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On soft T-continuous mapping

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Abstract

In this paper, the concepts of T-closed set and T-open set in soft topological space are introduced, after that we define a new mapping called soft T-continuous by using these sets. We study some important characterizations and several properties concerning this mapping, also we explain a relationship between T-continuous mapping on the one hand and other important maps such as a soft continuous, soft T-irresolute and soft δ-continuous.

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Keywords: *Soft set, t-set, Soft continuous mapping, T-continuous mapping.*

1. Introduction

A soft set was studied in1991 by Molodtsov [1]. Solving problems in different scientific disciplines is the purpose of developing this theory. After that, the concept for soft topology (which is defined on a universal set with a set of parameters) was formed in 2011 by Shabir and Naz [2]. They studied some important types of soft sets such as interior set and closure set, also they

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discussed basic topics such as soft subspace and soft separation axioms.On the other hand, The idea of soft continuous mapping was given by Cigdem and Ayes [3] in 2013. Other researchers have worked on topology, and some of them can be listed [4, 3]. We use $\mathfrak{f}^{\rightarrow}(\mathcal{J},\mathfrak{D})$and $\mathfrak{f}^{\leftarrow}(\mathcal{J},\mathfrak{D})$denote to the image and pre image of the soft set$(\mathcal{J},\mathfrak{D})$ respectively, also soft regular open set by (Sr-open).

Definition 1.1: [1] Let $\mathbb{W}$ be a common universe and $\mathfrak{D}$ set of parameters. then $(\mathcal{J},\mathfrak{D})$is soft set (for short Sset) on $\mathbb{W}$ where $\mathcal{J}$ is a mapping defined as: $\mathcal{J}:\mathfrak{D}\rightarrow P(\mathbb{W})$.

Definition 1.2: [5] A Sset $(\mathcal{J},\mathfrak{D})$ on a common universe$\mathbb{W}$ is called soft point if $\exists a\in\mathfrak{D}$ s.t $\mathcal{J}(a)$ is a singleton, say {h}, and $\forall\, a^c\in\mathfrak{D}-\{a\}$, $\mathcal{J}(a^c)=\emptyset$.

Definition 1.3: [2] The family τ of Sets on a common universe $\mathbb{W}$ called soft topology on$\mathbb{W}$ if:

1) $\emptyset_{\mathbb{W}},\widetilde{\mathbb{W}}\in\tau$;
2) $\forall(\mathcal{J},\mathfrak{D}),(\mathcal{K},\mathfrak{D})\in\tau$, then $(\mathcal{J},\mathfrak{D})\,\widetilde{\cap}\,(\mathcal{K},\mathfrak{D})\in\tau$;
3) $\forall\,\{(\mathcal{J}_i,\mathfrak{D}): i\in\triangle\}\subseteq\tau$, then $\widetilde{\cup}\,\{(\mathcal{J}_i,\mathfrak{D}): i\in\triangle\}\in\tau$.

Every set in this family called soft open and its complement which be called soft closed (for short Sopen and Sclosed respectively) and the triple $(\mathbb{W},\tau,\mathfrak{D})$ is called soft topological space (for short Stopological space).

Definition 1.4: [3] the soft map $\mathfrak{f}:(\mathbb{W},\tau,\mathfrak{D})\rightarrow(\mathbb{M},\acute{\tau},\mathfrak{D})$ named soft continuous if $\mathfrak{f}^{\leftarrow}(\mathcal{J},\mathfrak{D})$is Sopen set over a common universe $\mathbb{W}$ for any Sopen set $(\mathcal{J},\mathfrak{D})$over a common universe $\mathbb{M}$.

Proposition 1.5: [6-7] A soft mapping $\mathfrak{f}:(\mathbb{W},\tau,\mathfrak{D})\rightarrow(\mathbb{M},\acute{\tau},\mathfrak{D})$ is δ-continuous if for every soft point $w_a$in $\mathbb{W}$ and any Sopen set $(\mathcal{J},\mathfrak{D})$over $\mathbb{M}$ and $\mathfrak{f}^{\rightarrow}(w_a)\,\widetilde{\in}\,(\mathcal{J},\mathfrak{D})$,there exists Sr-open set $(\mathcal{K},\mathfrak{D})$ over $\mathbb{W}$ containing w_a such that $\mathfrak{f}^{\rightarrow}(\mathcal{K},\mathfrak{D})\,\widetilde{\subseteq}\,(\mathcal{J},\mathfrak{D})$.

2. Main results

Definition 2.1: Let $(\mathbb{W},\tau,\mathfrak{D})$ be Stopological space and $(\mathcal{J},\mathfrak{D})$ be a Sset on a common universe$\mathbb{W}$, then$(\mathcal{J},\mathfrak{D})$ is called soft t-set (for short St-set) if $Int(\mathcal{J},\mathfrak{D})=IntCl(\mathcal{J},\mathfrak{D})$.

Proposition 2.2: Let $(\mathbb{W},\tau,\mathfrak{D})$ be Stopological space and $(\mathcal{J},\mathfrak{D})$ be a Sset on a common universe$\mathbb{W}$, then:

i- If $(\mathcal{J},\mathfrak{D})$is Sr-open set, then $(\mathcal{J},\mathfrak{D})$is St-set.
ii- If $(\mathcal{J},\mathfrak{D})$is Sclosed set, then $(\mathcal{J},\mathfrak{D})$ is St-set.

Proof: Obvious.

A reverse for Proposition 2.2 isn't always complete.

Examples 2.3: Let $\mathbb{W} = \{w_3, w_2, w_1\}$ and $\mathfrak{D} = \{a_2, a_1\}.\tau = \{\emptyset_{\mathbb{W}}, \widetilde{\mathbb{W}}, (\mathcal{J}_1, \mathfrak{D}),$ $(\mathcal{J}_2, \mathfrak{D}), (\mathcal{J}_3, \mathfrak{D}), (\mathcal{J}_4, \mathfrak{D})\}$, where $\mathcal{J}_1(a_1) = \{w_2\}, \mathcal{J}_1(a_2) = \{w_2\}$,$(\mathcal{J}_1, \mathfrak{D}) = \{\{w_2\}\}$, $\mathcal{J}_2(a_1) = \{w_3\}, \mathcal{J}_2(a_2) = \{w_1\}, (\mathcal{J}_2, \mathfrak{D})$=$\{\{w_3\}, \{w_1\}\}, \mathcal{J}_3(a_1) = (\mathcal{J}_1 \cap \mathcal{J}_2)(a_1) =$ $\emptyset, \mathcal{J}_3(a_2) = (\mathcal{J}_1 \cap \mathcal{J}_2)(a_2) = \emptyset, (\mathcal{J}_3, \mathfrak{D})$=$\{\emptyset\}$,$\mathcal{J}_4(a_1) = \mathcal{J}_1 \cup \mathcal{J}_2(a_1) = \{w_2, w_3\}$, $\mathcal{J}_4(a_2) = \mathcal{J}_1 \cup \mathcal{J}_2(a_2) = \{w_2, w_1\}, (\mathcal{J}_4, \mathfrak{D}) = \{\{w_2, w_3\}, \{w_2,w_1\}\}$.

If we take $(\mathcal{K}, \mathfrak{D})$ such that $\mathcal{K}(a_1) = \{\{w_1, w_3\}\}, \mathcal{K}(a_2)$=$\{w_1\}, (\mathcal{K}, \mathfrak{D})$ = $\{\{w_1, w_3\}, \{w_1\}\}$. $Int(\mathcal{K}, \mathfrak{D}) = (\mathcal{J}_2, \mathfrak{D})$= $\{w_3\}, \{w_1\}\}, Cl(\mathcal{K}, \mathfrak{D}) = \mathcal{J}_1{}^c\,(a_1) =$ $\{\{w_1, w_3\}, \{w_1, w_3\}\}$, Thus, $(\mathcal{K}, \mathfrak{D})$ is St-set but not Sr-open set also, not Sclosed.

Definition 2.4: Let $(\mathbb{W}, \tau, \mathfrak{D})$ be Stopological space and $(\mathcal{J}, \mathfrak{D})$ be a Sset on a common universe$\mathbb{W}$.Then, a soft point $w_a \widetilde{\in} (\mathbb{W}, \tau, \mathfrak{D})$ is called soft T-cluster of $(\mathcal{J}, \mathfrak{D})$if $(\mathcal{J}, \mathfrak{D}) \widetilde{\cap} (\mathcal{K}, \mathfrak{D}) \neq \emptyset_{\mathbb{W}}$, for each St-set $(\mathcal{K}, \mathfrak{D})$ contain w_a.

A set for soft T-cluster of $(\mathcal{J}, \mathfrak{D})$named soft T-closure for $(\mathcal{J}, \mathfrak{D})$ and its denoted by $Cl^{sT}(\mathcal{J}, \mathfrak{D})$.

Definition 2.5: Let $(\mathbb{W}, \tau, \mathfrak{D})$ be Stopological space and $(\mathcal{J}, \mathfrak{D})$ be a Sset on a common universe$\mathbb{W}$, then:

- $(\mathcal{J}, \mathfrak{D})$is called ST-closed if $(\mathcal{J}, \mathfrak{D})$= $Cl^{sT}(\mathcal{J}, \mathfrak{D})$.
- $(\mathcal{J}, \mathfrak{D})$is called ST-open ($\mathrm{STO}(\mathbb{W})$ denoted to family of ST-open set) if $(\mathcal{J}, \mathfrak{D})^c$ is ST-closed.
- The soft T-interior of $(\mathcal{J}, \mathfrak{D})$ denoted by $Int^{sT}(\mathcal{J}, \mathfrak{D})$ is largest ST-open set contained in $(\mathcal{J}, \mathfrak{D})$ (or $w_a \widetilde{\in} Int^{sT}(\mathcal{J}, \mathfrak{D})$ iff $\exists (\mathcal{K}, \mathfrak{D}) \widetilde{\in} \mathrm{STO}(\mathbb{W})$ such that $w_a \widetilde{\in} (\mathcal{K}, \mathfrak{D}) \widetilde{\subset} (\mathcal{J}, \mathfrak{D})$.

Proposition 2.6: Let $(\mathbb{W}, \tau, \mathfrak{D})$ be Stopological space and $(\mathcal{J}, \mathfrak{D})$ be a Sset on a common universe $\mathbb{W}$, then:

i- If $(\mathcal{J}, \mathfrak{D})$ is St-set, then $(\mathcal{J}, \mathfrak{D})$is ST-open set.

ii- If $(\mathcal{J}, \mathfrak{D})$ is Sclosed set, then $(\mathcal{J},\mathfrak{D})$ is ST-open set.

iii- $(\mathcal{J}, \mathfrak{D})$ is ST-open iff $(\mathcal{J}, \mathfrak{D}) = Int^{sT}(\mathcal{J}, \mathfrak{D})$.

iv- If $(\mathcal{J}, \mathfrak{D})$ and $(\mathcal{K}, \mathfrak{D}) \in \mathrm{STO}(\mathbb{W})$, then $(\mathcal{J}, \mathfrak{D}) \widetilde{\cap} (\mathcal{K}, \mathfrak{D})$is ST-open set.

Proof:

i- Let $w_a \widetilde{\in} Cl^{sT}(\mathcal{J}, \mathfrak{D})^c$, if $w_a \widetilde{\notin} (\mathcal{J}, \mathfrak{D})^c$ then $w_a \widetilde{\in} (\mathcal{J}, \mathfrak{D})$, Qut $(\mathcal{J}, \mathfrak{D})$ is St-set and $(\mathcal{J}, \mathfrak{D}) \widetilde{\cap} (\mathcal{J}, \mathfrak{D})^c = \emptyset$, this implies to introduction, thus $w_a \widetilde{\in}$ $(\mathcal{J}, \mathfrak{D})^c$and so $Cl^{sT}(\mathcal{J}, \mathfrak{D})^c \widetilde{\subset} (\mathcal{J}, \mathfrak{D})^c$, we have $(\mathcal{J}, \mathfrak{D})^c \widetilde{\subset} Cl^{sT}(\mathcal{J}, \mathfrak{D})^c$. There-fore $(\mathcal{J}, \mathfrak{D})^c = Cl^{sT}(\mathcal{J}, \mathfrak{D})^c$. Hence, $(\mathcal{J}, \mathfrak{D})$ is ST-open set.

ii- From Proposition 2.2(ii) and Proposition 2.6(i). (**iii** and **iv** are Obvious).

Proposition 2.7: Let $\{(\mathcal{J}_i, \mathfrak{D}): i \in\triangle\}$ be a family of all ST-open sets, then $\widetilde{\cup}_{i\in\Delta}$ $\{(\mathcal{J}_i, \mathfrak{D}): i \in\triangle \}$ is ST-open set.

Proof: Let $w_a \tilde{\in} Int^{sT} \tilde{\cup}_{i\in\Delta} \{(\mathcal{J}_i,\mathfrak{D}): i \in\Delta\}$, then there exists $(\mathcal{J}_j,\mathfrak{D}): j \in\Delta$ s.t $w_a \tilde{\in} (\mathcal{J}_j,\mathfrak{D})$, so we have St-set $(\mathcal{K},\mathfrak{D})$ with $w_a \tilde{\in} (\mathcal{K},\mathfrak{D}) \tilde{\subset} (\mathcal{J}_j,\mathfrak{D})$, thus $w_a \tilde{\in} (\mathcal{K},\mathfrak{D}) \tilde{\subset} (\tilde{\cup}_{i\in\Delta} \{(\mathcal{J}_i,\mathfrak{D}): i \in\Delta\})$. Hence $\tilde{\cup}_{i\in\Delta} \{(\mathcal{J}_i,\mathfrak{D}): i \in\Delta\}$ is ST-open set.

Proposition 2.8: Let $(\mathbb{W},\tau,\mathfrak{D})$ be soft T_1-space. Then any soft set $(\mathcal{J},\mathfrak{D})$ on a common universe$\mathbb{W}$ is ST-open set.

Proof: Since $\mathbb{W}$ is soft T_1-space and $(\mathcal{J},\mathfrak{D})$ is Sset on a common universe$\mathbb{W}$, then for any soft point $w_a \tilde{\in} (\mathcal{J},\mathfrak{D})$is Sclosed set. Thus, w_a is ST-open set due to Proposition 2.6(ii), so $(\mathcal{J},\mathfrak{D}) = \tilde{\cup}_{a\in\mathcal{J}}\{w_a\}$ is ST-open by Proposition 2.7.

Definition 2.9: A mapping $\mathfrak{f}$ from $(\mathbb{W},\tau,\mathfrak{D})$ into $(\mathbb{M},\acute{\tau},\mathfrak{D})$ is called soft T-continuous (soft T-irresolute) if $\mathfrak{f}^{\leftarrow}(\mathcal{J},\mathfrak{D})$is ST-open set over a common universe $\mathbb{W}$ for every Sopen (ST-open) set $(\mathcal{J},\mathfrak{D})$over a common universe $\mathbb{M}$.

Proposition 2.10: Let $\mathfrak{f}: (\mathbb{W},\tau,\mathfrak{D}) \to (\mathbb{M},\acute{\tau},\mathfrak{D})$, then following statement are equivalent:

(a) The soft mapping $\mathfrak{f}$ is T-continuous;

(b) For each $w_a$in $\mathbb{W}$ and Sopen set$(\mathcal{J},\mathfrak{D})$over $\mathbb{M}$ such that $\mathfrak{f}^{\rightarrow}(w_a) \tilde{\in} (\mathcal{J},\mathfrak{D})$, there exists ST-open $(\mathcal{K},\mathfrak{D})$ over $\mathbb{W}$ containing w_a and $\mathfrak{f}^{\rightarrow} (\mathcal{K},\mathfrak{D}) \tilde{\subset} (\mathcal{J},\mathfrak{D})$;

(c) $\mathfrak{f}^{\leftarrow}(\mathcal{J},\mathfrak{D})$ is ST-closed set over $\mathbb{W}$ for any Sclosed set $(\mathcal{J},\mathfrak{D})$over $\mathbb{M}$.

(d) For each Sset $(\mathcal{J},\mathfrak{D})$ over $\mathbb{W}$, $\mathfrak{f}^{\rightarrow} Cl^{sT}(\mathcal{J},\mathfrak{D}) \tilde{\subset} Cl(\mathfrak{f}^{\rightarrow}(\mathcal{J},\mathfrak{D}))$.

(e) For each Sset $(\mathcal{K},\mathfrak{D})$ over $\mathbb{M}$, $Cl^{sT}(\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})) \tilde{\subset} \mathfrak{f}^{\leftarrow}(Cl(\mathcal{K},\mathfrak{D}))$

Proof: $(\boldsymbol{a})\rightarrow(\boldsymbol{b})$ Let w_a be soft point in $\mathbb{W}$ and $(\mathcal{J},\mathfrak{D})$ be Sopen set over $\mathbb{M}$ containing $\mathfrak{f}(w_a)$, then $\mathfrak{f}^{\leftarrow}(\mathcal{J},\mathfrak{D})$is ST-open set over $\mathbb{W}$ because of $\mathfrak{f}$ is soft T-continuous, hence $\mathfrak{f}^{\rightarrow}(\mathfrak{f}^{\leftarrow}(\mathcal{J},\mathfrak{D})) \tilde{\subset} (\mathcal{J},\mathfrak{D})$.

$(\boldsymbol{b})\rightarrow(\boldsymbol{c})$ Let $(\mathcal{K},\mathfrak{D})$ be Sclosed set over $\mathbb{M}$, then $(\mathcal{K},\mathfrak{D})^c$ is ST-open set. For any soft point $w_a \tilde{\in} \mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})^c$ then $\mathfrak{f}(w_a) \tilde{\in} (\mathcal{K},\mathfrak{D})^c$and so, there exists ST-open set $(\mathcal{J},\mathfrak{D})$ containing w_a such that $\mathfrak{f}^{\rightarrow} (\mathcal{J},\mathfrak{D}) \tilde{\subset} (\mathcal{K},\mathfrak{D})^c$ therefore, $(\mathcal{J},\mathfrak{D}) \tilde{\subset} \mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})^c$. Since $(\mathcal{J},\mathfrak{D})$ is ST-open set, thus we have St-set $(L,\mathfrak{D})$ and $w_a \tilde{\in} (L,\mathfrak{D}) \tilde{\subset} (\mathcal{J},\mathfrak{D}) \tilde{\subset} \mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})^c$. Hence $\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})^c$ is ST-open set, so $\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})$ is ST-closed set.

$(\boldsymbol{c})\rightarrow(\boldsymbol{d})$ Let$(\mathcal{J},\mathfrak{D})$ be a Sset over a common universe $\mathbb{W}$, then $(\mathcal{J},\mathfrak{D}) \tilde{\subset} \mathfrak{f}^{\leftarrow}\big(\mathfrak{f}^{\rightarrow} (\mathcal{J},\mathfrak{D})\big) \tilde{\subset} \mathfrak{f}^{\leftarrow}\big(Cl\,(\mathfrak{f}^{\rightarrow}(\mathcal{J},\mathfrak{D}))\big)$ $so, Cl^{sT} (\mathcal{J},\mathfrak{D}) \tilde{\subset} Cl^{sT}(\mathfrak{f}^{\leftarrow} (Cl\,(\mathfrak{f}^{\rightarrow} (\mathcal{J},\mathfrak{D})))$. From $(\boldsymbol{c})$, $\mathfrak{f}^{\leftarrow}(Cl(\mathfrak{f}^{\rightarrow}(\mathcal{J},\mathfrak{D})))$ is ST-closed set in $\mathbb{W}$, therefore, $Cl^{sT}(\mathcal{J},\mathfrak{D}) \tilde{\subset} \mathfrak{f}^{\leftarrow}(Cl(\mathfrak{f}^{\rightarrow}(\mathcal{J},\mathfrak{D})))$. Hence, $\mathfrak{f}^{\rightarrow} (Cl^{sT}(\mathcal{J},\mathfrak{D})) \tilde{\subset} Cl(\mathfrak{f}^{\rightarrow}(\mathcal{J},\mathfrak{D}))$.

$(\boldsymbol{d})\rightarrow(\boldsymbol{e})$ Let $(\mathcal{K},\mathfrak{D})$ be a Sset over $\mathbb{M}$. Due to $(\boldsymbol{d})$, $\mathfrak{f}^{\rightarrow} (Cl^{sT} (\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D}))) \tilde{\subset} Cl(\mathfrak{f}^{\rightarrow}(\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D}))) \tilde{\subset} Cl(\mathcal{K},\mathfrak{D})$. Hence $Cl^{sT}(\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})) \tilde{\subset} \mathfrak{f}^{\leftarrow} (Cl(\mathcal{K},\mathfrak{D}))$.

$(\boldsymbol{e})\rightarrow(\boldsymbol{a})$ Let $(\mathcal{K},\mathfrak{D})$ be Sopen set over $\mathbb{M}$. By $(\boldsymbol{e})$, $Cl^{sT}(\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})^c) \tilde{\subset} \mathfrak{f}^{\leftarrow}(Cl(\mathcal{K},\mathfrak{D})^c)$, since $(\mathcal{K},\mathfrak{D})^c$ is Sclosed set, then $Cl^{sT}(\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})^c)$

$\tilde{\subseteq}$ $\mathfrak{f}^{\leftarrow}$ $((\mathcal{K},\mathfrak{D})^c)$, but we have$\mathfrak{f}^{\leftarrow}((\mathcal{K},\mathfrak{D})^c) \tilde{\subseteq} Cl^{sT}(\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})^c$, thus $\mathfrak{f}^{\leftarrow}$ $((\mathcal{K},\mathfrak{D})^c)$= $Cl^{sT}(\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})^c)$ and so, $\mathfrak{f}^{\leftarrow}((\mathcal{K},\mathfrak{D})^c)$ is ST-closed set. Hence, $\mathfrak{f}$ is soft T-continuous mapping because $\mathfrak{f}^{\leftarrow}(\mathcal{K},\mathfrak{D})$ is ST-open set.

Definition 2.11: Let $(\mathbb{W},\tau,\mathfrak{D})$ be Stopological space. Then $\mathbb{W}$ called T*-regular space if for each soft point w_a in $\mathbb{W}$ and ST-open set $(\mathcal{J},\mathfrak{D})$ contained w_a, there exists Sopen set $(\mathcal{K},\mathfrak{D})$ s.t $w_a \tilde{\in} (\mathcal{K},\mathfrak{D}) \tilde{\subseteq} Cl(\mathcal{K},\mathfrak{D}) \tilde{\subseteq} (\mathcal{J},\mathfrak{D})$.

Proposition 2.12: Let $\mathfrak{f}$: $(\mathbb{W},\tau,\mathfrak{D}) \rightarrow (\mathbb{M},\acute{\tau},\mathfrak{D})$is soft mapping, then:

(1) $\mathfrak{f}$ soft T-continuous, if its soft continuous & $\mathbb{W}$ is soft T_1-space.

(2) $\mathfrak{f}$ soft T-continuous, when its soft T-irresolute & $\mathbb{M}$ soft T_1-space.

(3) $\mathfrak{f}$ soft continuous, where its soft δ-continuous.

(4) $\mathfrak{f}$ is soft T-continuous, when its soft δ-continuous.

(5) $\mathfrak{f}$ soft δ-continuous, when its soft T-continuous & $\mathbb{M}$ is soft T^*-regular space.

(6) $\mathfrak{f}$ soft δ-continuous, if its soft T-irresolute and $\mathbb{M}$ is soft T^*-regular and T_1-space.

(7) $\mathfrak{f}$ is soft continuous, if it is soft T-irresolute and $\mathbb{M}$ is soft T_1-space.

Proof: **(1), (2)** and **(3)** are obvious.

(4) Let $\mathfrak{f}$: $(\mathbb{W},\tau,\mathfrak{D}) \rightarrow (\mathbb{M},\acute{\tau},\mathfrak{D})$ be soft δ-continuous mapping and w_a is a soft point in $(\mathbb{W},\tau,\mathfrak{D})$. If $(\mathcal{J},\mathfrak{D})$ is Sopen set over $\mathbb{M}$ such that $\mathfrak{f}^{\rightarrow}(w_a) \tilde{\in} (\mathcal{J},\mathfrak{D})$, there exists Sr-open set $(\mathcal{K},\mathfrak{D})$ over $\mathbb{W}$ containing w_a such that $\mathfrak{f}^{\rightarrow}(\mathcal{K},\mathfrak{D}) \tilde{\subseteq} (\mathcal{J},\mathfrak{D})$. But $(\mathcal{K},\mathfrak{D})$ is ST-open set by Proposition 2.2(i) and Proposition 2.6(i). Hence $\mathfrak{f}$ is soft δ-continuous.

(5) Let $w_a \tilde{\in} (\mathbb{W},\tau,\mathfrak{D})$ and $(\mathcal{J},\mathfrak{D})$ be soft open set over $\mathbb{M}$ such that $\mathfrak{f}^{\rightarrow}(w_a) \tilde{\in} (\mathcal{J},\mathfrak{D})$. We have $\mathfrak{f}$ is a soft T-continuous mapping, thus from Proposition 2.10(b), there exists ST-open set $(L,\mathfrak{D})$ over $\mathbb{M}$ contained w_a such that $\mathfrak{f}^{\rightarrow}(w_a) \tilde{\in} \mathfrak{f}^{\rightarrow}(L,\mathfrak{D}) \tilde{\subseteq} (\mathcal{J},\mathfrak{D})$. Since $\mathbb{M}$ is a soft T^*-regular space and $w_a \tilde{\in} (L,\mathfrak{D})$then there exists Sopen set$(\mathcal{K},\mathfrak{D})$ over $\mathbb{M}$ such that $w_a \tilde{\in} (\mathcal{K},\mathfrak{D})$ $\tilde{\subseteq} Cl(\mathcal{K},\mathfrak{D}) \tilde{\subseteq} (L,\mathfrak{D})$. So,$w_a \tilde{\in} (\mathcal{K},\mathfrak{D}) \tilde{\subseteq} IntCl(\mathcal{K},\mathfrak{D}) \tilde{\subseteq} (L,\mathfrak{D})$. Now, $\mathfrak{f}^{\rightarrow}(w_a) \tilde{\in} (\mathcal{K},\mathfrak{D}) \tilde{\subseteq} \mathfrak{f}^{\rightarrow}(IntCl(\mathcal{K},\mathfrak{D})) \tilde{\subseteq} \mathfrak{f}^{\rightarrow}(L,\mathfrak{D})$, therefor $\mathfrak{f}^{\rightarrow}$(*IntCl* (Q,R)) $\tilde{\subseteq} (\mathcal{J},\mathfrak{D})$. But $IntCl(\mathcal{K},\mathfrak{D})$ is Sr-open set, Hence by Proposition 1.5, $\mathfrak{f}$ is δ-continuous.

(6) From (2) + (5).

(7) From (3) + (6).

Proposition 2.13: If $\mathfrak{f}$: $(\mathbb{W},\tau,\mathfrak{D}) \rightarrow (\mathbb{M},\acute{\tau},\mathfrak{D})$ soft T-continuous map & $(\mathbb{M},\acute{\tau},\mathfrak{D})$ soft T_2-space, then a Sset $\Delta = \{(w_a,w_b): \mathfrak{f}^{\rightarrow}(w_a) = \mathfrak{f}^{\rightarrow}(w_b)\}$ over a universe $\mathbb{W} \times \mathbb{W}$ is ST-closed set.

Proof: Suppose that $(w_a,w_b) \tilde{\notin} \Delta$, thus $\mathfrak{f}^{\rightarrow}(w_a) \neq \mathfrak{f}^{\rightarrow}(w_b)$, since $(\mathbb{M},\acute{\tau},\mathfrak{D})$ is a soft T_2-space, then there exists two Sopen sets $(\mathcal{J}_1,\mathfrak{D})$and $(\mathcal{J}_2,\mathfrak{D})$ over $\mathbb{M}$ are containing $\mathfrak{f}^{\rightarrow}(w_a)$ and $\mathfrak{f}^{\rightarrow}(w_b)$ respectively, such that $(\mathcal{J}_1,\mathfrak{D}) \tilde{\cap} (\mathcal{J}_2,\mathfrak{D}) \neq \emptyset_{\mathbb{W}}$.

But $\mathfrak{f}$ is a soft T-continuous, therefore there is two ST-open sets $(\mathcal{K}_1, \mathfrak{D})$ and $(\mathcal{K}_2, \mathfrak{D})$ over $\mathbb{W}$ are containing w_a and w_b respectively, such that $\mathfrak{f}^{\rightarrow}(\mathcal{K}_1, \mathfrak{D}) \tilde{\subseteq} (\mathcal{I}_1, \mathfrak{D})$ and $\mathfrak{f}^{\rightarrow}(\mathcal{K}_2, \mathfrak{D}) \tilde{\subseteq} (\mathcal{I}_2, \mathfrak{D})$. Consequently, $(w_a, w_b) \tilde{\in} (\mathcal{K}_1, \mathfrak{D}) \times (\mathcal{K}_2, \mathfrak{D}) \tilde{\subseteq} \Delta^c$ and so, $(w_a, w_b) \tilde{\in} Int^{ST}(\Delta^c)$, this implies $(w_a, w_b) \tilde{\notin} (Int^{ST}(\Delta^c))^c = Cl(\Delta)$. Now, we have $\Delta = Cl^{ST}(\Delta)$ and so, Δ is ST-closed set.

3. Conclusion

First, a new point called soft T-cluster is defined, which to form the concept of soft T-closed and soft T-open. Then we define T-continuous mapping and discussed equivalents of this definition and its relation to other types of maps.

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