whereby P represents an N-dimensional column [10]. This can be accomplished by introducing the two-dimensional in nature subspace Vr, J 2 Vr J Vr J L2() is:

$$\left\{\phi_{J.b}^k \phi_{J.b'}^{k'} : b.b' \in \mathcal{B}_J . k.k' \in \mathcal{R}\right\}. \tag{8}$$

On the basis of this supposition, we can use the projection operators Pr J to construct an approximation of the variable p L2.

$$p(s.t) \approx \mathcal{P}_J^r(p)(s.t) = \Phi_J^{rT}(s) P \Phi_J^r(t). \tag{9}$$

where elements of the order N square matrices P are obtained by:

$$p(s.t) \approx \mathcal{P}_J^r(p)(s.t) = \Phi_J^{rT}(s) P \Phi_J^r(t). \tag{10}$$

Here ((k + 1)/2) is k. Take into consideration p's 2r-th partial derivatives: R is continuous [3, 10]. Using this premise, the error of this estimate can be constrained to the following:

$$\|\mathcal{P}_J^r p - p\| \leq \mathcal{M}_{max} \frac{2^{1-rJ}}{4^r r!}\left(2 + \frac{2^{1-Jr}}{4^r r!}\right) \tag{11}$$

The 2-dimensional Friedholm integral equation's Gallerken wave is presented in operator format so that the solution can be expressed as follows:

$$(I - R)w = f \tag{12}$$

in which one is the Fredholm player and I is the identity operator.

$$\mathcal{K}(w)(xy) := \int_0^1 \int_0^1 k(x.y.s.t) g\big(w(s.t)\big) \mathrm{d}s\mathrm{d}t \tag{13}$$

on L2(D) is taken to be compact.

Assume that the integrating scaling functions may be expanded to roughly represent the unique solution as:

$$w(x.y) \approx \mathcal{P}_J^r(w)(x.y) = \Phi_J^{rT}(x) W \Phi_J^r(y) \tag{14}$$

The projection operator Pr J projects L2(D) on the domain Vr J, and W a N N matrix which is components is located. Additionally, we can express function f using the same expansion [10, 22].

The Galerkin technique is used to figure out the coefficient W after substituting *ese into (13). Introduce the residue as needed afterwards.

$$r_J^r := \left(I - \lambda \mathcal{K}_J^r\right) w_J^r - f_J^r \tag{15}$$

wr J Pr J(w), f J Pr J(f), are the variables. must rr J, [r,J]i[r,J]j 0, for i, j 1: N. according to Galerkin technique. the study a set of our equs involving algebra to arrive at the approximated solution.

$$\left\langle \left(I - \lambda \mathcal{K}_J^r\right) w_J^r . \left[\Phi_J^r\right]_i \left[\Phi_J^r\right]_j \right\rangle = 0. \tag{16}$$

It relies on the value of g whether this system is either nonlinear or linear therefore how to approximation K(w) is a crucial stage in this procedure [5].

$$k(x.y.s.t)g\big(w(s.t)\big) := \Phi_J^r(s)\Gamma(x.y)\Phi_J^r(t).(x.y.s.t) \in \mathcal{S}. \tag{17}$$

If (x, y) is a N by N matrix, and (10) provides the entries. In place of (13), one can write (17) like:

$$\begin{aligned}\mathcal{K}(w)(x.y) &:= \int_0^1 \int_0^1 k(x.y.s.t)g(w(s.t))\mathrm{d}s\mathrm{d}t \\ &= \int_0^1 \int_0^1 \Phi_J^r(s)\Gamma(x.y)\Phi_J^r(t)\mathrm{d}s\mathrm{d}t \\ &= \Big(I_\phi \Phi_J^r(1)\Big)^T \Gamma(x.y)\big(I_\phi \Phi_J^r(1)\big) \\ &= \Phi_J^r(x) G \Phi_J^r(y)\end{aligned} \tag{18}$$

where I is the operationally fundamental matrix for scaling factor interpolation established G is a N N matrix [5, 22]. This approximate representation allows system (17) to be expressed as:

$$W - G = F \tag{19}$$

3. Affinities Analysis Research

Theory 3.1: *Assume that the sequence Kr J: L2(S) L2(S) is converge from point to K and collectively consolidated, and that I K is an injection-dependent factor [6]. Since (I Kr J)1 is consistently finite and exists, the answer to (13) fulfills the error estimate.*

$$\|w - w_J\| \le \frac{\left\|\left(I-\mathcal{K}_J^r\right)^{-1}\right\|}{1-\varepsilon_{J_0}\left\|\left(I-\mathcal{K}_J^r\right)^{-1}\right\|}\left(\|w - \mathcal{P}_J^r w\| + \left\|\left(\mathcal{P}_J^r\mathcal{K} - \mathcal{P}_J^r\mathcal{K}_J^r\right)w\right\|\right). \tag{20}$$

Proof: In light of the theorem's theory, one can say that:

$$\| \left(\mathcal{F}_J^r - \mathcal{F}'\mathcal{F}_J^r\right) \| \longrightarrow 0. \text{as } J \longrightarrow \infty. \tag{21}$$

For all enough J, we may write using (21):

$$\left\|(I-\mathcal{K})^{-1}\left(\mathcal{K}_J^r - \mathcal{K}\right)\mathcal{K}_J^r\right\| < 1 \tag{22}$$

I Kr J is hence reversible as a result. Choose JJ0 so that:

$$\varepsilon_{J_0} \equiv \sup_{J\ge J_0}\left\|\mathcal{K}_J^r - \mathcal{P}_J^r\mathcal{K}_J^r\right\| < \frac{1}{\left\|\left(I-\mathcal{K}_J^r\right)^{-1}\right\|} \tag{23}$$

(I + (I Kr J) 1 (Kr J Pr JKr J)) 1 exists & is consistently bounded as a result of the geometric sequence theorem, that is,

$$\left\|\left(I + \left(I-\mathcal{K}_J^r\right)^{-1}\left(\mathcal{K}_J^r - \mathcal{P}_J^r\mathcal{K}_J^r\right)\right)^{-1}\right\| \le \frac{1}{1-\varepsilon_{J_0}\left\|\left(I-\mathcal{K}_J^r\right)^{-1}\right\|} \tag{24}$$

The estimate of I Kr J by I Pr JKr J serves as the foundation for the study of convergence analyses.

$$\begin{aligned}I - \mathcal{P}_J^r\mathcal{K}_J^r &= \left(I-\mathcal{K}_J^r\right) + \left(\mathcal{K}_J^r - \mathcal{P}_J^r\mathcal{K}_J^r\right) \\ &= \left(I-\mathcal{K}_J^r\right)\left(I + \left(I-\mathcal{K}_J^r\right)^{-1}\left(\mathcal{K}_J^r - \mathcal{P}_J^r\mathcal{K}_J^r\right)\right).\end{aligned} \tag{25}$$

Using the norm of (25) and (24), one may get:

$$\left\|\left(I-\mathcal{P}_J^r\mathcal{K}_J^r\right)^{-1}\right\| \leq \frac{\left\|\left(I-\mathcal{K}_J^r\right)^{-1}\right\|}{1-\varepsilon_{J_0}\left\|\left(I-\mathcal{K}_J^r\right)^{-1}\right\|} \tag{26}$$

Using the function Pr J to (13)'s both sides, then rearranging to obtain.

$$\left(I-\mathcal{P}_J^r\mathcal{K}_J^r\right)w = \mathcal{P}_J^r f + \left(w-\mathcal{P}_J^r w\right) + \left(\mathcal{P}_J^r\mathcal{K}-\mathcal{P}_J^r\mathcal{K}_J^r\right)w \tag{27}$$

When we take (27) out of the initial Equation (12), we get:

$$\left(I-\mathcal{P}_J^r\mathcal{K}_J^r\right)\left(w-w_J\right) = \left(w-\mathcal{P}_J^r w\right) + \left(\mathcal{P}_J^r\mathcal{K}-\mathcal{P}_J^r\mathcal{K}_J^r\right)w \tag{28}$$

if wJ be series for continous function, with the result wJ w as J . because Pr J 1 is satisfied by the orthonormal representation, we may obtain

$$\begin{aligned}\|w-\mathcal{P}_J^r w\| &\leq \|w-w_J\| + \|w_J-\mathcal{P}_J^r w_J\| + \|\mathcal{P}_J^r\left(w-w_J\right)\| \\ &\leq 2\|w-w_J\| + \|w_J-\mathcal{P}_J^r w_J\|.\end{aligned} \tag{29}$$

4. Example Application

To show off the plan's precision and effectiveness. The greatest absolute value mistake in this section

$$e_j = \left|w-w_j\right| \tag{30}$$

and L2-error are utilized to demonstrate the suggested method's precision [7, 9]. Let's have a look at the aforementioned 2-dimensional linear Fredholm integrated equation:

$$\begin{aligned}k(x.y.s.t) &:= \frac{x}{(8+y)(1+s+t)} \\ f(x.y) &:= (1+x+y)^{-2} - \frac{x}{48+6y}.\end{aligned} \tag{31}$$

W(x, Y) ($\frac{1}{(1+x+y)2}$) is the exact solution [5, 7]. In Table 1, the proposed approach is contrasted with the integration mean values theorems [5] & the 2D-TFs methods [7]. As can be seen from the findings in Table 1, the suggested technique is more accurate than other methods. Where r 4, J:2, Figure1 depicts the surfaces for approximation & absolute error [12]. We display the impact for parameter r & J on L2-error in Figure 2. Its evident decreasing error requires raising the parameters J and r.

Table 1
Maximum absolute value inaccuracy in relation to others

r	*R,j=(4,3)*	*Y=32*	*M=* 32=N=y	*R,j=(3,4)*	*Y=32*	*M=* 32=N=y
(0.2, 0.19)	2.10e - 06	3.61e	8.60e - 03	6.24e	3.61e	8.60e - 03
(0.4, 0.4)	3.80e - 07	5.04e	9.08e - 03	1.43e - 06	5.04e	9.08e - 03
(0.6, 0.6)	1.47e - 07	7.41e	1.60e - 04	5.89e - 07	7.41e	1.60e - 04
(0.8, 0.8)	1.56e - 08	9.65e	2.64e - 04	3.67e - 07	9.65e	2.64e - 04

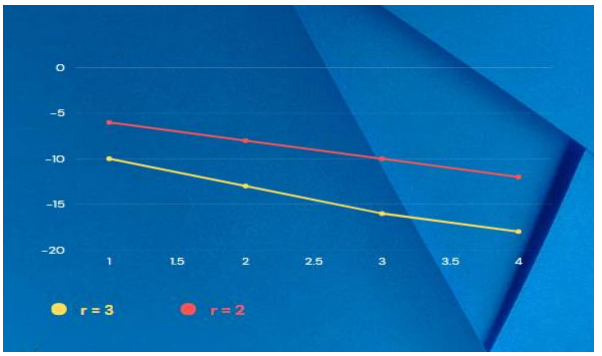


Figure 1

org Effects of the refining level J and the number of parameters parameter r

4.1 *Second Application*

Consider the subsequent two-dimensional linear FIE now

$$w(x.y) = \frac{(\cos(1))^3 xy^2}{36} - \frac{\cos(1)xy^2}{12} + \frac{xy^2}{18} + \sin(y) - \int_0^1 \int_0^1 \frac{xy^2 s}{6} w^3(s.t)\mathrm{d}s\mathrm{d}t \tag{32}$$

It shows that the proposed method provides significantly greater accuracy while requiring less calculations at r 4 and J 2 [13, 23],

The consequences for optimization level J & the multiplicity parameters r are illustrated of the example. In Table2, the numerical order of convergence is shown for application 2. The exact answer for w(x, y) sin(y)) [24] .

Table 2

Correlation of the suggested strategy for application 2 with the greatest absolute value mistakes

	(0.1, 0.1)	(0.3, 0.3)	(0.5, 0.5)	(0.7, 0.7)	(0.9, 0.9)
WGM ($r = 7$, $J = 1$)	1.39e - 10	9.62e - 12	2.97e - 10	1.88e - 09	3.82e
(n equal to 100)	8.20e - 9	2.24e - 08	1.03e - 06	2.81e	5.95e

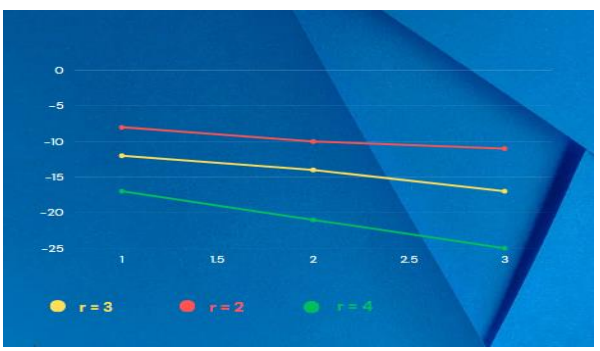


Figure 2

Impact for fineness level J & the multiplicity variable r of application 2

4.2 *Third Application*

Take taken into account the next nonlinear 2-Dimention -FIE supplied:

$$w(x.y) - \int_0^1 \int_0^1 \frac{x(1-s^2)}{(y+1)(t^2+1)}\left(1 - e^{-w(s.t)}\right)\mathrm{d}s\mathrm{d}t = -\log\left(1 + \frac{xy}{y^2+1}\right) + \frac{x}{16(y+1)} \tag{33}$$

The exact answer is provided by:

$$w(x.y) = -\log\left(1 + \frac{xy}{y^2+1}\right) \tag{34}$$

The computational results of the suggested technique,

Table 3
Maximum absolute value errors are compared to those of others

J	1	2	3	4	5
Nystrom method	1.4e	3.4e	8.3e	2.1e	5.2e
WGS	7.6e	4.7e	2.9e	1.8e	1.1e
Haar method	2.6e	7.3e	1.9e	5.0e	1.3e

5. Conclusion

The approximation of the 2-Dimention Fredholm integral problems is obtained by interpolating scaled functions. An investigation for convergence analysis was done, and numerical outcomes are assessed against current approaches. A results demonstrate proposed approach is substantially more accurate than existing methods and that it is also lot more adaptable. Contrary to bases like these and Legendre in time, it is not required to increase the complexity of polynomials to increase precision. We can boost accuracy by raising J's level of fineness. Additionally, the interpolation characteristic of these bases allows us to avoid using the direct summation to obtain the coefficients, which lowers the computing cost.

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