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Certain types of topological groupoids

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Abstract

Our main the goal is to work upon certain types of the topological groupoid, source correct groupoid, submersion groupoid, source proper groupoid-space and submersion groupoid-space to get some outcomes of topological groupoid. We are concerned to study the relations of source proper groupoid-space and submersion groupoid, source proper groupoid-space and submersion groupoid-space respectively.

Subject Classification: *Primary 93A30, Secondary 49K15.*

Keywords: *Groupoid, Topological group, Source proper groupoid, Submersion groupoid.*

1. Introduction

The research objective is to study specific topological groupoid types, this is the source appropriate groupoid, denoted by (SR- groupoid), submersion groupoid denoted by (SSR- groupoid), appropriate source groupoid-space denoted by (SRR- space) and submersion groupoid-space denoted by (SSRR-space), and the connections between them to acquire new attributes.

2. Topological groupoid

Within this section, Our present the core ideas of this research. A groupoid is a topological groupoid. (R, S) in addition to topologies on R & S to the extent that the maps α, β, w, γ & σ are ongoing where R×R The subspace topology is derived from R×R. [5], [4]. Let (R, S) be a groupoid with topology, N be a space

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with topology, π: N→S be an ongoing map and allow R*N continuous map and permit α & π above that:

$\pi\big(\theta^*(r,z)\big) = \beta(r)$ for each $(r,z) \in R * N$. $\theta^*\big(w\big(\pi(z)\big),z\big) = Z$for each $z \in M$. $\theta^*\big(r,\theta^*(\acute{r},z)\big) = \theta^*(\gamma(r,\acute{r}),z)$ for every $(r,\acute{r}) \in R * R$ and $(r,z) \in R * N$, The package (N,π,S) Besides taking actionθ^* It is known as a groupoid space, and it is represented by R-space. Likewise, we can specify the proper course of action for R upon (N,π,S). [12-14] Allow (N,π,S) be an R-space, then the portion that is not $R_{(Z)} = \{\theta^*(r,z)|r \in R_{\pi(Z)}\}$ is z's orbit beneath R. The behavior of R upon (N,π,S) Its cost-free if each $(r,z) \in R \times N$, the connection $\theta^*(r,Z) = Z$ demonstrates that r is unity.(ii) What the action was R upon (N,π,S) functions as a transitive if for every $(z,\acute{z}) \in N \times N$, In addition to $r \in R$ so that. $\acute{r} = \theta^*(r,z)$.[10]. Allow me to (N,π,S) being R-space. Afterward: It is what we say (N,π,S) R-space is available for free. This paper declares the (N,π,S) is moving R-space when activity of R upon (N,π,S) be moving. [7]. A groupoid with topology (R, S) referred to as the source appropriate groupoid ((SR- SR-groupoid)) if: The original map $\alpha: R \to S$ be proper. The foundational area S is Hausdorff. [9], [6]. The transitive SR-groupoid (R, S) is known as submersion the groupoid (SSR- groupoid), if the chart $\beta_x: R_x \to S$ is immersion for each $s \in S$. [8], [11-14]. A R-space (N,π,S) source appropriate groupoid space is the term for it (SRR-space) if :(i) the groupoid of cations $(R * N,N)$ be SR- SR-groupoid (ii) (N,π,S) be a transitive and free R-space. [2], [3], A SRR-space (N,π,S) is known as submersion space groupoid " SSR-space" Should (R, S) is submersion groupoid. [1]

3. SR-Groupoid, SSR- Groupoid, SSR-Space, SSRR-Space

In this part introduce new properties of certain types of topological groupoid.

Proposition 1: Let (R, S) be an SSR- groupoid then (N, π, S) is SSR-Space for every x ∈ S, where R_x is the α – fiber space.

Proof: (N,π,S) is R-Space by the subgroup $R(Z) = \{\theta^*(r,z)|r \in R_{\pi(Z)}\}$ represents z's orbit under R. The action that R took on (N,π,S) Its cost-free if each $(r,z) \in R \times N$, The relationship $\theta^*(r,z) = z$ suggests r is unity. The act of R on (N,π,S) is a transitive verb if for Every one $(z,\acute{z}) \in N \times N$, There's $r \in R$ so that $\acute{r} = \theta^*(r,z)$ & (N,π,S) is R-space Consequently, we state that (N,π,S) is without charge. We state that (N,π,S) be in transitive R-space If there is activity of R upon (N,π,S) transitive. Let the action groupoid $(R * N,N)$ is SR- groupoid. The foundational area N be Hausdorff. The source map $\alpha_x R * N \to N$, $\alpha_x(r,\acute{r}) = \acute{r}$ is proper since α-fiber space N is compact (R, S) is SR- groupoid). And then the map $Pr_1: R \times N \to R$ is proper map. And then $P_1 = Pr_1|_{R*N} : R * N \to N$ is proper since $R * N = (\alpha \times \pi)^{-1}(\Delta S)$ is closed subspace of $R \times N(S\ is\ Hausdorff)$ and by using the following commutative Figure 1 fiber spaces .

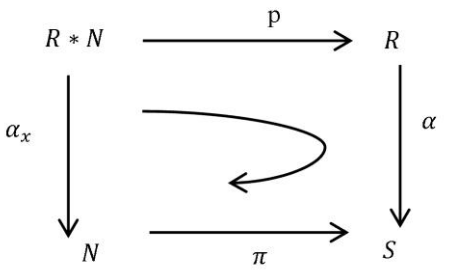


Figure 1
Fiber spaces

In which α, P_1 and π are proper maps Afterward $\pi \circ \alpha_x$ be proper a map & then α_x is proper a map (N is Hausdorff).

Proposition 2: Let (R_x, β_x, S) be an SSRR then the map $\beta_x: R_x \to S$ is submersion map, for every $x \in B$.

Proof: Let $r \in R_x$, $\beta_x(r) = s \in S$ After it, there's an open neighborhood. $\coprod_s$ of s within S along with constant right invers $v: \coprod_s \to R_x$ such that $v \circ \beta_x(r) = r\big((R, S)\big)$ be an SSR- groupoid and after that $\beta_x: R_x \to S$ is submersion. Now , define $\psi_r(\acute{s}) = \theta^*\left(\gamma\left(v(\acute{s}), \sigma\big(\beta_x(r)\big)\right), r\right)$, for all $\acute{s} \in \coprod_s$, wherein θ^* is considered as action law of R on (R_x, β_x, S), θ^* is continuous and $\beta_x \circ \psi_r = \theta^*(\gamma\big(v(\acute{s}), \sigma\big(\beta_x(r)\big), \gamma\big) = \beta(\gamma\big(v(\acute{s}), \sigma\big(\beta_x(r)\big), r\big) \quad = \beta\left(v(\acute{s})\right) = \big(\beta \circ v(\acute{s})\big) = \acute{s} = I_{\coprod_s}(\acute{s})\big)$. for all $\acute{s} \in \coprod_s$, and $\psi_r \circ \beta_x(r) = \psi_r\big(\beta_x(r)\big) = \theta^*(v\big(\beta_x(r)\big), \sigma(v\big(\beta_x(r)\big), r) = \theta^*\big(w\big(\beta_x(r)\big), r\big) = r$, for all $r \in R$ .Hence a map $\beta_x: R_x \to S$ is submersion.

Proposition 3: Let the map $\pi: N \to S$ be submersion then N is $SSR_{\pi(r)}R_{\pi(r)}$ −space & for everyone $z \in N$.

Proof: N be $SR_{\pi(r)}R_{\pi(r)}$ −space & for all $z \in N$ since if (R, π, S) be an SRR-space Afterward, N be $SR_{\pi(Z)}R_{\pi(Z)}$ −room for everyone $z \in N$.To show that the map $P_z: N \to N/_{\pi(Z)}R_{\pi(Z)}$ is submersion .The maps $P_z: N \to N/_{\pi(Z)}R_{\pi(Z)}$ and $\pi: N \to S$ are both recognized (π is appropriate for surjective), since if (R,S) is an SR- groupoid & (N, π, S) is SRR-space then a map $\pi: N \to S$ be proper and constant on each other's fiber. Thus, the following's dotted arrows Figure 2 submersion map.

Exist and are distinct in T because to the identification map's universal characteristic, and the map η_Z is provided by $\eta_Z\big(P_z(\acute{z})\big) = \pi(\acute{z})$. now, let $\acute{z} \in N$, $P(\acute{z}) \in N/_{\pi(Z)}R_{\pi(Z)}$, $\eta_Z\big(P_Z(\acute{z})\big) = \pi(\acute{z}) \in S$. Next is a neighborhood that is open $\coprod_Z$, off $\pi(\acute{z})$ within S and unbroken right invers $\psi_{\acute{z}}: \coprod_{\acute{z}} \to N$ to $\pi: N \to S$ such that $\psi_{\acute{z}} \circ \pi(\acute{z}) = \acute{z}$ (π is submersion by proposition(1)). Now

$\eta_z^{-1}(\coprod_s)$ is an open neighborhood of $P(\acute{z})$ in $N/_{\pi(z)}R_{\pi(z)}$ and define $V_{\acute{z}}: \psi_{\acute{z}} \circ \eta_Z^{-1}(\coprod_{\acute{z}}) \overset{\eta z}{\to} \coprod_{\acute{z}} \overset{\psi z}{\to} N$, by $V_{\acute{z}}(\acute{r}) = \psi_{\acute{z}} \circ \eta_Z(\acute{r})$, $V_{\acute{z}}$ is continuous and $P_Z \circ \psi_{\acute{z}}(\acute{r}) = P_Z \circ \psi_{\acute{z}} \circ \eta_Z(\acute{r}) = \eta_Z^{-1} \circ \eta_z \circ P_z \circ \psi_{\acute{z}} \circ \eta_Z(\acute{r}) = \eta_Z^{-1} \circ \pi \circ \psi_{\acute{z}}$, $\circ \eta_Z(\acute{r}) = \eta_Z^{-1} \circ \eta_Z(\acute{r}) = r \; and \; V_{\acute{z}} \circ P_Z(\acute{z}) = \psi_{\acute{z}} \circ \eta_z \circ P_z(\acute{z}) = \psi_{\acute{z}} \circ \pi(\acute{z}) = \acute{z}$.Hence $V_{\acute{z}}$ is continuous right invers to P_z and then P_z is submersion.

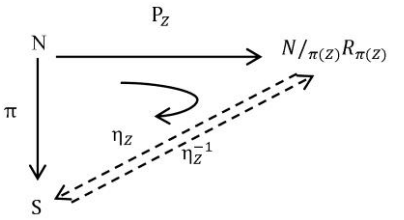


Figure 2
Submersion map

Proposition 4: Let (R_x, β_x, S) be an SSRR-Space then Ehresman groupoid $(N \times N/_{\pi(Z)}R_{\pi(Z)})$ is SSR- groupoid for every $\in N. x \in S$.

Proof: Let $\acute{R} = N \times N/_{\pi(Z)}R_{\pi(Z)}$, $\acute{S} = N/_{\pi(Z)}R_{\pi(Z)}$ $(\acute{R}, \acute{S})$ be transitive SP-groupoid since if (N, π, S) b an SRR-Space Afterward N be $SP_{\pi(Z)}R_{\pi(Z)}$ −space for everyone $z \in N$. Now to prove $\acute{\beta}_{\acute{x}}: \acute{R}_{\acute{x}} \to \acute{S}$ is submersion for every $\acute{x} \in \acute{S}$. Let $r = [(\acute{z}, \acute{\acute{z}})] \in \acute{R}_{\acute{x}}, \acute{\beta}_{\acute{x}}(r) \in \acute{S}$ and then the neighborhood is open $\coprod of \; P_Z(z)$ as well as ongoing invers $v: \coprod_{\acute{z}} \to S$ to $P_Z: N \to \acute{S}$ such that $V \circ P_Z(\acute{z}) = \acute{z}$. (P_Z is submersion by proposition (2). Now define $\acute{\psi}_{\acute{x}}: \coprod \to \acute{R}_{\acute{x}}$ by: $\acute{\psi}_{\acute{x}}(t) = [(V(t), \acute{\acute{z}})]$ for every $t \in \coprod$. $\acute{\beta}_{\acute{x}} \circ \acute{\psi}_{\acute{x}} = \acute{\beta}[(V(t)z)] = P_Z \circ V(t) = t$ and

$$\acute{\psi}_{\acute{x}} \circ \acute{\beta}_{\acute{x}} = \acute{\psi}_{\acute{x}} \circ \beta_x[(\acute{z}, \acute{\acute{z}})] = \acute{\psi}_{\acute{x}} \circ P(\acute{z}) = [(V(P(\acute{z}), \acute{\acute{z}})] = [(\acute{z}, \acute{\acute{z}})] = r .$$

Hence $\acute{\beta}_{\acute{x}}: \acute{R}_{\acute{x}} \to \acute{S}$ is submersion and then $(\acute{R}, \acute{S})$ is SSR- groupoid.

4. Conclusion

Recent research on specialized classes of topological groupoids—namely, source groupoids (denoted as SR-groupoids) and submersion groupoids (denoted as SSR-groupoids)—along with their associated spaces (SRR-spaces for source groupoid-spaces and SSRR-spaces for submersion groupoid-spaces), has uncovered novel properties and relationships among these structures.

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