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$\acute{P}$-compact and $\acute{P}$-closed spaces and some of its properties

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Abstract

This work introduces a new class of spaces called $\acute{P}$-compact and $\acute{P}$-closed spaces, and various characterizations of $\acute{P}$-set and $\acute{P}$-compact subspace in topological spaces.

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Keywords: *$\acute{P}$-compact, $\acute{P}$-closed spaces, $\acute{P}$-set and $\acute{P}$-compact subspace.*

1. Introduction

Not just general topology, also other fields of mathematics. Nowadays, compactness indicates the most major, advantageous and substantial concept. Additionally, the main characteristics of compactness have been discussed completely by many academics' researchers. However, there are many results in the academic institutions educational books concerning the analysis and general topology. Moreover, many mathematicians have ended their researches with particular conclusions concerning this topic. The class of semi-generalized closed sets was established by [8].A subset K of a space $(X^*,\mathcal{T})$ is said to be a semi open (shortened: S-O) set [7] (resp. Regular open (shortened: R-O)set [3], [14] if K $\subseteq$ c l (int(K))(resp, K=int (cl(K)). A semi open set D of topological space $(X^*,\mathcal{T})$ is said to be $\boldsymbol{S_{gs}}^*$ -open[16], if for each $x \in D$, $\exists$ a gs*-closed set F s.t. $x \in F \subseteq D$,then $D \subseteq X^*$is $\boldsymbol{S_{gs}}^*$ -closed set ,if W^c is an $\boldsymbol{S_{gs}}^*$ -open set. A topological space ($X^*,\mathcal{T}$) is said to be almost regular[13], [1] (resp.s-regula [3], [4]) if and only if for each R-O set (resp. open) set M containing d,

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∃ a open (resp. S-O)set H^* s.t. $d \in H^* \subseteq cl(H^*) \subseteq$ M(resp. $d \in H^* \subseteq scl(H^*) \subseteq$ M)**.** If every open subset of X^* is closed, then X^* is called locally indiscrete (shortened: Loc-ind.) [6]. The space X^* is called to be semi compact [9], [12], [15] (resp. nearly compact [10], [2] if every semi open (resp. regular open) cover of X^* has a finite subcover. The researchers adopted a new category of spaces that are known as P′-compact and P′-closed spaces based on the notion of $S_{gs}{}^*$-closed sets, obtaining new characterizations of P′-compact and P′-Closed spaces, as well as more preservation theorems. Many researchers have studied the compactness property, and some conclusions have been drawn from their work recently, see:[5], [11, 17]). In this paper we refer by topological space, open cover, closed cover, finite subcover, such that, semi open, regular open (for shutting by T.SP, O.C.,CL.C.,F.S.C.,s.t .,S-O.,R-O.)

2. Preliminaries

Now go over the definitions that are relevant in the research and will be utilized in the subsequent parts:

Definition 1: [16] f: $(X^*, \mathcal{T}) \to (Y^*, \mathcal{S})$ is said to be $S_{gs}{}^*$-continuous if the inverse image $f^{-1}(\mathcal{P})$ is a $S_{gs}{}^*$-O set in X^* for each open set $\mathcal{P}$ of Y^*.

Definition 2: [16] f : $(X^*,\mathcal{T}) \to (Y^*, \mathcal{S})$ is said to be $\boldsymbol{S_{sg}}^*$-irresolute,if for each $S_{gs}{}^*$-O set $\mathcal{K}$ of Y^*, the inverse image $f^{-1}(\mathcal{K})$ is a $S_{gs}{}^*$-O set in X^*.

Definition 3: [16] f: $(X^*, \mathcal{T}) \to (Y^*, \mathcal{S})$ is said to be almost $\boldsymbol{S_{gs}}^*$-continuous function ,if for each regular open set V of Y^*, the inverse image $f^{-1}(V)$ is a $\boldsymbol{S_{gs}}^*$-O set in X^*.

Proposition 4: [16] Every S-O set is a $\boldsymbol{S_{gs}}^*$- O set if $(X^*,\mathcal{T})$is Loc. Ind.

Proposition 5: [16] If D is a clopen, $D \subseteq X^*$ and P is $\boldsymbol{S_{gs}}^*$ - open in X^***,** then $D \cap P$ is $\boldsymbol{S_{gs}}^*$ - open set in X^*.

Proposition 6: [7] Let N be a subset of T. SP $(X^*, \mathcal{T})$, if N$\in \mathcal{T}$ then scl(N)=int cl(N)

3. $\acute{\boldsymbol{P}}$- Compact and $\acute{\boldsymbol{P}}$-Closed spaces

Definition 7: A collection $\{U_\alpha : \alpha \in \Lambda\}$ of $S_{gs}{}^*$- open in a T.SP $(X^*, \mathcal{T})$ is called $S_{gs}{}^*$- open cover(briefly, $S_{gs}{}^*$ **-**O.C)of a subset p^* of X^* if $p^* \subseteq \{U_\alpha : \alpha \in \Lambda\}$.

Definition 8: A T. SP $(X^*, \mathcal{T})$ is said to be $\acute{P}$ -compact spaces if for every $S_{gs}{}^*$ -O.C $\{U_\alpha : \alpha \in \Lambda\}$ of X^*, ∃ a finite $\Lambda_\circ$ subset of Λ s.t. $X^* = \cup \{U_\alpha : \alpha \in \Lambda_\circ\}$.

Definition 9: A T.SP $(X^*, \mathcal{T})$ is said to be $\acute{P}$ - closed spaces if for every $\boldsymbol{S_{gs}}^*$ - O.C $\{U_\alpha : \alpha \in \Lambda\}$ of X^*, ∃ a finite $\Lambda_\circ$ subset of Λ s.t. $X^* = \cup \{S_{gs}{}^* clU_\alpha : \alpha \in \Lambda_\circ\}$

Proposition 10: If every gs^*-*CL.C.* of a space X^* has a *F.S.C,* then X^* is $\boldsymbol{\acute{P}}$ – compact

Proof: Assuming that $b^* \in X$ and $\{U_\alpha : \alpha \in \Lambda\}$ be any $S_{gs}{}^*$ -O.C of X^*, after that for each $b^* \in U_{\alpha(b^*)}$, ∃ a sg*-closed set $G_{\alpha(b^*)}$ s.t $b^* \in G_{\alpha(\upsilon)} \subseteq U_{\alpha(b^*)}$, then the family $\{G_{\alpha(b^*)} : \upsilon \in X^*\}$ is a sg*- CL.C.of X^*. By hypothesis $\{G_{\alpha(\upsilon)} : \upsilon \in X^*\}$ has F.S.C. $X^* = \cup \{G_{\alpha(b^* i)} : i = 1,2 \ldots . n\} \subseteq \cup \{U_{\alpha(b^* i)} : i = 1,2,3 \ldots\ldots ,n\}$.Thus X^* is $\acute{P}$ -compact. Then $X^* = \{U_{\alpha(b^* i)} : i = 1,2,3 \ldots ,n\}$

Proposition 11:

1. If $(X^*, \mathcal{T})$ is semi-compact ,then it is an $\acute{P}$-compact spaces.
2. If $(X^*, \mathcal{T})$ is s-closed, then it is an Ṕ-closed spaces.

Proof:

1. Let $\{U_\alpha : \alpha \in \Lambda\}$ be any $S_{gs}{}^*$- O.C. of X^*.Then $\{U_\alpha : \alpha \in \Lambda\}$ semi O.C. of X^*. Because is $X^{\cdot}$ is semi-compact , $sclU_\alpha \subseteq S_{gs}{}^* clU_\alpha$ for each $\alpha \in \Lambda$, hence ∃ a finite $\Lambda_\circ$ subset of Λ s.t. $X^* = \cup \{U_\alpha : \alpha \in \Lambda_\circ\} \subseteq \cup \{S_{gs}{}^* clU : \alpha \in \Lambda_\circ\}$,then $X^{\cdot}$ is $\acute{P}$ -compact spaces.
2. Let $\{M_\alpha : \alpha \in \Lambda\}$ be any $S_{gs}{}^*$- O.C. of X^*.Then $\{M_\alpha : \alpha \in \Lambda\}$ semi O.C. of X^*. Since $X^{\cdot}$ is s-closed ,since $sclM_\alpha \subseteq S_{gs}{}^* clM_\alpha$ for each $\alpha \in \Lambda$,hence ∃ a finite $\Lambda_\circ$ subset of Λ s.t. $X^* = \cup \{sclM_\alpha : \alpha \in \Lambda_\circ\} \subseteq \cup \{S_{gs}{}^* clM_\alpha : \alpha \in \Lambda_\circ\}$),then X^* is $\acute{P}$ -closed spaces.

The succeeding example shows Proposition (11) part (1) converse to be true in general

Example 12: Let X^*=IR and $\mathcal{J} = \{p \in \text{IR}; p < 0\} \cup \{X, \varphi\}$, $SO(X^*) = \{p \in \text{IR}; p < 0\} \cup \{X^*, \varphi\}$ and $S_{gs}{}^* O(X^*) = \{X^*, \varphi\}$. Then $(X^*, \mathcal{J})$ is an $\acute{P}$ - compact (resp. $\acute{P}$ -closed) space, but not s-compact.

Proposition 13: If X^* is an s-regular and $S_{gs}{}^*$- closed and T_1- space, then So X^* is $\acute{P}$-compact

Proof: Let $\{B_\alpha : \alpha \in \Lambda\}$ be any O.C. of an s-regular and $S_{gs}{}^*$- closed and T_1- space,then for each $z \in X^*$ ∃ $\alpha(z) \in \Lambda$ s.t. $z \in B_{\alpha(Z)}$. Since X^* is an semi-regular and T_1- space ,there exist, $P_z \in SO(X^{*\cdot})$ and $z \in P_z \subseteq Scl\ P_z \subseteq B_{\alpha(Z)}$,then $P_z \in S_{gs}{}^* O(X^*)$ and $S_{gs}{}^* cl\ P_z \subseteq B_{\alpha(Z)}$, implies that the family$\{P_z : z \in X^*\}$ is an $S_{gs}{}^{*}$- O.C. of X^* . Because X^* is an $S_{gs}{}^{*}$ - closed space, { P_{zi}: i=1,2……n} is subfamily s.t. $X^* = \bigcup_{i=1}^{n} S_{gs}{}^{*}cl P_{zi} \subseteq \bigcup_{i=1}^{n} B_{\alpha(Zi)}$. Therefore X^* is $\acute{P}$-compact.

The succeeding example clarifies needles of proposition (13) converse to be generally true

Example 14: In Example **(12)**, since the only non –empty $S_{gs}{}^{*}$- open set in(X^*,$\mathcal{J}$) is X^* itself, (X^*,$\mathcal{J}$)is neither s-regular nor -compact ,but (X^*,$\mathcal{J}$) is $S_{gs}{}^{*}$- closed.

Proposition 15: If $(X^*, \mathcal{J})$ is Loc. Ind and X^* is $\acute{P}$-compact, then it is semi-compact.

Proof: Concludes with Proposition 4**.**

Proposition 16: If X^* is $\acute{P}$ **-**compact and almost regular space , then $X^*$is compact.

Proof: Let $\{\beta_\alpha : \alpha \in \Lambda\}$ be any regular O.C. of X^*, since X^* is an almost regular space, then for any g∈ X^* and each regular open set $\beta_{\alpha(g)}$, then ∃an open set E_g s.t. $g \in E_g \subseteq clE_g \subseteq \beta_{\alpha(g)}$,but clE_g is regular closed for any g∈ $X^{\cdot}$,it followes X^* =$\bigcup_{g\in X} clE_g$ =$\bigcup_{\alpha(g)\in\Lambda} U_{\alpha(g)}$and {$E_g$: g∈ X }is an $S_{gs}{}^{*}$- O.C. of X^*. Since X^* is $\acute{P}$ **-**compact, then {clE_{gi}: i = 1, 2,..n} is subfamily s.t. $X^{\cdot}$ = $\bigcup_{i=1}^{n} clE_{gi} \subseteq \bigcup_{i=1}^{n} \beta_{\alpha(gi)}$. Therefore $X^{\cdot}$ is compact.

Proposition 17: If X^* be a *s*- regular -closed space, then X^* is $\acute{P}$ –compact

Proof: Let $\{Y_\alpha : \alpha \in \Lambda\}$ be any $S_{gs}{}^{*}$- O.C. of X^*, then Y_α is semi-open for each $\alpha \in \Lambda$. Since X^* be a regular semi for each $m \in X^*$ and $Y_{\alpha(m)}$, then ∃ a semi-open set D_m s.t. $m \in D_m \subseteq sclD_m \subseteq Y_{\alpha(m)}$. So, the family $\{ D_m : m \in X^* \}$ is a semi-O.C. of $X^{\cdot}$. Since X^* is an s-closed space, then $\{D_{(mi)}$: i = 1,2,..n } is subfamily s.t. $X^* = \bigcup_{i=1}^{n} sclD_{(mi)} \subseteq \bigcup_{i=1}^{n} Y_{\alpha(m)}$.Therefore X^* is a $\acute{P}$ -compact.

Proposition 18: If *(X^*,$\mathcal{J}$)* is T_1- space and $S_{gs}{}^{*}$**-** closed, then it is nearly compact.

Proof: Let $\{Z_\alpha : \alpha \in \Lambda\}$ be O.C. of $X^{\cdot}$. Then by Proposition (6), we have $\text{int cl}Z_\alpha = S_{gs}{}^{*}\text{cl}Z$ for each $\alpha \in \Lambda$. Since X is $S_{gs}{}^{*}$- closed space ,∃ a finite $\Lambda_\circ$ subset of Λ s.t. X^* =$\cup\{int\ clZ_\alpha : \alpha \in \Lambda_\circ\}$. So X^* is nearly compact.

Proposition 19: Let H:$X^* \to Y^*$ be a surjective almost $S_{gs}{}^{*}$**-** continuous function. If X^* is $\acute{P}$ -compact, then Y^* is nearly compact**.**

Proof: Let $\{U_\alpha: \alpha \in \Lambda \}$ be any regular O.C. of Y^*.Since H is almost $S_{gs}{}^*$ - continuous function, then $H^{-1}(U_\alpha)$ is $S_{gs}{}^*$-open in X^* for each $\alpha \in \Lambda$, Since H is surjective ,Then $H(X^*) = Y^*{=}\cup\{H^{-1}(U_\alpha): \alpha \in \Lambda\}$and $\{H^{-1}(U_\alpha): \alpha \in \Lambda\}$is an $S_{gs}{}^*$-O.C. of X^* .Since X^* is $S_{gs}{}^*$- compact, then $\exists$ a finite $\Lambda_\circ$ subset of Λ s.t. $X^* = \cup\{H^{-1}(U_\alpha): \alpha \in \Lambda_\circ\}$ .Then $Y^* = \cup \{U_\alpha: \alpha \in \Lambda_\circ \}$ .After which Y^* is nearly compact.

Proposition 20: Let $d{:}X^* \to Y^*$ be a surjective $S_{gs}{}^*$-continuous function. If X^* is $\acute{P}$-compact, then Y^* is compact.

Proof: First let $\{t_\alpha: \alpha \in \Lambda\}$ be O.C. of Y^* .Since P^* is $S_{gs}{}^*$ -continuous function, so by Definition (7),$d^{-1}(t_\alpha)$ is $S_{gs}{}^*$-open in X^* for each $\alpha \in \Lambda$, So that $\{t_\alpha: \alpha \in \Lambda \}$is $S_{gs}{}^*$-O.C. of X^*.As X^* is $\acute{P}$ - compact, then $\exists$ a finite $\Lambda_\circ$ subset of Λ s.t. $X^* = \cup\{d^{-1}(t_\alpha): \alpha \in \Lambda_\circ\}$, then $Y^* = d\ (X^*) = d(\cup \{d^{-1}(t_\alpha): \alpha \in \Lambda_\circ\} = (\cup\{d(d^{-1}(t_\alpha): \alpha \in \Lambda_\circ)\} = \cup \{t_\alpha: \alpha \in \Lambda_\circ \}$. As a result, Y^* is $\acute{P}$-compact

Proposition 21: Let $h{:}X^* \to Y^*$ be a surjective $S_{gs}{}^*$- irresolute function. If X^* is semi-compact, then Y^* is $\acute{P}$- compact.

Proof: Now let $\{U_\alpha: \alpha \in \Lambda \}$ be any O.C. of Y^*, implies that $Y^* = \cup\{U_\alpha: \alpha \in \Lambda\}$, since h is $S_{gs}{}^*$ - irresolute function, then by Definition (1)$h(U_\alpha)$ is semi-open in X^* for each $\alpha \in \Lambda$, Since h is surjective ,$X^* = h^{-1}(Y^\cdot) = h^{-1}(\cup (U_\alpha): \alpha \in \Lambda\}) = \cup\{h^{-1}(U_\alpha): \alpha \in \Lambda\}$. So $\{h^{-1}(U_\alpha): \alpha \in \Lambda\}$is an semi-O.C. of X^*.When X^* is semi- compact, therefore $\exists$ a finite $\Lambda_\circ$ subset of Λ s.t. $X^* = \cup\{h^{-1}(U_\alpha) : \alpha \in \Lambda_\circ\}$. Then $X^* = h^{-1}(H) = h(\cup \{h^{-1}(U_\alpha): \alpha \in \Lambda_\circ\} = \cup \{h(h^{-1}(U_\alpha): \alpha \in \Lambda_\circ)\} = \cup \{U_\alpha: \alpha \in \Lambda_\circ \}$. Thus Y^* is $\acute{P}$ -compact

Definition 22: A subset T^* of a T.SP $(X^*, \mathcal{J})$ is called $\acute{P}$-set ,if for every cover$\{q_\alpha: \alpha \in \Lambda \}$ of T^* by $S_{gs}{}^*$-O subsets of X^*, $\exists$ a finite subset $\Lambda_\circ$ of Λ s.t. $S^* \subseteq \cup\{q_\alpha: \alpha \in \Lambda_\circ\}$.

Definition 23: A subset V^* of a T.SP $(X^*,\mathcal{J})$ is is called $\acute{P}$ -compact subspace if for every cover $\{U_\alpha: \alpha \in \Lambda\}$ of V^* by $S_{gs}{}^*$- O subsets relative to V^*, $\exists$ a finite subset $\Lambda_\circ$ of Λ s.t. $V^* = \cup\{U_\alpha: \alpha \in \Lambda_\circ\}$.

Proposition 24: If every cover of B by gs^*-closed subset of X^* *(resp.,* gs^*-closed subset of P)has a F.S.C., then $\c{U}$ is $\acute{P}$ -set (resp. $\acute{P}$ - compact subspace)

Proof: Let $\{\acute{M}_\alpha: \alpha \in \Lambda\}$ be any O.C. of B by gs^*-closed subset of X^* (resp., gs^*-closed subset of B), then for each $j \in X^*$(resp., $j \in B$) , $\exists\ \alpha \in \Lambda$, $j \in \acute{M}_{\alpha(j)}$ and ${H^*}_{\alpha(j)}$ is gs^*-closed subset of X^* s.t. , $j \in {H^*}_{\alpha(j)} \subseteq \acute{M}_{\alpha(j)}$ and $\{ {H^*}_{\alpha(j)} : j \in X \}$is a cover of B by gs^*-closed subset of X^* (resp., gs^*-closed subset of B), the family has a F.S.C. s.t. $B \subseteq \{{H^*}_{\alpha(ji)} : i = 1,2, \dots, n\} \subseteq$

$\{\acute{M}_{\alpha(ji)}: i = 1,2, \dots, n\}$ .Therefore B⊆∪ $\{\acute{M}_{\alpha(ji)}: i = 1,2, \dots, n\}$ (resp., P=∪ $\{\acute{M}_{\alpha(ji)}: i = 1,2, \dots, n\}$).Thus B is $\acute{P}$ –set (resp.$\acute{P}$ - compact subspace)

Proposition 25: Let K ⊆ X^* be a clopen set, K is $\acute{P}$ -compact subspace of X^* iff K is $\acute{P}$-set.

Proof: Suppose that K is an $\acute{P}$ -set and $\{M_\alpha: \alpha \in \Lambda\}$ be any O.C. of K with $M_\alpha \in S_{gs}{}^*$ O(K) for every $\alpha \in \Lambda$ .Since K is clopen, then $M_\alpha \in S_{gs}{}^*$ O(X^*) for every $\alpha \in \Lambda$,also since K is and $\acute{P}$ -set, then ∃ a finite $\Lambda_\circ$ subset of Λ s.t. K⊆∪$\{M_\alpha: \alpha \in \Lambda_\circ\}$,but $M_\alpha \in S_{gs}{}^*$ O(K) for every $\alpha \in \Lambda$, This gives K = ∪ $\{M_\alpha: \alpha \in \Lambda_\circ\}$.Hence K is $\acute{P}$ - compact subspace.

Conversely, Let K be $\acute{P}$ -compact subspace of X^* and $\{M_\alpha: \alpha \in \Lambda\}$ be any O.C. of K and $M_\alpha \in S_{gs}{}^*$ O(X^*) for every $\alpha \in \Lambda$. Since K is clopen, then by Proposition (5), K ∩ $U_\alpha \in S_{gs}{}^*$ O(X) for every $\alpha \in \Lambda$,also since K is open,then K ∩ $M_\alpha \in S_{gs}{}^*$ O(K) for every $\alpha \in \Lambda$,its followes that the family { K ∩ $U_\alpha: \alpha \in \Lambda$ } is O.C. of K by gs*-closed subset of K. Since K is an $\acute{P}$ - compact subspace of X, then ∃ a finite $\Lambda_\circ$ subset of Λ s.t. K =∪{ K ∩ $M_\alpha: \alpha \in \Lambda_\circ$ }⊆∪{ $M_\alpha: \alpha \in \Lambda_\circ$ }. Thus, K is an $\acute{P}$ -set.

Proposition 26: Let D⊆ K^* and $K^* \subseteq X^*$ be a clopen set. Then D is an $\acute{P}$ -set of X^* if and only if D is an $\acute{P}$ -set of K^*

Proof: First let $\{O_\alpha: \alpha \in \Lambda\}$ be any O.C. of D. Since K^* is clopen, then by Proposition (5), for every $o \in \Lambda$, then ∃ an $S_{gs}{}^*$-open set S_α of X^* s.t. $O_\alpha = S_\alpha \cap K^*$, Furthermore D⊆∪$\{O_\alpha: \alpha \in \Lambda\}$ = ∪$\{S_\alpha \cap K^*: \alpha \in \Lambda\}$⊆∪$\{S_\alpha: \alpha \in \Lambda\}$.Then the family $\{S_\alpha: \alpha \in \Lambda\}$ is a cover of D and S_α is an $S_{gs}{}^*$-O set of X^*. Since D is an $\acute{P}$ - set of X^*, ∃ finite $\Lambda_\circ$ subset of Λ s.t. D ⊆∪$\{S_\alpha: \alpha \in \Lambda_\circ\}$,since D ⊆ K^* , then D⊆∪$\{S_\alpha \cap K^*: \alpha \in \Lambda_\circ\}$ ⊆∪$\{O_\alpha: \alpha \in \Lambda_\circ\}$.Then D is an $\acute{P}$ -set of K^*

Conversely: Assuming that D is an $\acute{P}$ -set of K^* and $\{S_\alpha: \alpha \in \Lambda_\circ\}$ be O.C. of D where S_α is an $\acute{P}$ -set of X^*.Since D ⊆ K^* ,then D ⊆∪ $\{S_\alpha: \alpha \in \Lambda\} \cap K^*$ =$\{S_\alpha \cap K^*: \alpha \in \Lambda\}$.Since K^* is clopen, then then by Proposition (5), ∃ $S_{gs}{}^*$-open set O_α of K^*s.t. $O_\alpha = S_\alpha \cap K^*$ for every $\alpha \in \Lambda$. Then the family $\{O_\alpha: \alpha \in \Lambda\}$ is a cover of D and O_α is an $S_{gs}{}^*$-open set of K^*. Since D is an $S_{gs}{}^*$-O set , ∃ finite $\Lambda_\circ$ subset of Λ s.t. D⊆∪ $\{O_\alpha: \alpha \in \Lambda_\circ\}$ ⊆$\{S_\alpha \cap K^*: \alpha \in \Lambda_\circ\}$⊆∪ $\{S_\alpha: \alpha \in \Lambda_\circ\}$. Thus, D is an $\acute{P}$ -set of X^*.

Proposition 27: Every proper $S_{gs}{}^*$- closed subset of a space X^* is $\acute{P}$- set iff X^* is $\acute{P}$- compact.

Proof: ⟹ First let T be any proper $S_{gs}{}^*$- closed set of X^*, let $\{D_\alpha: \alpha \in \Lambda\}$ be any O.C. of T. Since F is an $S_{gs}{}^*$- closed set, then X^* - T is an $S_{gs}{}^*$- open set, then the family $\{\ D_{\alpha}: \alpha\ \in \Lambda\ \} \cup\ X^* - T$ is an $S_{gs}{}^*$- O.C. of T. Since $X^*$is $\acute{P}$ - compact, $\exists$ a finite substet $\Lambda_\circ$ of Λ s.t. $X^*= \{\ D_{\alpha}: \alpha\ \in \Lambda_\circ\ \} \cup (\ X^* - T)$. We get $T \subseteq \{D_{\alpha}: \alpha\ \in \Lambda_\circ\}$. Hence F is an $\acute{P}$ - set.

$\Leftarrow$ suppose that every proper $S_{gs}{}^*$- closed subset of X^* is an $S_{gs}{}^*$-set and let$\{D_{\alpha}: \alpha\ \in \Lambda\ \}$ be any O.C. of X^* with $D_{\alpha} \in S_{gs}{}^*$ O(X^*) for every $\alpha\ \in \Lambda$, and $X^* \neq\ D_{\alpha S} \neq \varphi$ for every $\alpha\ \in \Lambda$, then X^* - D_{α} is a proper $S_{gs}{}^*$- closed subset of X^*. By hypothesis, $\exists$ a finite subset $\Lambda_\circ$ of Λ s.t. X^*- $D_{\alpha}\ \subseteq\ \cup \{D_{\alpha}: \alpha\ \in \Lambda_\circ\}$. Now, we obtain $X^*=\ \cup\{D_{\alpha}: \alpha\ \in \Lambda_\circ\ \cup\{\alpha_S\}\ \}$. This gives X^* is $\acute{P}$ - compact.

4. Conclusion

We created types for of compact spaces and closed spaces in our work. Additionally, by employing $S_{gs}{}^*$ -open, we were able to derive a number of weak and strong relation of compact spaces and closed spaces. These relations, if defined, will be powerful formulas for the concepts of of $\acute{P}$-compact and $\acute{P}$-closed spaces, and they will directly relate to the compact's paces and closed spaces in subsequent research.

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