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Application of the Shehu transform and science and technology

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Abstract

The paper investigates the Laplace Transform's effectiveness in solving mathematical models for engineering problems involving simple electric circuits and mechanical analysis and population dynamics and economic systems. The paper uses Shehu Transform Method as an investigation technique for studying beta & gamma functions & their application to simultaneous differential equation systems. The research displays useful examples to demonstrate how well the proposed technique performs in practice.

Subject Classification: *46N10, 47N10, 90C30.*

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1. Introduction

Various initial or boundary conditions present in differential equations can be solved with the help of integral transforms. Operational transformations are extended mathematical quantifiers that are broadly applied in numerous areas of physical science, astronomy, engineering, and so on [4]. There are numerous published papers dealing with theory & application for integral transform like as Mellin & Laplace & Hankel transforms & Fourier [3– 4]. The Shehu Transform is a very general class of integral transforms and this transform has shown to be very effective in solving linear ordinary and partial differential equation. In our daily life, mathematical models and applications are present or used in electrical engineering & signal processing & physics & optics & control systems, and mathematics [8] and [9]. Since Shehu Transform is easily applicable in several differential equations with initial or boundary conditions we directly employed the relationship in the transform of a given function. While the Laplace Transform has a different definition while the Shehu Transform is in some way more relaxed in many mathematical and engineering practices.

Comparison Between the Shehu and Laplace Transforms: While both the Shehu and Laplace transforms are widely used to solve ordinary differential equations, they differ significantly in their formulation, properties, and areas of application. Laplace Transform define as: $F(s) = L\{\mathfrak{f}(\mathrm{t})\} = \int_0^{\infty} e^{-st}\mathfrak{f}(\mathrm{t})d(\mathrm{t})$.

On a set for function, Shehu transform for $u(\mathrm{t})$ of exponential order define as:

$$A = \left\{u(\mathrm{t}) \colon \exists N, \eta_{1,}\eta_2 \ > 0, |u(\mathrm{t})|\ N\ U\ U\ \left(\frac{|\mathrm{t}|}{\eta_i}\right)\ , where\ \mathrm{t} \in (-1)^i \times [0, \infty)\right\},$$

$$S[u(\mathrm{t})] = U(q, v) = \int_0^{\infty}\ U\ U\ \left(\frac{-q\mathrm{t}}{v}\right) u(\mathrm{t})dt \tag{1}$$

Dirichassume that Conditions for Shehu Transformability: For a time-domain function u(t) to be eligible for the Shehu Transform, it must satisfy a set of conditions similar in nature to the classical Dirichassume that conditions, which ensure the existence and convergence of the integral transform. These conditions are essential to guarantee the mathematical validity and applicability of the transform. The function u(t) must meet the following criteria [1] and assume that exp exp $\quad = U$:

1. when t > 0, u(t) has finite numbers for the isolated finite discontinuities but should be singleton value since it piecewise continuous.
2. when u(t) is of a exponential order, and its less than $\left(\frac{-q\mathrm{t}}{v}\right)$ when t→ ∞ where *S* positive constant. & $q\ and\ v$ true positive amount.

I If a function u(t) fulfill, l Dirich assume that requirement, so

$U(q, v) = \int_0^\infty U\, U \left(\frac{-qt}{v}\right) u(\mathrm{t})dt$ write $S[u(\mathrm{t})]$ and named Shehu transformations for $u(\mathrm{t})$. An integral $\int U\, U \left(\frac{-qt}{v}\right) u(\mathrm{t})dt$ converges if $\int\left|U\, U \left(\frac{-qt}{v}\right) u(\mathrm{t})\right| dt < \infty$ and is especially powerful in analyzing linear time-invariant systems, particularly in control theory, electrical circuit analysis, and mechanical vibrations. On one hand, the known transforms and the inverse are listed in its table making ATA an some of the most used method in most classical engineering issues. On the other hand the Shehu Transform is a comparatively more recent integral transform which has been tested widely for versatility in dealing with a set of differential equations especially with the Beta and Gamma function. Equally important, it becomes effective in solving the equations with some special boundary conditions or where the Laplace transforms is most onerous or even less efficient. Further, Shehu Transform is most general in the sense that more kernel functions may be used or the initial/boundary conditions are beyond of those assumed by Laplace transformation. Thus, while the Laplace Transform remains foundational in engineering and applied mathematics, the Shehu Transform offers a powerful complementary technique with distinct advantages in modern mathematical modeling.

2. Some Important Properties of Shehu Transforms

Since the Shehu transforms of various standard functions are already extensively documented in existing mathematical and engineering literature, this paper will not focus on re-deriving or listing them in detail. Instead, the reader is referred to foundational references for comprehensive tables and derivations of these transforms [1, 2–7]. However, for the purposes of this study, we highlight several essential properties of the Shehu Transform that will be instrumental in the application sections that follow. These properties provide the theoretical foundation for solving differential equations using the Shehu method and facilitate its implementation in practical engineering and mathematical modeling problems.

Property 1: Assume that the functions $au(\mathrm{t})$ and $b\omega(\mathrm{t})$ be in set A, then $\left(au(\mathrm{t}) + bv(\mathrm{t})\right)\mathcal{E}A, a, b > 0$ arbitrary constants,

$$S[au(\mathrm{t}) + bv(\mathrm{t})] = aS[u(\mathrm{t})] + bS[v(\mathrm{t})] \tag{2}$$

Proof: By definition of Shehu transform.

$$S[au(\mathrm{t}) + bv(\mathrm{t})] = \int_0^\infty U\, U \left(\frac{-qt}{v}\right)(a\, u(\mathrm{t}) + bv(\mathrm{t}))dt$$
$$= \int_0^\infty U\, U \left(\frac{-qt}{v}\right)(\, au(\mathrm{t}))dt + \int_0^\infty U\, U \left(\frac{-qt}{v}\right)(\, bv(\mathrm{t}))dt$$
$$= a\int_0^\infty U\, U \left(\frac{-qt}{v}\right) u(\mathrm{t})dt + b\int_0^\infty U\, U \left(\frac{-qt}{v}\right) v(\mathrm{t})dt$$
$$= av\int_0^\infty U\, U\, (-q\mathrm{t})\, u(\, v\mathrm{t})dt + bv\int_0^\infty U\, U\, (-q\mathrm{t})\; v(\, v\mathrm{t})dt$$
$$= aS[u(\mathrm{t})] + bS[v(\mathrm{t})]$$

Lemma 1: *When* $u(\mathrm{t})$ *is a* n^{th} *derivative for* $u(\mathrm{t}) \in A$ *related to* t, *so its Shehu transformations define as:*

$$S[u'(\mathrm{t})] = \frac{q}{v} U(q, v) - \varphi u(0) \tag{3}$$

$$S[u''(\mathrm{t})] = \frac{q^2}{v^2} U(q, v) - \frac{q}{v} u(0) - u'(0) \tag{4}$$

$$S[u'''(\mathrm{t})] = \frac{q^3}{v^3} U(q, v) - \frac{q^2}{v^2} u(0) - \frac{q}{v} u'(0) - u''(0) \tag{5}$$

Proof: We assume that equation (1.5) true when *n=r*, we obtain

$$S\left[u^{(r)}(\mathrm{t})\right] = \frac{q}{v} S[u^{(r)}(\mathrm{t})] - u^{(r)}(0)$$

$$= \frac{q}{v}\left[\frac{q^r}{v^r} S[\ u(\mathrm{t})\ ]\ -\ \sum_{i=0}^{r-1} \left(\frac{q}{v}\right)^{r-(i+1)} u^{(i)}(0)\right] - u^{(r)}(0)$$

$$= \left(\frac{q}{v}\right)^{r+1} S[u(\mathrm{t})] - \sum_{i=0}^{r} \left(\frac{q}{v}\right)^{r-i} u^{(i)}(0)\ ,$$

By applying the principle of mathematical induction, we can extend the validity of the preceding equation. Assuming that the relation holds true for an arbitrary positive integer n=r, we aim to demonstrate that it also holds for n=r+1. This step is achieved by substituting n=r+1 into the recursive formulation and confirming that the left-hand and right-hand sides remain equivalent under the assumptions made. Therefore, for induction, the equation is true for all n∈N and that completes the proof.

S.No	$\boldsymbol{u(\mathrm{t})}$	$\boldsymbol{S[u(\mathrm{t})]}$
1	1	$\frac{s}{v}$
2	T	$\frac{s^2}{v^2}$
3	$Ua(\mathrm{t})$	$\frac{v}{s - av}$
4	$sin(at)$	$\frac{av^2}{s^2 + a^2v^2}$
5	$cos(at)$	$\frac{vq}{s^2 + a^2v^2}$
6	$\mathrm{t}\, a(\mathrm{t})$	$\frac{v^2}{(s - av)^2}$
7	$\frac{U\ (bt)\ sin(at)}{a}$	$\frac{v^2}{(s - bv)^2 + a^2v^2}$
8	$\frac{U\ (at)}{b - a}$	$\frac{v^2}{(s - av)(s - bv)}$

II. Application for Shehu transforms

This part describes application for the Shehu transform across multiple scientific and engineering domains. The Shehu Transform also turns out to be a quite effective method providing solutions to problems which are associated with differential equations, some particular functions and other complicated boundary value. Because of this, it has been proved to be a useful tool in simplifying and coming up with easier ways of solving various problems in applied mathematics as well as engineering.

1. The Shehu Transform in the analysis of Electric Circuits

It applies for witching transient phenomena of series and parallel RL, RC or RLC circuits and can be transformed to solve them [4]. This application follows next

Consider: The electrical components L, R, and C connect series in a circuit with an electromotive force ϵ that contains resistance value R and inductance element L together with capacitive component C. The circuit utilizes a built-in switch. The circuit evaluation using Kirchhoff's Voltage Law results in this mathematical expression: $LI + +rI + \frac{Q}{C} = \mathcal{E}$

Example 1: A circuit consisting of R=16Ω and L=2H together with C=0.02F forms a series connection. The circuit accepts a constant voltage source of ϵ=300V which appears in Figure 1. The circuit starts with t=0 while the capacitor holds zero charge and there is no current flow throughout the circuit. The circuit setup leads to a second-order linear differential equation that describes its operation yet the initial conditions represent the uncharged capacitor and the nonflow of current through the circuit at the beginning of the analysis.

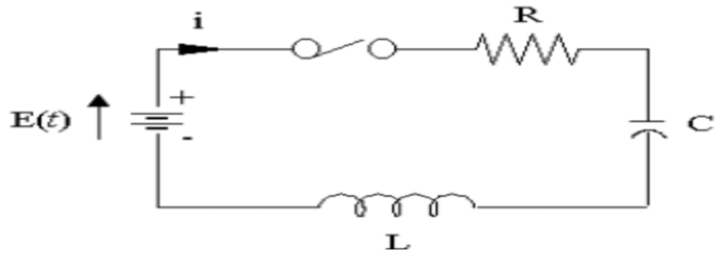

Figure 1
Shehu transforms in analysis for electric circuits

get charge & current at a time, we have:

$LI' + +rI + \frac{Q}{C} = \mathcal{E}$ that is $LQ'' + 8Q + 25Q = 150$ where the initial conditions $Q = 0$ & $I = O$ when t=0

Applying Shehu transforms of the both side for above equation, so:

$$LS[Q''] + +8S[Q'] + 25S[Q] = S[150]$$

$$\frac{q^2}{v^2}\, Q(q,v) + \frac{q}{v} q(0) + q'(0) + 8\left[\frac{q}{v}\, Q(q,v) + q(0)\right] + 25Q(q,v) = 150\frac{1}{q}$$

Substitute the initial conditions implies:

$$\frac{q^2}{v^2}\, Q(q,v) + 8\left[\frac{q}{v}\, Q(q,v)\right] + 25Q(q,v) = 150\frac{1}{q}.$$

thus: $Q(q,v) = \frac{150v^2}{q^2+8q+25}$

The inverse transform given: $Q = 6 - 6e^{-4t}\, cos\, cos\, 3t - 8\, sin\, sin\, 3t$ and $I = 50e^{-4t}\, cos\, cos\, 3t$

By above equations, Q & I can be extracted at arbitrary time t> 0.

2. Applications of solve simultaneous differential equation

In engineering disciplines, many real-world systems are inherently complex and cannot be accurately described by a single differential equation. Linear differential equations systems appear when multiple interconnected variables dynamically affect each other throughout continuous periods of time. As an example electrical engineers need simultaneous differential equations to analyze multiple branches in harmonically connected RLC circuits. Several masses connected by springs produce a coupled system because the displacement of one object impacts all others necessitating single-step solution methods that become more complex when initial factors or boundary requirements exist. The Shehu Transform alongside Laplace corresponds as a conversion technique for converting systems into transform-domain algebraic expressions that become simpler to ease. The solution starts by applying inverse transformation after solving in the transform domain to find a response for initial problems. The paper demonstrates Shehu Transform applications for simultaneous systems while emphasizing its usefulness for solving initial value problems containing special functions and complex couplings.

Example 2: Assuming that a particle moves along the plane path. At t & the particle has coordinate (w, z) move along a path express as w'+2 z =sin(2 t) & z'-2w=cos(2 t) , (t > 0). where t = 0, w = 1 , z = 0, we get from Laplace transforms particle move through a path . so from Shehu transforms we proof a particle move through a path $4z^2 + 4wz + 2w^2 = 1$ put Shehu transforms for given simultaneous equation, implies:

$$\frac{q}{v} W(q,v) - W(0) + 2Z(q,v) = \frac{2v}{q^2+v^2}$$

or
$$2Z(q,v) + \frac{q}{v} W(q,v) = q^2 + 4v^2$$

So

$$W(q,v) = -\frac{2v}{q^2+4v^2}\ , w(\mathrm{t}) = -sin2\mathrm{t}$$

from (3.2.1) $Z(q,v) = \frac{1}{2}\left[\frac{2v}{q^2+4v^2} + \frac{2qv}{q^2+4v^2}\right]$

Invers Shehu transform, we obtain:

$$z(\mathrm{t}) = \frac{1}{2}(sin2\mathrm{t} + cos2\mathrm{t})$$

Hence $2z(\mathrm{t}) = \frac{1}{2}(sin2\mathrm{t} + cos2\mathrm{t})$

$$2z + w = cos2\mathrm{t}$$

$$(2z + w)^2 = 1 - 2\mathrm{t} = 1 - w^2$$

$$4z^2 + 4zw + 2w^2 = 1$$

Example 3: Shehu transform in a vibrating mechanical system, assume us consider a vehicle and mass m. A main factor for check vehicle's suspension systems are mass car has the damper & springs that used to connect car body to the suspension links. Transitive Mechanical structures have three main components: mass (Mkg), spring stiffness K, in Nm−1, and dampers D, in N S M - 1.

$$y'' + Dy' + Ky = F(\mathrm{t}) \quad or \quad y'' + 2y' + 5y = 2sinwt \tag{6}$$

Taking Shehu transforms in (6) gives merging properties of Shehu transform, we get:

$$S[(y')] + 2S[y'] + 5S[(x)] = 2S(sin\ sin\ (wt)$$

i.e . $\left(\frac{q^2}{v^2} + 2\frac{q}{v} + 5\right)S[y(\mathrm{t})] = \frac{q}{v}y(0) + y'(0) + 2y(0) + \frac{2vw}{q^2+v^2w^2}$

with y(0) = y'(0) = 0 , hence $\mathrm{Y}(q,v) = \frac{2Wv^2}{(q^2+2vq+5v^2)(q^2+W^2v^2)}$

By use partition fraction of different value for w (say w =1), implies:

$$\mathrm{Y}(q,v) = \frac{2v^2}{(q^2+2vq+5v^2)(q^2+v^2)} = \frac{Aq+B}{q^2+v^2} + \frac{Cq+D}{q^2+2vq+5v^2}$$

Now, on Taking inverse Shehu transforms, we have:

$$y = \frac{-1}{4}cost - \frac{3}{8}sint + e^{-\mathrm{t}}\left(\frac{1}{4}cos2\mathrm{t} + \frac{23}{8}cos3\mathrm{t}\right)$$

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