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Using the Kamal decomposition method for solving nonlinear partial differential equations

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Abstract

This survey, prepare a new techniques through coupled integral transform (IT) as Kamal integral transformation(KIT) with Adomian decomposition method (ADM)in which assigned Kamal Decomposition Method(KDM).This technique is attractive and useful to confirmed the exact solution (ES) by using simple steps of procedure obtained from produce primary of (KIT) and derivion (KDM) for solving nonlinear partial differential equations (NPDEs) is the perfect results acquired. Also to understand (KDM) presented (ES) through equip applications show that effective techniques.

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1. Introduction

In the past, many researchers presented applied mathematics by using major role as differential equations. Their powerful techniques for solving (PDEs) by (IT), but to find solution of (NPDEs) not directly must drive new techniques coupled with (IT) as (ADM). Moreover, to drive solution for (NPDEs) applied (KDM) obtained (ES). In [1] informed the new transform us (KIT). Also prepared dualities between (KIT) and some integral transforms in [2]. the literature for solving (differential and Integral) equations with support (KIT) in [3]-[5].There are various cases in the field of science is effective used (IT) combined with approximate method that offered in [6]-[13]. In this task prepared a new techniques us (KIT) for solved (NPDEs) to find the (ES) assist with (ADM) preformed as (KDM) that displayed through major concepts of (KIT) and derive on (KDM). Moreover solving several applications from exhibit professional techniques to offered (ES) with simple steps of procedure for settling (NPDEs).

2. Preliminaries of (KIT)

The (KIT) is derived in [1] by operator $\mathcal{K}(.)$ defined as:

$$Ķ(ʄ(ţ)) = g(\upsilon) = \int_0^\infty ʄ(ţ)e^{(-\frac{ţ}{\upsilon})}\,dţ, ţ \geq 0, \kappa_1 \leq \upsilon \leq \kappa_2 \quad (1)$$

$$Ɏ = \left\{ʄ(ţ): \exists\, ɱ,\ \kappa_1, \kappa_2 \geq 0.\ |ʄ(ţ)| < ɱ e^{\frac{|ţ|}{k_ɟ}}, if\ \ ţ \in (-1)^ɟ \times [0, \infty)\right\}. \quad (2)$$

In [3], prepared derivatives of (PDEs) get:

$$Ķ\left[\frac{\partial ʄ(\varkappa,ţ)}{\partial ţ}\right] = \upsilon^{-1}\, g(\varkappa,\upsilon) - ʄ(\varkappa,0) \quad (3)$$

$$Ķ\left[\frac{\partial^2 ʄ(\varkappa,ţ)}{\partial ţ^2}\right] = \upsilon^{-2}\, g(\varkappa,\upsilon) - \upsilon^{-1}\, ʄ(\varkappa,0) - \frac{\partial ʄ(\varkappa,ţ)}{\partial ţ} \quad (4)$$

$$Ķ\left[\frac{\partial^n ʄ(\varkappa,ţ)}{\partial ţ^n}\right] = \upsilon^{-n} g(\varkappa,\mathcal{V}) - \upsilon^{-(n-1)} ʄ(\varkappa,0) - \upsilon^{-(n-2)}\frac{\partial ʄ(\varkappa,ţ)}{\partial ţ} - \cdots - \mathscr{f}(\mathcal{X},0). \quad (5)$$

Now, write $ʄ(\varkappa,ţ)$ of exponential order find:

$$Ķ\left[\frac{\partial ʄ(\varkappa,ţ)}{\partial \varkappa}\right] = \int_0^\infty e^{(-\frac{ţ}{\upsilon})}\,\frac{\partial ʄ(\varkappa,ţ)}{\partial \varkappa}\,dţ = \frac{d}{d\,\varkappa}(\,g(\varkappa,\upsilon)) \quad (6)$$

$$Ķ\left[\frac{\partial^2 ʄ(\varkappa,ţ)}{\partial \varkappa^2}\right] = \frac{d^2}{d\,\varkappa^2}\left(\,g(\varkappa,\upsilon)\right), \quad (7)$$

$$Ķ\left[\frac{\partial^n ʄ(\varkappa,ţ)}{\partial \varkappa^n}\right] = \frac{d^n}{d\,\varkappa^n}(\,g(\varkappa,\upsilon)) \quad (8)$$

3. Derivation of (KDM)

In this part, prepared the non-homogenous of (NPDEs) as:

$$\mathcal{L}\, ʄ(\varkappa,ţ) + \mathfrak{K}\, ʄ(\varkappa,ţ) + \mathcal{N}\, ʄ(\varkappa,ţ) = \hbar\,(\varkappa,ţ). \quad (9)$$

With premier condition (PC):

$$ʄ(\varkappa,0) = r(\varkappa),\ \ ţ(\varkappa,0) = \wp(\varkappa). \quad (10)$$

Where operators are defined as $\mathcal{L}$: The 2nd order linear differential.
$\mathcal{L}^{-1}$: The inverse, $\mathfrak{K} < \mathcal{L}$: The remaining is linear.
$\mathcal{N}$: It is nonlinear, $\hbar\,(\varkappa, \mathfrak{t})$: The non-homogenous.
Now, take (KIT) of (8) resulted:

$$Ꞩ[\mathcal{L}\, \mathfrak{f}(\varkappa, \mathfrak{t})] + Ꞩ[\,\mathfrak{K}\, \mathfrak{f}(\varkappa, \mathfrak{t})] + Ꞩ[\mathcal{N}\, \mathfrak{f}(\varkappa, \mathfrak{t})] = Ꞩ[\,\hbar\,(\varkappa, \mathfrak{t})] \tag{11}$$

By Utilizing (4) get:

$$\upsilon^{-2}\, Ꞩ[\,\mathfrak{f}(\varkappa, \mathfrak{t})] - \upsilon^{-1}\, \mathfrak{f}(\varkappa, 0) - \frac{\partial \mathfrak{f}(\varkappa,\mathfrak{t})}{\partial\, \mathfrak{t}}] + Ꞩ[\,\mathfrak{K}\, \mathfrak{f}(\varkappa, \mathfrak{t})] + Ꞩ[\mathcal{N}\, \mathfrak{f}(\varkappa, \mathfrak{t})] = Ꞩ[\,\hbar\,(\varkappa, \mathfrak{t})] \tag{12}$$

$$Ꞩ[\,\mathfrak{f}(\varkappa, \mathfrak{t})] - \upsilon\, \mathfrak{f}(\varkappa, 0) - \upsilon^2\, \frac{\partial \mathfrak{f}(\varkappa,\mathfrak{t})}{\partial\, \mathfrak{t}}] + \upsilon^2 Ꞩ[\,\mathfrak{K}\, \mathfrak{f}(\varkappa, \mathfrak{t})] + \upsilon^2 Ꞩ[\mathcal{N}\, \mathfrak{f}(\varkappa, \mathfrak{t})] = \upsilon^2 Ꞩ[\,\hbar\,(\varkappa, \mathfrak{t})] \quad (13), \quad Ꞩ^{-1}[\upsilon^2] = \mathfrak{t}. \tag{14}$$

Now, replace $\mathfrak{f}(\varkappa, \mathfrak{t})$ by $\mathfrak{f}_m$ as: $\mathfrak{f}(\varkappa, \mathfrak{t}) = \sum_{m=0}^{\infty} \mathfrak{f}_m(\varkappa, \mathfrak{t})$. (15)

The Adomian polynomial A_m introduce the nonlinear part by infinite series as:

$$\mathcal{N}\, \mathfrak{f}(\varkappa, \mathfrak{t}) = \sum_{m=0}^{\infty} \mathfrak{T}_m(\mathfrak{f}_0, \mathfrak{f}_1, \mathfrak{f}_2, \dots)\ , m = 0,1,2, \dots \tag{16}$$

$$\mathfrak{T}_m = \frac{1}{m!} \frac{d^n}{d\vartheta^n}\, [\mathcal{F}(\textstyle\sum_{\sigma=0}^{\infty} \vartheta^{\sigma} \mathfrak{f}_{\sigma})]_{\sigma=0}\ , m = 0,1,2, \dots \tag{17}$$

After substituting (15) and (16) in (13) get:

$$Ꞩ[\,\textstyle\sum_{m=0}^{\infty} \mathfrak{f}_m(\varkappa, \mathfrak{t})] = \upsilon\, \mathfrak{f}(\varkappa, 0) + \upsilon^2\, \frac{\partial \mathfrak{f}(\varkappa,\mathfrak{t})}{\partial\, \mathfrak{t}}] - \upsilon^2 Ꞩ[\,\sum_{m=0}^{\infty} \mathfrak{K}\mathfrak{f}_m(\varkappa, \mathfrak{t})] - \upsilon^2 Ꞩ[\sum_{m=0}^{\infty} \mathfrak{T}_m(\varkappa, \mathfrak{t})] + \upsilon^2 Ꞩ[\,\hbar\,(\varkappa, \mathfrak{t})]. \tag{18}$$

Finaly, by comparing both sides of (18) and substituting (IC) give in (10) also making inverse (KIT) get the (ES):

$$\mathfrak{f}_0(\varkappa, \mathfrak{t}) = r(\varkappa) + t\, p(\varkappa) + Ꞩ^{-1}\, \upsilon Ꞩ[\,\hbar\,(\varkappa, \mathfrak{t})], m = 0 \tag{19}$$

$$\mathfrak{f}_m(\varkappa, \mathfrak{t}) = -Ꞩ^{-1}\{\upsilon^2\{Ꞩ[\textstyle\sum_{m=0}^{\infty} \mathfrak{K}\mathfrak{f}_m(\varkappa, \mathfrak{t}) + \sum_{m=0}^{\infty} \mathfrak{T}_m(\varkappa, \mathfrak{t})]\}\}\ , m > 0 \tag{20}$$

4. Proceduer of (KDM)

i. Taking (KIT) for both sides of (9).
ii. Making inverse of (KIT) to obtain $\mathfrak{f}(\mathcal{X}, \mathfrak{t})$.
iii. Find $\mathfrak{T}_m(\mathcal{X}, \mathfrak{t})$ for nonlinear part of (9).
iii. Finally get the (ES).

5. Applications

In this part, displayed three test to show that effective and accurate technique of (KDM) for solving (NPDEs) to obtain (ES).

1. *Let the (NPDE)*

$$\mathfrak{f}_{\mathfrak{t}} + \mathfrak{f}\,\mathfrak{f}_{\varkappa} = \mathfrak{f}_{\varkappa\varkappa} \tag{21}$$

with (PC)

$$\mathfrak{f}(\varkappa, 0) = 2\,\varkappa\,,\ \mathfrak{t} > 0 \tag{22}$$

Sol.: by applying (KDM) of (21) and make (KIT) of (21) as:

$$Ꝁ[ʄ_{ţ}] = Ꝁ[ʄ_{ϰϰ} - ʄ\,ʄ_{ϰ}]\,, \tag{23}$$

$$υ^{-1}\,Ꝁ[\,ʄ(ϰ, ţ)] - ʄ(ϰ, 0) = Ꝁ[ʄ_{ϰϰ} - ʄ\,ʄ_{ϰ}] \tag{24}$$

$$Ꝁ[\,ʄ(ϰ, ţ)] = υ\,ʄ(ϰ, 0) + υ\,Ꝁ[ʄ_{ϰϰ} - ʄ\,ʄ_{ϰ}]. \tag{25}$$

Yet, by comparing both sides of (25) and substituting (IC) give in (22) also making inverse (KIT) get the (ES):

$$ʄ_0(ϰ, ţ) = ʄ(ϰ, 0)\,, m = 0 \tag{26}$$

$$ʄ_{m+1}(ϰ, ţ) = Ꝁ^{-1}\{\,υ\,Ꝁ[ʄ_{mϰϰ} - \mathfrak{T}_m]\}\,, m > 0, \tag{27}$$

$$\mathfrak{T}_m = (ʄ.\,ʄ_{ϰ})_m. \tag{28}$$

When subsituting (28) in (27) find:

$$ʄ_0(ϰ, ţ) = 2\,ϰ \tag{29}$$

$$ʄ_1(ϰ, ţ) = -4\,ϰ\,\,ţ, \tag{30}$$

$$ʄ_2(ϰ, ţ) = 8\,ϰ\,\,ţ^2, \tag{31}$$

$$ʄ_3(ϰ, ţ) = -\frac{96\,ϰ\,\,ţ^3}{3!}. \tag{32}$$

Now, subsituting (29)-(32) in (15) ylides the (ES): $ʄ(ϰ, ţ) = \frac{2\,ϰ}{1+2\,ţ}$
(33)

2. *Consider the (NPDE)*

$$ʄ_{ţţ} + ʄ\,ʄ_{ϰ} = -sin\,ţ\,, \tag{34}$$

With (PC) $\quad ʄ(ϰ, 0) = 0\,\,,\,\, ʄ_{ţ}(ϰ, 0) = 1 \qquad (35)$

Sol: by applying (KDM) and (KIT) of (34) as:

$$Ꝁ[ʄ_{ţţ}] = Ꝁ[-\,ʄ\,ʄ_{ϰ} - sin\,ţ] \tag{36}$$

$$υ^{-2}\,Ꝁ[\,ʄ(ϰ, ţ)] - υ^{-1}\,ʄ(ϰ, 0) - \frac{\partial ʄ(ϰ,ţ)}{\partial\,ţ} = Ꝁ[-\,ʄ\,ʄ_{ϰ} - sin\,ţ] \tag{37}$$

$$Ꝁ[\,ʄ(ϰ, ţ)] = υ\,ʄ(ϰ, 0) + υ^2\,\frac{\partial ʄ(ϰ,ţ)}{\partial\,ţ} + υ^2\,Ꝁ[-\,ʄ\,ʄ_{ϰ} - \boldsymbol{sin}\,ţ] \tag{38}$$

Yet, by comparing both sides of (38) and substituting (IC) give in (35) also making inverse (KIT) get the (ES):

$$ʄ_0(ϰ, ţ) = υ\,ʄ(ϰ, 0) + υ^2\,\frac{\partial ʄ(ϰ,ţ)}{\partial\,ţ} + Ꝁ^{-1}\big[υ^2\,Ꝁ[-\boldsymbol{sin}\,ţ]\big]\,, m = 0 \tag{39}$$

$$ʄ_0(ϰ, ţ) = ţ + υ^2\,Ꝁ^{-1}\left[υ^2\,\left[-\frac{υ^2}{1+υ^2}\right]\right] = \boldsymbol{sin}ţ\,, m = 0 \tag{40}$$

$$ʄ_{m+1}(ϰ, ţ) = -Ꝁ^{-1}\{\,υ^2\,Ꝁ[\,\mathfrak{T}_m]\}\,, m > 0 \tag{41}$$

where $\quad \mathfrak{T}_m = (ʄ.\,ʄ_{ϰ})_m, \qquad (42)$

when solution (42) find: $\quad ʄ_1(ϰ, ţ) = 0 \qquad (43)$

$$ʄ_2(ϰ, ţ) = 0, \tag{44}$$

$$ʄ_3(ϰ, ţ) = ʄ_4(ϰ, ţ) = ʄ_5(ϰ, ţ) = \cdots = 0. \tag{45}$$

Now, subsituting (43)-(46) in (15) produce the (ES): $\mathfrak{f}(\varkappa, \mathfrak{t}) = sin\mathfrak{t}$ (46)

3. Let the (NPDE)

$$\mathfrak{f}_{\mathfrak{t}} = \frac{1}{2}\, \mathcal{X}^2\, \mathfrak{f}\, \mathfrak{f}_{\varkappa\varkappa} \tag{47}$$

with (PC)

$$\mathfrak{f}(\varkappa, 0) = \varkappa^2 \tag{48}$$

Sol: Making (KIT) of both sides of (47) as:

$$\mathrm{K}[\mathfrak{f}_{\mathfrak{t}}] = \mathrm{K}\left[\frac{1}{2}\, \mathcal{X}^2 \mathfrak{f}\, \mathfrak{f}_{\varkappa\varkappa}\right] \tag{49}$$

$$\mathrm{K}[\, \mathfrak{f}(\varkappa, \mathfrak{t})] = \upsilon\, \mathfrak{f}(\varkappa, 0) + \upsilon\, \mathrm{K}\left[\frac{1}{2}\, \mathcal{X}^2 \mathfrak{f}\, \mathfrak{f}_{\varkappa\varkappa}\right] \tag{50}$$

$$\rightarrow \mathfrak{f}(\varkappa, \mathfrak{t}) = \upsilon\, \mathfrak{f}(\varkappa, 0) + \mathrm{K}^{-1}\left[\upsilon\, \mathrm{K}\left[\frac{1}{2}\, \mathcal{X}^2 \mathfrak{f}\, \mathfrak{f}_{\varkappa\varkappa}\right]\right]. \tag{51}$$

Yet, by comparing both sides of (51) and substituting (IC) give in (48) also making inverse (KIT) get the (ES):

$$\mathfrak{f}_0(\varkappa, \mathfrak{t}) = \mathfrak{f}(\varkappa, 0), m = 0\,, \tag{52}$$

$$\mathfrak{f}_{m+1}(\varkappa, \mathfrak{t}) = \mathrm{K}^{-1}\{\, \upsilon\, \mathrm{K}[\, \mathfrak{T}_m]\}\,, m > 0 \tag{53}$$

where

$$\mathfrak{T}_m = \left(\frac{1}{2}\, \varkappa^2\, \mathfrak{f}.\, \mathfrak{f}_{\varkappa\varkappa}\right)_m. \tag{54}$$

When subsituting (54) in (53) find:

$$\mathfrak{f}_0(\varkappa, \mathfrak{t}) = \varkappa^2 \tag{55}$$

$$\mathfrak{f}_1(\varkappa, \mathfrak{t}) = \mathfrak{t}, \tag{56}$$

$$\mathfrak{f}_2(\varkappa, \mathfrak{t}) = \mathfrak{f}_3(\varkappa, \mathfrak{t}) = \mathfrak{f}_4(\varkappa, \mathfrak{t}) = \cdots = 0 \tag{57}$$

Now, subsituting (55)-(57) in (15) obtain the (ES):

$$\mathfrak{f}(\varkappa, \mathfrak{t}) = \varkappa^2 + \mathfrak{t} \tag{58}$$

6. Conclusion

In this project, displyed a new an efficient technique to explore the accurate analytic solution of (NPDEs) said (KDM) is offered from the (KIT) assit with (ADM). This method is very important and attrictive for solving (NPDEs) with comparison other researcher and discussed steps of procedure by solving various application to understand this study.

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