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New weak open set in soft topological space with an ideal

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Abstract

In this study, we present and study *sIIpg**-closed soft sets in a soft topological space. In this article, *sIIpg**-closed soft sets are examined using soft set theory and examples. A suitable condition for recording *sIIpg**-open soft sets is determined that corresponds to the soft topology.

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1. Introduction

The notion of soft set theory was first developed by D. Molodtsov in [1] and has gained tremendous attention since. Molodtsov provided the essential principles of the new theory and effectively applied it in numerous domains, such as smoothness of functions, game theory, operations research. Recent years have been characterized by rapid advances in research into soft set theory and its applications in a variety of domains. Following the development of soft set operations [7, 5], the characteristics and applications of soft set theory have received increased attention [2, 20].

Recently, the horizon of soft set theory has broadened with the introduction of fuzzy set concepts [9, 18]. Building upon the foundational principles of soft

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set theory, researchers have refined the operational definitions and developed a cohesive decision-making process leveraging these redefined operations [3, 6]. It obtained some stability only after the introduction of soft topology [9] in 2011. Kandil et al. [10] established soft operations such semi-open, pre-open, and α-open. Kandil et al. [4, 14, 21, 23] were the first to introduce the concept of a soft ideal. They also discussed the concept of soft local functions. Kandil et al. investigated applications in many sectors [19, 12, 17]. El-Sheikh and Abd el-Latif [4] introduced the concept of b−open soft sets, which has since been used to support soft topological spaces [13]. The new weak soft open sets that have been reached and study are used in many applications, for example [10,16].

2. Preliminary

Definition 2.1: [1] Assume X is an initial universe, $X \neq \emptyset$ and K is a set of $parameters$. Let were $\Psi(X)$ the collection subsets of X and $P \neq \emptyset$ such that $P \subseteq K$. (F, K) (Briefly F_K) is a soft set over X whenever F is a function such that $F: P \to \Psi(X)$. So, $F_K = \{ F(d): d \in P \subseteq K, F: P \to \Psi(X) \}$.The $collection$ of all soft sets (briefly $ȘȘ(X)_K$).

Definition 2.2: [18] Assume $(F, \mu), (\Gamma, \mu) \in ȘȘ(X)_K$. Then (Γ, μ) continuing the soft set (F, μ) and denoted by $(F, \mu) \tilde{\sqsubseteq} (\Gamma, \mu)$, if $F(d) \tilde{\sqsubseteq} \Gamma(d)$, for all $d \in \mu$. Now (F, μ) is a soft subset of (Γ, μ) and (Γ, μ) is a soft super set of (F, μ).

Definition 2.3: [16] The relative complement of (F, K) (briefly $(F, K)'$) $(F, K)' = (F', K)$, $F': K \to \Psi(X)$ is a function where, $F'(d) = X - F(d)$, for all $d \in K$ and F' is called a soft complement of F.

Definition 2.4: [18] The difference for any two soft sets (F, K) and (Γ, K) over the common universe X, denoted by (F, K) - (Γ, K) is the soft set $(\mathcal{U}, K)$ were for all $d \in K$, $\mathcal{U}(d) = F(d) - g(d)$.

Definition 2.5: [15] Let (F, K) be a soft over X and $x \in X$. If $x \in F(d)$ for each $d \in K$, then $x \tilde{\in} (F, K)$.

Definition 2.6: [18] (F, K) is a NULL soft set denoted by $\tilde{\emptyset}$ or $\emptyset_K$ whenever, $\forall d \in K$, $F(d) = \emptyset$.

Definition 2.7: [18] (F, K) is an absolute soft set denoted by $\tilde{X}$ or X_K whenever, $\forall d \in K$, $F(d) = X$.

Definition 2.8: [24] Let (Υ, C), $(\Gamma, \text{Ԁ}) \in ȘȘ(X)$, $(F, K) = (\Upsilon, C) \tilde{\cup} (\Gamma, \text{Ԁ})$ where $K = C \cup \text{Ԁ}$ and $\forall d \in K$,

$$F(z) = \begin{cases} \Upsilon(z), & z \in C - \text{Ԁ}, \\ \Gamma(z), & z \in \text{Ԁ} - C, \\ \Upsilon(z) \cup \Gamma(z), & z \in C \cap \text{Ԁ}. \end{cases}$$

Definition 2.9: [22] Let $(\Upsilon, \mathrm{C}), (\Gamma, \mathrm{Ǝ}) \in$ ŞŞ$(\mathrm{X}), (\mathrm{F}, P) = (M, \mathrm{C}) \tilde{\cap} (\Gamma, \mathrm{Ǝ})$, where $P = \mathrm{C} \cap \mathrm{Ǝ}$ and for each $d \in \mathrm{K}$, $\mathrm{F}(d) = \Upsilon(d) \cap \Gamma(\mathrm{d})$.

Definition 2.10: [8,11] Assume that T is a collection of soft sets over X with the same K, then T is called a soft topology on X if the following condition is satisfied;

i. $\tilde{\mathrm{X}}, \tilde{\emptyset} \tilde{\in} \mathrm{T}$

ii. $\cup_{\alpha \in \Lambda} (\mathrm{G}_\alpha, \mathrm{K}) \tilde{\in} \mathrm{T}$ whenever, $(\mathrm{G}_\alpha, \mathrm{K}) \tilde{\in} \mathrm{T} \ \forall \ \alpha \in \Lambda$,

iii. $((\mathrm{F}, \mathrm{K}) \tilde{\cap} (\Gamma, \mathrm{K})) \tilde{\in} \mathrm{T}$ for each $(\mathrm{F}, \mathrm{K}), (\Gamma, \mathrm{K}) \tilde{\in} \mathrm{T}$.

The soft topological space is denoted by $(\mathrm{X}, \mathrm{T}, \mathrm{K})$.

Definition 2.11: [18] Let $(\mathrm{X}, \mathrm{T}, \mathrm{K})$be a soft topological space. (ψ, K) is called a soft closed set whenever $(\psi, \mathrm{K})' \tilde{\in} \mathrm{T}$. The family of all soft closed subsets is denoted by Ş$\mathrm{C}(\mathrm{X})$.

Definition 2.12: [18] If $(\mathrm{X}, \mathrm{T}, \mu)$ is a soft topological space and $(\mathrm{F}, \mu) \in$ ŞŞ$(\mathrm{X})_\mu$, then the soft closure of (F, μ), (briefly $cl(\mathrm{F}, \mu)$), $cl\big((\mathrm{F}, \mu)\big) = \tilde{\cap} \{ (\Upsilon, \mu) : (\Upsilon, \mu) \in$ Ş$\mathrm{C}(\mathrm{X}), (\mathrm{F}, \mu) \tilde{\subseteq} (\Upsilon, \mu) \}$.

Remark 2.13: [18] Let (Υ, K) and (Γ, K) are two soft subsets of a soft topological space (X ,T,K). Then the following hold.

i. $cl(cl(\Upsilon, \mathrm{K})) = cl((\Upsilon, \mathrm{K}))$

ii. $(\Upsilon, \mathrm{K}) \tilde{\subseteq} (\Gamma, \mathrm{K})$ implies $cl((\Upsilon, \mathrm{K})) \tilde{\subseteq} cl((\Gamma, \mathrm{K}))$

iii. $cl((\Upsilon, \mathrm{K})) \tilde{\cup} cl((\Gamma, \mathrm{K})) = cl((\Upsilon, \mathrm{K}) \tilde{\cup} (\Gamma, \mathrm{K}))$

iv. $cl((\Upsilon, \mathrm{K}) \tilde{\cap} (\Gamma, \mathrm{K})) \tilde{\subseteq} cl((\Upsilon, \mathrm{K})) \tilde{\cup} cl((\Gamma, \mathrm{K}))$.

Definition 2.14: [12] For any $(\mathrm{X}, \mathrm{T}, \mathrm{M})$. Let $(\mathrm{F}, \mathrm{M}) \in$ ŞŞ$(\mathrm{X})_{\mathrm{M}}$, then the soft interior of (Γ, M), (briefly $\mathrm{int}(\Gamma, \mathrm{M})$)), $\mathrm{int}(\Gamma, \mathrm{M}) = \tilde{\cup}\{ (\Upsilon, \mathrm{M}); (\Upsilon, \mathrm{M}) \tilde{\in} \mathrm{T}, (\Upsilon, \mathrm{M}) \tilde{\subseteq} (\Gamma, \mathrm{M})\}$.

Remark 2.15: [12] Let (Υ, Z) and (Γ, Z) are two soft sets in (X ,T,Z). Then the following hold.

i. $int(int(\Upsilon, \mathrm{Z})) = int((\Upsilon, \mathrm{Z}))$

ii. $(\Upsilon, \mathrm{Z}) \tilde{\subseteq} (\Gamma, \mathrm{Z})$ implies $int((\Upsilon, \mathrm{Z})) \tilde{\subseteq} int((\Gamma, \mathrm{Z}))$

iii. $int((\Upsilon, \mathrm{Z})) \tilde{\cap} int((\Gamma, \mathrm{Z})) = int((\Upsilon, \mathrm{Z}) \tilde{\cap} (\Gamma, \mathrm{Z}))$

iv. $int((\Upsilon, \mathrm{Z})) \tilde{\cup} int((\Gamma, \mathrm{Z})) \tilde{\subseteq} int((\Upsilon, \mathrm{Z}) \tilde{\cup} (\Gamma, \mathrm{Z}))$

Remark 2.16: [19]

i. $(\Upsilon,\mathrm{K}) \in$ Ş$\mathrm{C}(\mathrm{X}) \Leftrightarrow (\Upsilon, \mathrm{K}) = cl((\Upsilon, \mathrm{K}))$.

ii. $(\Upsilon,\mathrm{K}) \in$ Ş$\mathrm{O}(\mathrm{X}) \Leftrightarrow (\Upsilon, \mathrm{K}) = int((\Upsilon, \mathrm{K}))$.

Definition 2.17: [12] For each soft topological space $(\mathrm{X}, \mathrm{T}, \mathrm{K})$, then (F, K) is called a $soft\, pre$-open set and denoted by $Şp$-open set if $(\mathrm{F}, \mathrm{K}) \tilde{\subseteq} int(cl(\mathrm{F}, \mathrm{K}))$. X-$(\mathrm{F}, \mathrm{K})$ is a $soft$-pre-closed set. The collection of each pre-$soft$-open sets in $(\mathrm{X}, \mathrm{T}, \mathrm{K})$ and denoted by $\mathrm{SO}_{pre}(\mathrm{X})$). The family of all soft pre-closed sets denoted by $\mathrm{SC}_{pre}(\mathrm{X})$).

Remark 2.18: [2]

i. $(\Upsilon,\mathrm{K}) \in Ş\mathrm{O}(\mathrm{X})$ then $(\Upsilon, \mathrm{K}) \in \mathrm{SO}_{pre}(\mathrm{X})$

ii. $(\Upsilon,\mathrm{K}) \in Ş\mathrm{C}(\mathrm{X})$ then $(\Upsilon, \mathrm{K})\ \mathrm{SC}_{pre}(\mathrm{X})$.

Definition 2.19: [2] For a soft topological space $(\mathrm{X}, \mathrm{T}, \mathrm{K})$ and $(\Pi, \mathrm{K}) \in ŞŞ(\mathrm{X})_{\mathrm{K}}$. The soft-pre-closure (respectively, soft-pre-interior) of (Π, K) which is (briefly Şp-$cl\big((\Pi, \mathrm{K})\big)$) (respectively, Şp-$int\big((\Pi, \mathrm{K})\big)$) is defined by; Şp-$cl\big((\Pi, \mathrm{K})\big) = \tilde{\cap}\, \{ (\mathrm{U}', \mathrm{K}) \in ŞŞ(\mathrm{X})\mathrm{K}\, ;\, (\mathrm{U}', \mathrm{K}) \in \mathrm{SC}_{pre}(\mathrm{X})$. where $(\Pi, \mathrm{K}) \tilde{\subseteq} (\mathrm{U}', \mathrm{K})\}$, (respectively, Şp-$int((\Pi, \mathrm{K}))) = \tilde{\cup}\, \{ (\mathrm{U}', \mathrm{K}) \in ŞŞ(\mathrm{X})_{\mathrm{K}}$; $(\mathrm{U}', \mathrm{K})$ is soft-pre-open set where $(\mathrm{U}', \mathrm{K}) \tilde{\subseteq} (\Pi, \mathrm{K})\}$). Evidently, Şp-$cl((\Pi, \mathrm{K}))$ (respectively, Şp-$int((\Pi, \mathrm{K}))$) is the smallest soft-pre-closed (largest soft-pre-open) subset of X which contains (subset of) (Π, K).

Remark 2.21: [2]

1. The intersection of two soft-pre-open sets need not be soft pre-open set.
2. The union of two soft- pre-closed sets need not be $soft$-pre-closed set.

Definition 2.22: [2] Let (F, K) be a soft subset of soft space $(\mathrm{X}, \mathrm{T}, \mathrm{K})$, then (F, K) is called a $soft$-pre-g^*-closed set "$Şpg^*$-closed" if $cl(\mathrm{F}, \mathrm{K}) \tilde{\subseteq} (\Gamma, \mathrm{K})$ whenever, $(\mathrm{F}, \mathrm{K}) \tilde{\subseteq} (\Gamma, \mathrm{K})$ and $(\Gamma, \mathrm{K}) \in \mathrm{SO}_{pre}(\mathrm{X})$.

Definition 2.23: [16] Let II$\neq \emptyset$, $then$ II $\tilde{\subseteq}$ ŞŞ $(X)_{\mathrm{K}}$ is a soft ideal whenever,

i. If $(\mathrm{F}, \mathrm{K}) \tilde{\in}$ II and $(\Gamma, \mathrm{K}) \tilde{\in}$ II then, $(\mathrm{F}, \mathrm{K}) \tilde{\cup} (\Gamma, \mathrm{K}) \tilde{\in}$ II.

ii. If $(\mathrm{F}, \mathrm{K}) \tilde{\in}$ II and $(\Gamma, \mathrm{K}) \tilde{\subseteq} (\mathrm{F}, \mathrm{K})$ implies, $(\Gamma, \mathrm{K}) \tilde{\in}$ II.

Definition 1.24: [13] The space $(\mathrm{X}, \mathrm{T}, \mathrm{K})$ with a soft ideal II can be referred to be (X,T,K, II) which is a soft topological space with an ideal II.

Definition 2.25: [2] Let (F, K) be a soft subset of $(\mathrm{X}, \mathrm{T}, \mathrm{K}, \mathrm{II})$ which is soft topological space with an ideal, then (F, K) is a soft-II-pre-g-closedset (briefly s$IIpg$-closed). If (F, K) -$(\Pi, \mathrm{K}) \in$ II then , $cl(\mathrm{F}, \mathrm{K}) - (\pi, \mathrm{K}) \in$ II for each $(\Pi, \mathrm{K}) \in \mathrm{SO}_{pre}(\mathrm{X})$, and X$-(\mathrm{F}, \mathrm{K})$ is called a soft-II-pre-g-open set denoted by $sIIpg$-open set. The family of all $sIIpg$- closed sets denoted by $sIIpg$-C(X)) and we denoted for a family of all sIIpg-open soft sets by $sIIpg$-O(X)).

Remark 2.26: [2] For each $(\mathrm{X}, \mathrm{T}, \mathrm{K}, \mathrm{II})$ topological space, sIIpg-closed and $Şpg^*$-closed set are independent. See Example 2.1. It is clear that, $(\mathcal{H}, \mathrm{K})$;

$\mathcal{H}(\mathrm{d}) = \{\mathrm{e}\}(\forall \mathrm{d} \in \mathrm{K}) \tilde{\in} ŞŞC(X)_K$. Now $(\mathcal{H}, \mathrm{K}) \tilde{\subseteq} (\mathcal{H}, \mathrm{K})$, but $cl(\mathcal{H}, \mathrm{K}) = \tilde{X}$ and $\tilde{X} \tilde{\nsubseteq} (\mathcal{H}, \mathrm{K})$.

Definition 2.1.24: [22] Let $ŞŞ(X)_P$ and $ŞŞ(Y)_{ɰ}$ be the collection of soft sets, such that $\Gamma: X \to Y$ and $\Phi: P \to ɷ$ are two functions. Let $f_{\Phi\Gamma}: ŞŞ(X)_P \to ŞŞ(Y)_{ɰ}$ be a function then;

i. If $(\mathcal{M}, \mathrm{P}) \in ŞŞ(X)$. $f_{\Phi\Gamma}(\mathcal{M}, \mathrm{P}) = (f_{\Phi\Gamma}(\mathcal{M}), \Phi(\mathrm{P})) \in ŞŞ(Y)_{ɰ}$ whenever, for each $b \in ɷ$,

$$f_{\Phi\Gamma}(M)(b) = \begin{cases} \cup \{\Gamma(\mathcal{M})(a) : a \in \Phi^{-1}(\mathrm{b}) \cap P\} & \text{if } \Phi^{-1}(\mathrm{b}) \cap P \neq \emptyset \\ \emptyset & \text{Otherwise} \end{cases}$$

ii. If $(\mathcal{Z}, ɷ) \in ŞŞ(Y)_{ɰ}$. Then $f_{\Phi\Gamma}^{-1}(\mathcal{Z}, ɷ) = (f_{\Phi\Gamma}^{-1}(\mathcal{Z}), \Phi^{-1}(ɷ)) \in ŞŞ(X)_P$ whenever , for each $a \in \mathcal{P}$

$$f_{\Phi\Gamma^{-1}}(\mathcal{Z})(a) = \begin{cases} \Gamma^{-1}(\mathcal{Z}(\Phi(a)) & \text{if } \Phi(a) \in ɷ \\ \emptyset & \text{Otherwise} \end{cases}$$

Now, $f_{\Phi\Gamma}$ is a surjective (respectively, *injective*) if Φ and $\Gamma$are surjective (respectively, injective), functions.

3. On Soft Ideal Pre^*-g-Closed Set

This section presents a new weak open soft set in soft topological spaces termed soft-II–Pre^*-g-$closed$ set, which is introduced by soft ideal. There is further discussion of the connections between these concepts and other kinds of nearly open sets.

Definition 3.1: Let (F, K) be a soft subset of $(\mathrm{X}, \mathrm{T}, \mathrm{K}, \mathrm{II})$ which is soft topological space with an ideal, then (F, K) is a soft-II-Pre^*-g-closedset (briefly sIIp^*g-closed). If (F, K) -$(\Pi, \mathrm{K}) \in \mathrm{II}$ then , $\mathrm{spcl}(\mathrm{F}, \mathrm{K}) - (\Pi, \mathrm{K}) \in \mathrm{II}$ for each $(\Pi, \mathrm{K}) \in \mathrm{SO}_{pre}(\mathrm{X})$, and $\tilde{X} - (\mathrm{F}, \mathrm{K})$ is a soft-II–Pre^*-g-open set (briefly $sIIp^*g$-open set).

The family of all $sIIp^*g$- closed sets (briefly $sIIp^*g$-C(X)) and the family of all sIIp^*g-open soft sets (briefly $sIIp^*g$-O(X)).

Example 3.1: For a space $(\mathrm{X}, \mathrm{T}, \mathrm{K}, \mathrm{II})$, whenever $\mathrm{X} = \{\mathrm{e}, \mathrm{m}\}$, $\mathrm{K} = \{\mathrm{d}_1, \mathrm{d}_2\}$, $\mathrm{T} = \{\tilde{\emptyset}, \tilde{X}, (\mathrm{F}, \mathrm{K}), (\Pi, \mathrm{K})\}$, $II = \{\tilde{\emptyset}, (Y, \mathrm{K})\}$ such that $(\mathrm{F}, \mathrm{K}) = \{(\mathrm{d}_1, \emptyset), (\mathrm{d}_2, \{\mathrm{e}\})\}$, and $(\Pi, K) = \{(\mathrm{d}_1, \{\mathrm{e}\}), (\mathrm{d}_2, \{\mathrm{e}\})\}$ and $(M, K) = \{(\mathrm{d}_1, \emptyset), (\mathrm{d}_2, \{\mathrm{e}\})\}$ then $\mathrm{SO}_{pre}(\mathrm{X}) = \{\tilde{\emptyset}, \tilde{X}, (F, \mathrm{K}), (\Pi, \mathrm{K}), (\Gamma, \mathrm{K}), (\mathcal{H}, \mathrm{K}), (\mathcal{E}, \mathrm{K}), (\mathcal{N}, \mathrm{K}), (\mathcal{G}, \mathrm{K})\}$, sII$p^*gC(X) = \{\tilde{X}, \tilde{\emptyset}, (\mathrm{F}, \mathrm{K})', (\Pi, \mathrm{K})'\}$ such that $(\mathrm{F}, \mathrm{K})' = \{(\mathrm{d}_1, \mathrm{X}), (\mathrm{d}_2, \{\mathrm{m}\})\}$, $(\Pi, \mathrm{K})' = \{(\mathrm{d}_1, \{\mathrm{m}\}), (\mathrm{d}_2, \{\mathrm{m}\}\}$ where, $(\Gamma, \mathrm{K}) = \{(d_1, \emptyset), (d_2, \mathrm{X})\}$, $(\mathcal{H}, \mathrm{K}) = \{(\mathrm{d}_1, \{\mathrm{e}\}), (\mathrm{d}_2, \mathrm{X})\}$, $(\mathcal{E}, \mathrm{K}) = \{(d_1, \{\mathrm{m}\}), (d_2, \{\mathrm{e}\})\}$, $(\mathcal{N}, \mathrm{K}) = \{(d_1, \{\mathrm{m}\}), (d_2, \mathrm{X})\}$ and $(\mathcal{G}, \mathrm{K}) = \{(d_1, \mathrm{X}), (d_2, \{\mathrm{e}\})\}$, and $sIIp^*g\text{-O}(\mathrm{X}) = T \cup \{(\mathrm{F}_2, \mathrm{K})', (\mathrm{F}_8, \mathrm{K})'\}$.

Proposition 3.3:

1. Every $sIIpg$-closed sets is a $sIIp^*g$-closed.
2. Every $sIIpg$-open sets is a $sIIp^*g$-open.

Proof:

1. Let $(\mathrm{M},\mathrm{K}) \in ȘȘ(\mathrm{X})_{\mathrm{K}}$ and Let $(\Pi,\mathrm{K}) \in \mathrm{SO}_{pre}(\mathrm{X})_{\mathrm{K}}$ such that $(\Upsilon,\mathrm{K})-(\Pi,\mathrm{K}) \in II$, but (M,K) is $sIIpg$-closed set, $\mathrm{cl}(\Upsilon,\mathrm{K}) - (\Pi,\mathrm{K}) \in II$ Since $sp\mathrm{cl}(\Upsilon,\mathrm{K}) \tilde{\subseteq} \mathrm{cl}(\Upsilon,\mathrm{K})$, then $\mathrm{spcl}(\Upsilon,\mathrm{K})-(\Pi,\mathrm{K}) \tilde{\subseteq} \mathrm{cl}(\Upsilon,\mathrm{K})-(\Pi,\mathrm{K}) \in II$, implies $sp\mathrm{cl}(\Upsilon,\mathrm{K})-(\Pi,\mathrm{K}) \in \mathrm{II}$. Therefore, (Υ,K) is a $sIIp^*g$-closed set.
2. Let $(\mathrm{u},\mathrm{K}) \in sIIpg\text{-O}(\mathrm{X})$, so (u,K) is $sIIpg$-open. Hence $(\mathrm{u},\mathrm{K})^c$ is $sIIpg$-closed set. From (i) $(\mathrm{u},\mathrm{K})^c$ is a $sIIp^*g$-closed. Therefore (u,K) is a $sIIp^*g$-open. Hence, $(\mathrm{u},\mathrm{K}) \in sIIp^*g\text{-O}(\mathrm{X})$

Corollary 3.1: *Let (X, T, K, II), be a soft topological space then*

i. *Each soft closed set is a $sIIp^*g$-closed.*

ii. *Each soft open set is a $sIIp^*g$-open.*

Proof: Remark 1.25 and proposition 2.3.

Example 3.2: Consider $\mathrm{X} = \{\text{ę}, \text{ɱ}\}$, $\mathrm{K} = \{d_1, \mathrm{d}_2\}$, $\mathrm{T}=\{\tilde{\emptyset}, \tilde{\chi}, \mathrm{F(d)} = \{\text{ę}\}\ \forall \mathrm{d}\}$, then $\mathrm{SO}_{pre}(\mathrm{X})=\{\tilde{\emptyset}, \tilde{\mathrm{X}}, (\Upsilon,\mathrm{K}), (\Pi,\mathrm{K}), (\Gamma,\mathrm{K}), (\mathcal{H},\mathrm{K}), (\mathcal{E},\mathrm{K}), (\mathcal{N},\mathrm{K}), (\mathcal{G},\mathrm{K}), (\mathcal{C},K), (\omega,K), (\exists,K), (\alpha,K)\}$, such that $(\Upsilon,\mathrm{K}) = \{(d_1,\emptyset),(\mathrm{d}_2,\{\text{ę}\})\}$, $(\Pi,\mathrm{K}) = \{(d_1,\emptyset), (\mathrm{d}_2,\{\mathrm{X}\})\}$, $(\Gamma,\mathrm{K}) = \{(d_1,\{\text{ę}\}), (\mathrm{d}_2,\{\emptyset\})\}$, $(\mathcal{H},\mathrm{K}) = \{(d_1,\{\text{ę}\}), (\mathrm{d}_2,\{\text{ę}\})\}$, $(\mathcal{E},\mathrm{K})= \{(d_1,\{\text{ę}\}),(\mathrm{d}_2,\{\text{ɱ}\})\}$, $(\mathcal{N},\mathrm{K})=\{(d_1, \{\text{ę}\}),(\mathrm{d}_2,\{\mathrm{X}\})\}$, $(\mathcal{G},\mathrm{K})=\{(d_1,\{\text{ɱ}\}),(\mathrm{d}_2,\{\text{ę}\})\}$, $(\mathcal{C},K)=\{(d_1,\{\text{ɱ}\}),(\mathrm{d}_2,\{\mathrm{X}\})\}$, $(\omega,K)=\{(d_1,X),(\mathrm{d}_2,\{\emptyset\})\}$, $(\exists,K)=\{(d_1,X),(\mathrm{d}_2,\{\text{ę}\})\}$, $(\alpha,K)=\{(d_1,X),(\mathrm{d}_2,\{\text{ɱ}\})\}$, $\mathrm{II} = ȘȘ(\mathrm{X})\mathrm{K}$, $sIIp^*g\text{-c}(\mathrm{X})= sIIp^*g\text{-o}(\mathrm{X})= ȘȘ(\mathrm{X})_{\mathrm{K}}$.

i. Let $(\mathcal{E},\mathrm{K}) = \{(\mathrm{d}_1, \{\text{ę}\}), (\mathrm{d}_2, \{\text{ɱ}\})\}$ is a $\mathrm{sII}p^*g$-closed set, but $(\mathcal{E},\mathrm{K})$ is not $closed\ soft$ set.

ii. Let $(\mathcal{G},\mathrm{K})= \{(\mathrm{d}_1, \{\text{ɱ}\}), (\mathrm{d}_2, \{\text{ę}\})\}$ is a $\mathrm{sII}p^*g$-open set, but $(\mathcal{G},\mathrm{K}) \notin \mathbf{T}$.

Proposition 3.5: For a soft topological space $(\mathrm{X},\mathrm{T},\mathrm{K},II)$, if $II= ȘȘ(\mathrm{X})_{\mathrm{K}}$, then $s\mathrm{II}p^*g\text{-C}(\mathrm{X})= ȘȘ(\mathrm{X})_{\mathrm{K}}$.

Example 3.3: Let X={1,2,3,4,….}. K= $\{d_1,d_2\}$, T = $ȘȘ(X)_{\mathrm{K}}$ and $II= \{\tilde{\emptyset}\}$. So, $sIIp^*g\text{-O}(\mathrm{X}) = sIIp^*g\text{-C}(\mathrm{X}) = ȘȘ(X)_{\mathrm{K}}$.

Proposition 3.6: Let $(\Upsilon,\mathrm{K}) \in ȘȘ(\mathrm{X})_K$. If $\mathrm{cl}(\Upsilon,\mathrm{K}) \in II$, then (Υ,K) is a $sIIp^*g$-closed set.

Proof: Suppose that $(\Upsilon,\mathrm{K}) - (\Pi,\mathrm{K}) \in II$ for each, $(\Pi,\mathrm{K}) \in \mathrm{SO}_{pre}(\mathrm{X})$, then $\mathrm{spcl}(\Upsilon,\mathrm{K}) - (\Pi,\mathrm{K}) \tilde{\subseteq} \mathrm{cl}(\Upsilon,\mathrm{K}) \in II$, implies $sp\mathrm{cl}(\Upsilon,\mathrm{K}) - (\Pi,\mathrm{K}) \in \mathrm{II}$.

Therefore, (Υ, K) is a $sIIpg$-closed set. Now, for each $(\Pi, \mathrm{K}) \in sIIp^*g$-C(X) hence that, cl $(\Pi, K) \notin II$.

Theorem 3.7: $\widetilde{\cup}^{n}{}_{\alpha=1}\,(\mathcal{H}_\alpha, \mathrm{K})$ is sIIp*g-closed set, which $(\mathcal{H}_\alpha, \mathrm{K})$ is a sIIp*g-closed set $\forall \alpha = 1,2,..,\mathrm{n}$.

Proof: Let (Υ, K) and (Z, K) are two $sIIp^*g$-closed set in $(\mathrm{X}, \mathrm{T}, \mathrm{K}, \mathrm{II})$ and $(\varphi, \mathrm{K}) \in \mathrm{SO}_{pre}(\mathrm{X})$ where, $\left((\Upsilon, K) \,\widetilde{\cup}\, (\mathrm{Z}, \mathrm{K})\right) - (\varphi, \mathrm{K}) \in II$, then $(\Upsilon, K) - (\varphi, \mathrm{K}) \in II$ and $(\mathrm{Z}, \mathrm{K}) - (\varphi, \mathrm{K}) \in II$.

So, $spcl(\Upsilon, K) - (\varphi, \mathrm{K}) \in \mathrm{II}$ and $spcl(\mathrm{Z}, \mathrm{K}) - (\varphi, \mathrm{K}) \in \mathrm{II}$ therefore, $(spcl(\Upsilon, K) - (\varphi, \mathrm{K})) \,\widetilde{\cup}\, (spcl(\mathrm{Z}, \mathrm{K}) - (\varphi, \mathrm{K})) \in II$

So, $spcl((\Upsilon, K) \,\widetilde{\cup}\, (\mathrm{Z}, \mathrm{K})) - (\varphi, \mathrm{K}) \in II$ Hence, $(\Upsilon, K) \,\widetilde{\cup}\, (\mathrm{Z}, \mathrm{K})$ is a $sIIp^*g$-closed set.

Corollary 3.8: $\widetilde{\cap}^{n}{}_{\alpha=1}\,(\mathcal{H}_\alpha, \psi)$ *is* $sIIp^*g$*-open set, which* $(\mathcal{H}_\alpha, \psi)$ *is a* $sIIp^*g$*-open set* $\forall \alpha = 1,2,..,n$.

Proof: Let (Υ, ψ) and (Z, ψ) *are* two $sIIp^*g$-open soft *set* in $(\mathrm{X}, \mathrm{T}, \psi, \mathrm{II})$. So, $\widetilde{\mathrm{X}} - (\Upsilon, \psi)$ and $\widetilde{\mathrm{X}} - (\mathrm{Z}, \psi)$ are $sIIp^*g$ - closed *sets* therefore, $(\widetilde{\mathrm{X}} - (\Upsilon, \psi))$ $\widetilde{\cup}\, (\widetilde{\mathrm{X}} - (\mathrm{Z}, \psi))$ is *a* $sIIp^*g$-closed sets by Theorem 2.7 Hence, $\widetilde{\mathrm{X}} - ((\widetilde{\mathrm{X}} - (\Upsilon, \psi))$ $\widetilde{\cup}\, ((\widetilde{\mathrm{X}} - (\mathrm{Z}, \psi)))$ is a $sIIp^*g$-open set. So, $(\Upsilon, \psi) \,\widetilde{\cap}\, (\mathrm{Z}, \psi)$ is a $sIIp^*g$-open set.

Remark 3.1:

(i) $\widetilde{\cup}_{\alpha \in \Lambda}\,(\mathcal{H}_\alpha, \mathrm{K})$ need not be $sIIp^*g$-closed set, which $(\mathcal{H}_\alpha, \mathrm{K})$ is a $sIIp^*g$-closed set $\forall \alpha \in \Lambda$.

(ii) $\widetilde{\cap}_{\alpha \in \Lambda}\,(\mathcal{H}_\alpha, \mathrm{K})$ need not be $sIIp^*g$-open set, which $(\mathcal{H}_\alpha, \mathrm{K})$ is a $sIIp^*g$-open set $\forall \alpha \in \Lambda$.

Example 3.10: Let X ={1,2,3,4,….}. K= $\{d_1, d_2\}$,T = $\{\widetilde{\emptyset}\}\widetilde{\cup}\{(\Upsilon,\mathrm{K}): (\Upsilon,\mathrm{K}) \,\widetilde{\subseteq}\, \widetilde{\mathrm{X}}$ such that $(\Upsilon,\mathrm{K})'$ which is finite}, let II= $\{\widetilde{\emptyset}\}$.

Now, $\{(\Gamma_I,\mathrm{K}): \Gamma(\mathrm{r}) = \{I\}\ \forall \mathrm{r} \in \mathrm{K}$ and $I \in N - \{2\}\}$. Is the family of closed sets, so it is a $sIIp^*g$-closed soft sets, but $\widetilde{\cup}\{(\Gamma_\mathrm{I},\mathrm{K}): \Gamma_\mathrm{I}\ (\mathrm{r}) = \{\mathrm{I}\}$ for each $\mathrm{r} \in \mathrm{K}$ and $\mathrm{I} \in N - \{2\}\} = \{(\Gamma,K): \Gamma(\mathrm{r}) = N - \{2\}\ \forall \mathrm{r} \in \mathrm{K}\}$ where is not $sIIp^*g$-closed soft set because $\{(\Upsilon,\mathrm{K}); \Upsilon\ (\mathrm{r}) = N - \{2\}, \forall \mathrm{r} \in \mathrm{K}\} \in \mathrm{T}$ so, which is a sp-open set and (Γ,K)- $(\Upsilon,\mathrm{K}) \in II$, but $spcl(\Gamma, K)$- $(\Upsilon, K) = \widetilde{\mathrm{X}}$- $(\Upsilon, K) \notin II$

Theorem 3.11: *For any* (X, T, ψ, II), *let* $(\Upsilon, \Psi) \in ŞŞ(X)\Psi$. *Then* (Υ, Ψ) *is a* $sIIp^*g$*-open set, if and only if* $((\mathcal{U}, \Psi) - spint(\Upsilon, \Psi)) \in II$ *whenever,* $(\mathcal{U}, \Psi) - (\Upsilon, \Psi) \in II$ *and* $(\mathcal{U}, \Psi) \in ŞpC(X)\Psi$.

Proof: $(\Rightarrow)$ $Let(\Upsilon, \Psi)$ is a $sIIp^*g$-open set *and* $(\mathcal{U}, \Psi) - (\Upsilon, \Psi) \in II$ where $(\mathcal{U}, \Psi) \in Ş p\mathrm{C}(\mathrm{X})$. Since $\tilde{\chi} - (\Upsilon, \Psi)$ is *a* $sIIp^*g$*-closed* set, and $((\tilde{X} - (\Upsilon, \Psi))$

$-(\widetilde{X}-(\mathcal{U},\Psi))) \in$ II whenever, $(\widetilde{X}-(\mathcal{U},\Psi) \in SO_{pre}(\chi))$ implies, $cl((\tilde{X}-(\Upsilon,\Psi))$ $-(\widetilde{X}-(\mathcal{U},\Psi))) \in II$. Then $\widetilde{X}-int\big(\widetilde{X}-(\tilde{X}-(\Upsilon,\Psi))\big)-(\widetilde{X}-(\mathcal{U},\Psi)) \in II$.

So,$(\widetilde{X}-int(\Upsilon,\Psi))-(\widetilde{X}-(\mathcal{U},\Psi)) \in II$ implies $(\mathcal{U},\Psi)-int(\Upsilon,\Psi) \in$ II .

(⇐) Let $(\mathcal{U},\Psi)-$ spint$(\Upsilon,\Psi) \in II$ whenever, $(\mathcal{U},\Psi)-(\Upsilon,\Psi) \in II$ and $(\mathcal{U},\Psi) \in ŞpC(X)$. And let $(\tilde{X}-(\Upsilon,\Psi))-(\Pi,\Psi) \in II$ whenever, $(\Pi,\Psi) \in SO_{pre}(X)$ implies $(\widetilde{X}-(\Pi,\Psi))-(\Upsilon,\Psi) \in II$, so, $(\widetilde{X}-(\Pi,\Psi))-spint(\Upsilon,\Psi) \in II$, therefore $cl(\tilde{X}-(\Upsilon,\Psi))-(\Pi,\Psi) \in II$. So, $(\widetilde{X}-(\Upsilon,\Psi))$ is a $sIIp^{*}g$-closed set, and (Υ,Ψ) is a $sIIp^{*}g$-open set.

Proposition 3.12: Let a space (X,T,K,II), let $x_0 \in X$, K any set of parameters, $II = \{(\Upsilon,K) \in ŞŞ(X)K: int(cl(\Upsilon,K)) = \widetilde{\emptyset}\}$ and T= $\{\tilde{\chi}, \widetilde{\emptyset}, (F,K)\ ;\ F(d) = x_0$ $\forall$ d $\in$ K$\}$ then sII$p^{*}g$-C(X)= $\{(\Gamma,K);\ x_0 \tilde{\notin} \Gamma(d). \forall\ d\} \cup X$.

Proof: since T=$\{\tilde{\chi},\widetilde{\emptyset},(F,K),\ F(d)=x_0,\ \forall d \in K\}$ such that $x_0 \in X$. So, $SO_{pre}(X)=\{(\Upsilon,K) \in ŞŞ(X)K : x_0 \in \Upsilon(d).$ for some d $\in$ K$\}$.

Let $(\Gamma,K) \in ŞŞ(\chi)K$ such that $(\Gamma,K)-(\Pi,K) \in II$ whenever, $(\Pi,K) \in SO_{pre}(X)$. Now, since $\big((\Gamma,K)-(\Pi,K)\big) \tilde{\in} II$ this implies, $int(cl((\Gamma,K)-(\Pi,K))) = \widetilde{\emptyset}$. this means that $x_0 \tilde{\notin} ((\Gamma,K)-(\Pi,K)), \forall d \in K$ if $x_0 \tilde{\notin} (\Gamma,K)$, $\forall$ d $\in K$ implies, $spcl(\Gamma,K) = F'$. So, $int(cl(spcl(\Gamma,K)-(\Pi,K))) = \widetilde{\emptyset}$ implies $spcl((\Gamma,K)-(\Pi,K)) \in II$.

4. Soft Pre^{*}-g -open Function

In section introduces new forms of near soft open functions based on the $sIIp^{*}g$-open set notion.

Definition 4.1: The function $f_{\Phi\Gamma}$: $(X,T,K,II) \to (\Upsilon,\overline{T},\mathcal{S},I)$ is said to be

i. Soft-II-pre^{*}-g-$open\ function$, denoted by $sIIp^{*}$g-O-function , if $f_{\Phi\Gamma}((A,K)) \in sIpg$-O$(\Upsilon)$ whenever, $(A,K) \in sIIp^{*}g$-O(X).

ii. Soft-II^{*}-pre^{*}-g-$open\ function$, denoted by $sII^{*}p^{*}g$-O-function , if $f_{\Phi\Gamma}((A,K)) \in sIp^{*}g$-O$(\Upsilon)$ for each $(A,K) \in$ T.

iii. Soft-II^{**}-pre^{*}-g-$open\ function$, denoted by $sII^{**}p^{*}g$-O-function, if $f_{\Phi\Gamma}((A,K)) \in \overline{T}$ whenever,$(A,K) \in s$II$p^{*}g$-O(X).

Proposition 4.2: Let $f_{\Phi\Gamma}$: $(X,T,K,II) \to (\Upsilon,\overline{T},\mathcal{S},I)$ be a $function$, then

1. $f_{\Phi\Gamma}$ is a $sII^{*}p^{*}g$-O-$function$ if $f_{\Phi\Gamma}$ is a soft-open function.
2. $f_{\Phi\Gamma}$ is a soft-open $function$ if $f_{\Phi\Gamma}$ is a $sII^{**}p^{*}g$-O-function.
3. $f_{\Phi\Gamma}$ is a $sII^{*}p^{*}g$-O-$function$ if $f_{\Phi\Gamma}$ is a $sIIp^{*}g$-O-function.
4. $f_{\Phi\Gamma}$ is a sII$p^{*}g$-O-$function$ if $f_{\Phi\Gamma}$ is a $sII^{**}p^{*}g$-O-function.

Proof:

i. Let $(\mathrm{M}, \mathrm{K}) \in \mathrm{T}$ because $\mathrm{f}_{\Phi\Gamma}$ is a soft open function, then $\mathrm{f}_{\Phi\Gamma}$ $((\mathrm{M}, \mathrm{K})) \in \overline{\mathrm{T}}$. By Remark 3.1, $\mathrm{f}_{\Phi\Gamma}$ $((\mathrm{M}, \mathrm{K})) \in sIIp^*g$-O$(\Upsilon)$. Therefore $\mathrm{f}_{\Phi\Gamma}$ is a sII^*p^*go-function.

ii. Let $(\mathrm{M}, \mathrm{K}) \in \mathrm{T}$. By Remark 3.1, $(\mathrm{M}, \mathrm{K}) \in sIIp^*g$-O(X). Since $\mathrm{f}_{\Phi\Gamma}$ is a $sII^{**}p^*go$-function, then $\mathrm{f}_{\Phi\Gamma}((\mathrm{M}, \mathrm{K}))$ is open in $(\Upsilon, \overline{\mathrm{T}}, \mathcal{S})$. Therefore $\mathrm{f}_{\Phi\Gamma}$ is an open $function$.

iii. Let $(\mathrm{M}, \mathrm{K}) \in \mathrm{T}$. By Remark 3.1, (M, K) is a $sIIp^*g$-O(X), since $\mathrm{f}_{\Phi\Gamma}$ is a sIIp^*g-O-function, then $\mathrm{f}_{\Phi\Gamma}$ $(\mathrm{M}, \mathrm{K}) \in$ sIIp^*g-O(Υ). Therefore $\mathrm{f}_{\Phi\Gamma}$ is a sII^*p^*g-O-function.

iv. Let $(\mathrm{M}, \mathrm{K}) \in sIIp^*g$-O(Ӽ). Because $\mathrm{f}_{\Phi\Gamma}$ is a $sII^{**}p^*go\text{-}function$, then $\mathrm{f}_{\Phi\Gamma}((\mathrm{M}, \mathrm{K}))$ is soft open in $(\Upsilon, \overline{\mathrm{T}}, \mathcal{S})$. By Remark 3.1., $\mathrm{f}_{\Phi\Gamma}$ $((\mathrm{M}, \mathrm{K})) \in$ sIIp^*g-O(Υ). Therefore $\mathrm{f}_{\Phi\Gamma}$ is a $sIIp^*g$-o-function.

Example 3.3 and Example 3.4 shows the inverse of Figure 1 and Proposition 4.2 incorrect:

Example 4.3: Let $\mathrm{f}_{\Phi\Gamma}$: $(\mathrm{X}, \mathrm{T}, \mathrm{K}, II) \rightarrow (\mathrm{X}, \mathrm{T}, \mathrm{K}, II')$ be a function whenever, T= $\{ \widetilde{\mathrm{X}}, \widetilde{\emptyset}, (\mathrm{F}, \mathrm{K})$; $\mathrm{F}(\mathrm{r}) = \{\mathrm{a}\}$, for all $\mathrm{r} \in \mathrm{K}\}$ where, $\mathrm{X} = \{\mathrm{a}, \mathrm{h}, \mathrm{r}\}$, K=$\{d_1, d_2\}$, II= $\{\widetilde{\emptyset}\}$ and II'= ŞŞ$(\mathrm{X})_{\mathrm{K}}$ where, ψ: K → K So that $\psi(d_1)$=$\{d_1\}$, $\psi(d_2)$=$\{d_2\}$ and $\Gamma : \mathrm{X} \rightarrow \mathrm{X}$ where $\Gamma(\mathrm{a}) = \{\mathrm{h}\}$, $\Gamma(\mathrm{h}) = \{\mathrm{a}\}$, $\Gamma(\mathrm{r}) = \{\mathrm{r}\}$ then ŞpO(X) = $\{(\mathcal{M}, \mathrm{K})$; a$\in (\mathcal{M}, \mathrm{K})$ for some $(d_{\mathrm{i}} \in \mathrm{K})$, so that i =$\{1,2\}\}$, so $sIIp^*g$-O(X)=$\{(\mathrm{F}, \mathrm{D}), \mathrm{e} \in (\mathrm{F}, \mathrm{D})$ for some $d\}$, sIIp^*g-C(X)=$\{$ ŞŞ$(\mathrm{X})/(\mathrm{R}, \mathrm{D})$; $\mathrm{e} \in (\mathrm{R}, \mathrm{D})$, $(\mathrm{R}, \mathrm{D}) \neq \widetilde{\mathrm{X}}\}$ and $sIIp^*g$-C(X)= $sIIp^*g$-O-(X)= ŞŞ$(X)_{\mathrm{K}}$. Then $\mathrm{f}_{\Phi\Gamma}$ is a $sIIp^*g$-O-function and sII^*p^*g-O-function which is not $sII^{**}p^*g$-O-function and not soft open function. Because $(\mathrm{F}, \mathrm{K}) \in \mathrm{T}$, but $\mathrm{f}_{\Phi\Gamma}$ $((\mathrm{F}, K)) = (\mathrm{R}, \mathrm{K}) \in$ŞŞ $(\mathrm{X})_{\mathrm{K}}$ such that R(r) = {h}, ∀ r ∈ K, But it is not soft open set

Example 4.4: Let $\mathrm{f}_{\Phi\Gamma}$: $(\mathrm{X}, \mathrm{T}, \mathrm{K}, II) \rightarrow (\mathrm{X}, \mathrm{T}, \mathrm{K}, II)$ be a function where, T= $\{\widetilde{\mathrm{X}}, \widetilde{\emptyset}$, (F, K) ; $\mathrm{F}(\mathrm{r}) = \{\mathrm{a}\}$, for all $\mathrm{r} \in \mathrm{K}\}$ such that X= $\{\mathrm{a}, \mathrm{s}, \mathrm{l}\}$, K= $\{d_1, d_2\}$, II= $\{\widetilde{\emptyset}\}$ and II = ŞŞ$(\mathrm{X})_{\mathrm{K}}$ where Φ : K → K such that $\Phi(d_1)$ = $\{\mathrm{l}_1\}$, $\Phi(d_2)$=$\{\mathrm{l}_2\}$ and Γ: X → X where $\Gamma(\mathrm{a}) = \{\mathrm{a}\}$, $\Gamma(\mathrm{s}) = \{\mathrm{s}\}$, $\Gamma(\mathrm{l}) = \{\mathrm{l}\}$ then Şp^*O(X) = $\{(\mathcal{M}, \mathrm{K})$; a$\in (\mathcal{M}, \mathrm{K})$for some $(d_{\mathrm{i}} \in \mathrm{K})$such that i =$\{1,2\}$ }, $sIIp^*g$-C(X) = $sIIp^*g$-O(X)= ŞŞ$(\mathrm{X})_{\mathrm{K}}$ .So, sIp^*g-C-(X)= $\{ \widetilde{\emptyset}, \widetilde{\mathrm{X}}, (\mathrm{F}, \mathrm{K})'$; F' (r) = {s, l} for every (r ∈ K)} and sIp^*g-O(X)= T. then $\mathrm{f}_{\Phi\Gamma}$ is open function and sII^*p^*g-C-function which is not sIIp^*g-O-function and not $sII^{**}p^*g$-O-function. Because $(\mathcal{U}, K) \in sIIp^*g$-O(X) = ŞŞ$(X)_{\mathrm{K}}$ such that $\mathcal{U}$(r) ={l}, $(\forall d \in K)$but $\mathrm{f}_{\Phi\Gamma}$ $((\mathcal{U}, \mathrm{K})) = (\mathcal{U}, \mathrm{K}) \notin sIIp^*g$-O(X) = T.

Definition 4.5: Let $\mathrm{f}_{\Phi\Gamma}$: $(\mathrm{X}, \mathrm{T}, \mathrm{K}, II) \rightarrow (\mathrm{P}, \mathrm{T}', \mathcal{S}, I)$ be a function, then $\mathrm{f}_{\Phi\Gamma}$ is called

i. soft-II-pre^*-g-closed function, dented by $sIIp^*g$-C-function, if $\mathfrak{f}_{\Phi\Gamma}((M,K)) \in sIIp^*g$-C-$(\Upsilon)$ whenever, $(M,K) \in sIIp^*g$-C(X).

ii. soft-II^*-pre^*-g-closed function, denoted by sII^*p^*g-C-function, if $\mathfrak{f}_{\Phi\Gamma}((M,K)) \in sIp^*g$-C-$(\Upsilon)$ whenever, (M,K) is a soft-closed set in (X,T,K).

iii. soft-II^{**}-pre^*-g-closed function, denoted by $sII^{**}p^*g$-C-function, if $\mathfrak{f}_{\Phi\Gamma}(M,K))$ is a soft-closed set in $(P,T',\mathcal{S})$ for each, $(M,K) \in sIIp^*g$-C-(X).

Proposition 4.6: Let $\mathfrak{f}_{\Phi\Gamma}$: $(X,T,K,II) \to (\Upsilon,\overline{T},\mathcal{S},I)$ is a $function$

i. If $\mathfrak{f}_{\Phi\Gamma}$ is soft-closed $function\ then$ $\mathfrak{f}_{\Phi\Gamma}$ is a sII^*p^*g-C-$function$.

ii. If $\mathfrak{f}_{\Phi\Gamma}$ is $sII^{**}p^*g$-C-$function\ then$ $\mathfrak{f}_{\Phi\Gamma}$ is a $soft$-$closed\ function$.

iii. If $\mathfrak{f}_{\Phi\Gamma}$ is $sIIp^*g$-C-$function\ then$ $\mathfrak{f}_{\Phi\Gamma}$ is a sII^*p^*g-C-$function$.

iv. If $\mathfrak{f}_{\Phi\Gamma}$ is $sII^{**}p^*g$-C-$function\ then$ $\mathfrak{f}_{\Phi\Gamma}$ is a $sIIp^*g$-C-$function$.

5. Discussion and Conclusion

In this study, a new weak open soft set ($sIIp^*g$-open soft set) was presented based on soft set in soft topological space with an ideal, and its characteristics and relationship with other soft set were studied. The Figure 1 shows the relationship of the $sIIp^*g$-open soft set that was reached with other soft sets.

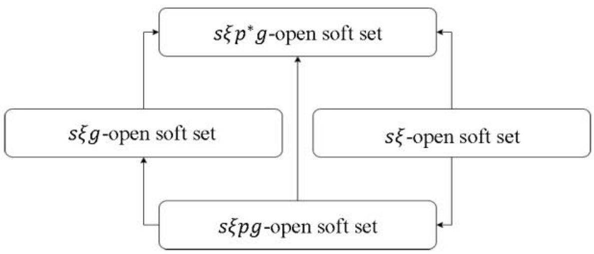

Figure 1
The relationship of the $sIIp^*g$-open soft set with other soft sets.

The following Figure 2 depicts the links between these sorts of near soft-closed functions.

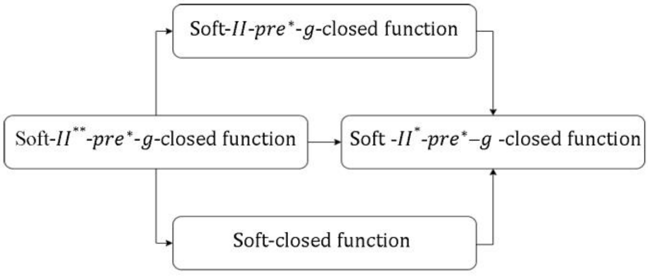

Figure 2
Soft-II-pre^*-g- closed function

The following Figure 3 demonstrates the relationships between the different concepts mentioned in definition 4.1.

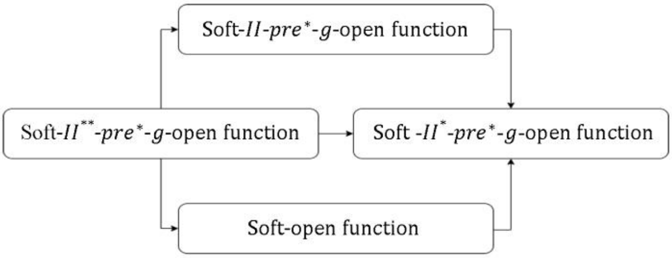

Figure 3
Soft-$\boldsymbol{II}$-$\boldsymbol{pre^*}$-$\boldsymbol{g}$-$\boldsymbol{open}$ function.

References

[1] D. A. Molodtsov, "Soft set theory—first results," *Computers & Mathematics with Applications*, vol. 37, pp. 19–31 (1999).

[2] A. A. Hammood and R. B. Esmaeel, "On Soft Pre-Generalized-Closed Sets with New near Soft function," in *Journal of Physics: Conference Series*, vol. 1897, no. 1, p. 012032 (2021).

[3] R. J. Mohammad and R. B. Esmaeel, "On Separation Axioms With Soft-J-Semi-g-Open Sets," in *IOP Conference Series: Materials Science and Engineering,* vol. 871, no. 1, p. 012052 (2020).

[4] A. M. Abd El-latif, "Supra soft topological spaces," *Jökull Journal,* vol. 8, no. 4, pp. 1731–1740 (2014).

[5] B. Ahmad and A. Kharal, "Mapping of soft classes," *Mathematical and Natural Computing*, vol. 7, no. 3, pp. 471–481 (2011).

[6] M. I. Ali, F. Feng, X. W. K. Liu, and M. Shaber, "On some new operations in soft set theory," *Computers & Mathematics with Applications*, vol. 57, no. 9, pp. 1547–1553 (2009).

[7] K. V. Babita and J. J. Sunil, "Soft set relations and functions," *Computers & Mathematics with Applications,* vol. 60, no. 7, pp. 1840–1849 (2010).

[8] N. Çağman, S. Karataş, and S. Enginoğlu, "Soft topology," *Computers & Mathematics with Applications,* vol. 62, pp. 351–358 (2011).

[9] S. A. El-Sheikh and A. M. Abd El-latif, "Decompositions of some types of supra soft sets and soft continuity," *International Journal of Mathematical Trends and Technology*, vol. 9, no. 1, pp. 37–56 (2014).

[10] T. H. Majeed, "Some types of topological groupoids and their action," in *American Institute of Physics Conference Series*, vol. 2834, no. 1, p. 080121 (2023).

[11] H. Hazra, P. Majumdar, and S. K. Samanta, "Soft topology," *Fuzzy Information and Engineering,* vol. 4, no. 1, pp. 105–115 (2012).

[12] S. Hussain and B. Ahmad, "Some properties of soft topological spaces," *Computers & Mathematics with Applications*, vol. 62, pp. 4058–4067 (2011).

[13] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif, "Soft ideal theory, soft local function and generated soft topological spaces," *Applied Mathematics & Information Sciences*, vol. 8, no. 4, pp. 1595–1603 (2014).

[14] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, and A. M. Abd El-latif, "γ-operation and decompositions of some forms of soft continuity in soft topological spaces," *Applied and Fundamental Mathematics and Informatics* (AFMI), vol. 7, pp. 181–196 (2014).

[15] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers & Mathematics with Applications,* vol. 45, pp. 555–562 (2003).

[16] T. H. Majeed, "On some results of topological groupoid," in *Journal of Physics: Conference Series*, vol. 1003, no. 1, p. 012067 (2018).

[17] H. I. Mustafa and F. M. Sleim, "Soft generalized closed sets with respect to an ideal in soft topological spaces," *Applied Mathematics & Information Systems,* vol. 8, no. 2, pp. 665–671 (2014).

[18] M. Shabir and M. Naz, "On soft topological spaces," *Computers & Mathematics with Applications,* vol. 57, pp. 1786–1799 (2011).

[19] J. Subhashini and C. Sekar, "On soft pre generalized closed sets in soft topological spaces," *International Journal of Engineering Trends and Technology,* vol. 12 (2014).

[20] I. Zorlutuna, M. Akdağ, W. K. Min, and S. Atmaca, "Remark on soft topological spaces," *Annals of Fuzzy Mathematics and Informatics*, vol. 3, no. 2, pp. 171–181 (2012).

[21] S. I. Mahmood, "On weakly soft omega open functions and weakly soft omega closed functions in soft topological spaces," *Iraqi Journal of Science*, pp. 1094–1106 (2017).

[22] A. J. Bidaiwi and N. A. Dawood, "Soft strongly generalized mapping with respect to an ideal in soft topological space," *Ibn Al-Haitham Journal for Pure and Applied Science*, vol. 28, no. 2, pp. 171–183 (2017).

[23] S. I. Mahmood and A. A. Abdul-Hady, "Weak forms of soft (1,2)*-omega open sets in soft bitopological spaces," *Iraqi Journal of Science*, pp. 572–580 (2018).

[24] R. B. Esmaeel, "Certain types of soft sets in the games theory," *Iraqi Journal of Science* (2025).

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