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On coupled fixed point theorem in partially fuzzy Fréchet space

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Abstract

We provide a linked fixed-point hypothesis for a combination of mappings with blended g-monotone properties in partly fuzzy Fréchet space, where the family of fuzzy seminorms fulfills the n-property.

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1. Introduction

Fixed point theory is a well-known traditional one in mathematics and it acquired this importance from its large number of applications. One of the most significant results of the analysis of this theory is the Banach contraction mapping. It is considered a famous measure to solve existing problems in various aspects of mathematics and it was utilized in various areas of literature. . Besides, it was expanded by Ran and Reurings [1] in sets which are in part requested with a few applications to direct and nonlinear network conditions. At that point, Nieto and Rod'riguez-López [2] amplified the about of their colleagues and connected their major hypotheses for the reason of the accomplishment of a critical arrangement related to, to begin with, standard differential conditions with intermittent boundary conditions. On the other hand, Bhaskar and Lakshmikantam [3] presented the guideline of blended monotone mappings and delivered a few coupled fixed points comes about. Additionally, they examined the application of them comes about on to begin with arranging differential conditions with occasional boundary conditions. Numerous other analysts also came up with issues related to fixed points in cone metric spaces, mostly requested metric spaces, and numerous other areas (see [1]-[8]). Ghananwi and Al-Nafie [9] first presented the concept of related fuzzy Frechet spaces. This paper points at showing a few related fixed points that come about in part requested fuzzy Frechet space for each of two mappings that have a monotone property and accomplish a contractive condition.

Definition 1.1: [8] Assume that Z be a non-empty set. A point $(w, v) \in \mathrm{Z} \times \mathrm{Z}$ is said to be a coupled coincidence point of the mappings $\mathcal{S}: \mathrm{Z} \times \mathrm{Z} \to \mathrm{Z}$ and $g: \mathrm{Z} \to \mathrm{Z}$ if $\mathcal{S}(w, v) = gw,\ \mathcal{S}(v, w) = gv, \forall w, v \in \mathrm{Z}$.

Definition 1.2: [8] Assume that Z be a non-empty set. The mappings $\mathcal{S}: \mathrm{Z} \times \mathrm{Z} \to \mathrm{Z}$ and $g: \mathrm{Z} \to \mathrm{Z}$ are commuting if $g\mathcal{S}(w, v) = \mathcal{S}(gw, gv), \forall w, v \in \mathrm{Z}$.

Definition 1.3: [8] Assume that $(\mathrm{Z}, \sqsubseteq)$ be a partially ordered set and $\mathcal{S}: \mathrm{Z} \times \mathrm{Z} \to \mathrm{Z}$ and $g: \mathrm{Z} \to \mathrm{Z}$ be two mappings. $\mathcal{S}$ is called to have the mixed $g-$ monotone property if $\mathcal{S}$ is monotone-$g - non - decreasing$ in the first dispute and is monotone-$g - \mathrm{non} - \mathrm{increasing}$ in the second argument, this mean, $\forall\ w, v \in \mathrm{Z}$,

$$w_1, w_2 \in \mathrm{Z}, g(w_1) \sqsubseteq g(w_2) \Longrightarrow \mathcal{S}(w_1, v) \sqsubseteq \mathcal{S}(w_2, v) \tag{1.1}$$

And,
$$v_1, v_2 \in \mathrm{Z}, g(v_1) \sqsubseteq g(v_2) \Longrightarrow \mathcal{S}(w, v_1) \sqsubseteq \mathcal{S}(\mathrm{w}, v_2) \tag{1.2}$$

Definition 1.4. [9] Assume that Z be a FTVspace. Z is said to be fuzzy Fréchet space (for short, $\boldsymbol{FFS}$) if whose FT $\tau_{\mathfrak{S}}$ caused by fuzzy seminorms with a countable dividing family $\mathfrak{S} = \{\mu_i\}_{i \in I}$.

The properties of fuzzy $Fréchet$ space, the $FF -$ sequentially continuous and [9] provides information on the ideas of convergence and Cauchy sequence in FFS.

2. Main results

We start with $n-property$ in FFS.

Definition 2.1: Assume that Z be a FFS. Then $\mathfrak{S}=\{\mu_i\}_{i\in I}$ defined as satisfying the $n-$ property on $\mathrm{Z}\times(0,\infty)$ if $\lim_{n\to\infty}[\mu_i(w,\ r^n s)]^{n^q}=1,\ \forall\mu_i\in\mathfrak{S}, w\in \mathrm{Z}, \mathrm{r}>1$ and $q>0$.

Example 2.2: Assume that Z be a Fréchet space whose topology–induced by a separating family of ordinary seminorms $\{\|.\|_i\}_{i\in I}$, $*$ is a continuous t-norm such that $c*d=cd, \forall c,d\in[0,1]$ and $\mu_i(w,\ s)=(\frac{s}{s+1})^{\|w\|_i}, i\in I, \forall w\in \mathrm{Z}$ and $t>0$. Then $\lim_{n\to\infty}[\mu_i(w,r^n s)]^{n^q}=\lim_{n\to\infty}(\frac{r^n s}{r^n s+1})^{\|w\|_i n^q}=e^B=1$, when $B=\lim_{n\to\infty}\left(\frac{r^n s}{r^n s+1}-1\right)\|w\|_i n^q=\lim_{n\to\infty}\frac{-\|w\|_i n^q}{r^n s+1}=0$. Hence $\{\mu_i\}_{i\in I}$ satisfy the $n-property$ on $\mathrm{Z}\times(0,\infty)$.

Example 2.3: Assume that Z be a Fréchet space whose topology–induced by a separating family of ordinary seminorms $\{\|.\|_i\}_{i\in I}$, $*$ is a continuous t-norm such that $c*d=cd, \forall c,d\in[0,1]$ and $\mu_i(w,\ s)=\left(\frac{s}{s+\|w\|_i}\right),\ i\in I, \forall w\in \mathrm{Z}$ and $t>0$. Then $\lim_{n\to\infty}[\mu_i(w,r^n s)]^{n^q}=\lim_{n\to\infty}(\frac{r^n s}{r^n s+\|w\|_i})^{n^q}=e^B=1$,when $B=\lim_{n\to\infty}\left(\frac{r^n s}{r^n s+\|w\|_i}-1\right)n^q=\lim_{n\to\infty}\frac{-\|w\|_i n^q}{r^n s+\|w\|_i}=0$. Hence $\{\mu_i\}_{i\in I}$ satisfy the $n-property$ on $\mathrm{Z}\times(0,\infty)$.

Lemma 2.4: *Assume that Z be a* FFS. *Assume that* $c*d=cd,\forall c,d\in[0,1]$ *and* $\mathfrak{S}=\{\mu_i\}_{i\in I}$ *satisfy the* $n-property$. *Assume that* $\{w_n\}$ *is a sequence in* Z *such that* $\mu_i(w_{n+1}-w_n,rs)\geq\mu_i(w_n-w_{n-1},s),\forall\mu_i\in\mathfrak{S},\forall s>0,n\in\mathbb{N}$, *where* $0<r<1$. *Then* $\{w_n\}$ *is a Cauchy sequence in* Z.

Proof: $\forall\, w_n,w_{n+1}\in \mathrm{Z}$ and $\forall\mu_i\in\mathfrak{S}$, we get $\mu_i(w_{n+1}-w_n,s)\geq\mu_i\left(w_1-w_0,\frac{s}{k^n}\right)$. If $m>n$, we have $(1-r)(1+r+\cdots+r^{m-n-1})=1-r^{m-n}<1$

Since $s>0$ then $s(1-r)(1+r+\cdots+r^{m-n-1})=s(1-r^{m-n})s<s$

Now, applying the triangle inequality, we obtain

$$\begin{aligned}\mu_i(w_n-w_m,s)&\geq\mu_i\big(w_n-w_m,s(1-r)\,(1+r+\cdots+r^{m-n-1})\big)\\&\geq\mu_i\big(w_n-w_{n+1},s(1-r)\big)*\mu_i(w_{n+1}-w_{n+2},s(1-r)r)\\&\quad*\ldots*\mu_i(w_{m-1}-w_m,s(1-r)r^{m-n-1})\\&\geq\mu_i\left(w_0-w_1,(1-r)\frac{s}{r^n}\right)*\mu_i\left(w_0-w_1,(1-r)\frac{s}{r^n}\right)*\ldots\\&\quad*\mu_i\left(w_0-w_1,(1-r)\frac{s}{r^n}\right)\ \geq\left[\mu_i\left(w_0-w_1,(1-r)\frac{s}{r^n}\right)\right]^{m-n}\end{aligned}$$

$$\geq \left[\mu_i\left(w_0 - w_1, (1-r)\frac{s}{r^n}\right)\right]^m \geq \left[\mu_i\left(w_0 - w_1, (1-r)\frac{s}{r^n}\right)\right]^{n^q} \to 1,$$
$p > 0$ and $m < n^q$. Hence $\{w_n\}$ is a Cauchy sequence in Z.

Theorem 2.5: *Assume that Z be a FFS. Assume that $c * d \geq cd, \forall c, d \in [0,1]$, $\mathfrak{S} = \{\mu_i\}_{i\in I}$ satisfy the $n-property$ and suppose that $(Z, \sqsubseteq)$ be a partially ordered set. Assume that $\mathcal{S}: Z \times Z \to Z$ and $g: Z \to Z$ be two mappings, $\mathcal{S}$ has the $mixed\ g - monotone\ property$ and*

$$\mu_i(\mathcal{S}(w,v) - \mathcal{S}(x,y), rs) \geq \mu_i(gw - gx, s) * \mu_i(gv - gy, s), \quad (2.1)$$

$\forall \mu_i \in \mathfrak{S}$,where $0 < r < 1$, for which $gw \sqsubseteq gx$ and $gv \sqsupseteq gy$, $\mathcal{S}(\mathrm{Z} \times \mathrm{Z}) \subseteq g(\mathrm{Z})$ and g commuting with $\mathcal{S}$ and $FF-$ sequentially continuous. Suppose that either $\mathcal{S}$ and $FF-$ sequentially continuous or Z possesses the following property;

i. If $\lim_{n\to\infty} w_n = w$ and $\{w_n\}$ is a non-decreasing sequence, then
$$gw_n \sqsubseteq gw, \forall n \in \mathbb{N}, \quad (2.2)$$
ii. If $\lim_{n\to\infty} v_n = v$ and $\{v_n\}$ is a non-increasing sequence, then
$$gv_n \sqsupseteq gv, \forall n \in \mathbb{N}. \quad (2.3)$$

If there are $w_0, v_0 \in \mathrm{Z}$ and $gw_0 \sqsubseteq \mathcal{S}(w_0, v_0), gv_0 \sqsupseteq \mathcal{S}(v_0, w_0)$ then there are $w, v \in \mathrm{Z}$ and $gw = \mathcal{S}(w,v)$, $gv = \mathcal{S}(v,w)$. This mean, $g, \mathcal{S}$ have a coupled coincidence in Z.

Proof: Assume that $w_0, v_0 \in \mathrm{Z}$ such that $gw_0 \sqsubseteq \mathcal{S}(w_0, v_0)$ and $gv_0 \sqsupseteq \mathcal{S}(v_0, w_0)$. Since $\mathcal{S}(\mathrm{Z}\times\mathrm{Z}) \subseteq g(\mathrm{Z})$, we can select $w_1, v_1 \in \mathrm{Z}$ such that $gw_1 = \mathcal{S}(w_0, v_0)$ and $gv_1 = \mathcal{S}(v_0, w_0)$. Also, from $\mathcal{S}(\mathrm{Z}\times\mathrm{Z}) \subseteq g(\mathrm{Z})$, we can select $w_2, v_2 \in \mathrm{Z}$ such that $gw_2 = \mathcal{S}(w_1, v_1)$ and $gv_2 = \mathcal{S}(v_1, w_1)$. We can generate two sequences $\{w_n\}$ and $\{v_n\}$ in Z such that

$$gw_{n+1} = \mathcal{S}(w_n, v_n)\ ,\ gv_{n+1} = \mathcal{S}(v_n, w_n), \forall n \geq 0 \quad (2.4)$$

Now, we prove that

$$gw_n \sqsubseteq gw_{n+1,}\ \forall n \geq 0, \quad (2.5)$$

And
$$gv_n \sqsupseteq gv_{n+1},\ \forall n \geq 0 \quad (2.6)$$

To achieve this, we employ mathematical induction. Assume that $n = 0$. since $gw_0 \sqsubseteq \mathcal{S}(w_0, v_0)$, $gv_0 \sqsupseteq \mathcal{S}(v_0, w_0)$ and $gw_1 = \mathcal{S}(w_0, v_0)$ and $gv_1 = \mathcal{S}(v_0, w_0)$, we get $gw_0 \sqsubseteq gw_1$ and $gv_0 \sqsupseteq gv_1$. Then (2.5) and (2.6) satisfy for $n = 0$. Now, suppose that (2.5) and (2.6) hold for a fixed $n \geq 0$. Thus, since $gw_n \sqsubseteq gw_{n+1,}\ gv_n \sqsupseteq gv_{n+1}$ and $\mathcal{S}$ has the $mixed\ g - monotone\ property$, it follows from equations (2.4) and (1.1) that

$$gw_{n+1} = \mathcal{S}(w_n, v_n) \sqsubseteq \mathcal{S}(w_{n+1}, v_n), \quad gv_{n+1} = \mathcal{S}(v_n, w_n) \sqsupseteq \mathcal{S}(v_{n+1}, w_n) \quad (2.7)$$

Additionally, from equations (2.4) and (1.2), we obtain

$$gw_{n+2} = \mathcal{S}(w_{n+1}, v_n) \sqsubseteq \mathcal{S}(w_{n+1}, v_{n+1}),$$
$$gv_{n+2} = \mathcal{S}(v_{n+1}, w_n) \sqsupseteq \mathcal{S}(v_{n+1}, w_{n+1}). \quad (2.8)$$

Thus, by using equations (2.7) and (2.8), we have

$$gw_{n+1} \sqsubseteq gw_{n+2,}\ gv_{n+1} \sqsupseteq gv_{n+2} \tag{2.9}$$

Therefore, by induction, we get $gw_n \sqsubseteq gw_{n+1}$ and $gv_n \sqsupseteq gv_{n+1}$, $\forall n \geq 0$

Consequently,

$$gw_0 \sqsubseteq gw_1 \sqsubseteq gw_2 \sqsubseteq gw_3 \sqsubseteq \cdots \sqsubseteq gw_n \sqsubseteq gw_{n+1} \sqsubseteq \cdots \tag{2.10}$$

And, $$gw_0 \sqsupseteq gw_1 \sqsupseteq gw_2 \sqsupseteq gw_3 \sqsupseteq \cdots \sqsupseteq gw_n \sqsupseteq gw_{n+1} \sqsupseteq \cdots \tag{2.11}$$

Now, Assume that us define $\alpha_n^1(s) = \mu_i(gw_n - gw_{n+1}\ ,\ s) * \mu_i(gv_n - gv_{n+1}\ ,\ s)$, $\forall \mu_i \in \mathfrak{S}$. Then applying equations (2.4) and (2.1), we have

$$\mu_i(gw_n - gw_{n+1}, rs) = \mu_i(\mathcal{S}(w_{n-1}, v_{n-1}) - \mathcal{S}(w_n, v_n), rs) \geq \mu_i(gw_{n-1} - gw_n\ , s) * \mu_i(gv_{n-1} - gv_n\ , s) = \alpha_{n-1}^1(s) \tag{2.12}$$

And $\mu_i(gv_n - gv_{n+1}, rs) = \mu_i(\mathcal{S}(v_{n-1}, w_{n-1}) - \mathcal{S}(v_n, w_n), rs) \geq \mu_i(gv_{n-1} - gv_n\ , s) * \mu_i(gw_{n-1} - gw_n\ , s) = \alpha_{n-1}^1(s)$ (2.13)

Thus $$\alpha_n^1(rs) \geq \alpha_{n-1}^1\ (s) * \alpha_{n-1}^1(s) \geq [\alpha_{n-1}^1(s)\ ]^2 \tag{2.14}$$

And so, $$\alpha_n^1(s) \geq \left[\alpha_{n-1}^1\left(\frac{s}{r^n}\right)\right]^2 \geq \cdots \geq \left[\alpha_0^1\left(\frac{s}{r^n}\right)\right]^{2^n}, \tag{2.15}$$

That is, $\mu_i(gw_n - gw_{n+1}\ , rs) * \mu_i(gv_n - gv_{n+1}, rs)$

$$\geq \left[\mu_i\left(gw_0 - gw_1\ , \frac{s}{r^n}\right)\right]^{2^n} * \left[\mu_i\left(gv_0 - gv_1\ , \frac{s}{r^n}\right)\right]^{2^n}, \forall \mu_i \in \mathfrak{S}. \tag{2.16}$$

Conversely, we have $s\,(1-r)(1 + r + \cdots + r^{m-n-1}) < s, \forall\, m > n, 0 < r < 1$. Appling the triangle inequality, we obtain

$$\begin{aligned}
&\mu_i(gw_n - gw_m\ , s) * \mu_i(gv_n - gv_m, s) \\
&\geq \mu_i\big(gw_n - gw_m\ , s(1-r)\ (\ 1 + r + \cdots + r^{m-n-1})\big) \\
&\quad * \mu_i\big(gv_n - gv_m, s(1-r)(1 +\ r + \cdots + r^{m-n-1})\big) \\
&\geq \mu_i\big(gw_{n+1} - gw_n\ , s(1-r)\big) * \mu_i\big(gv_{n+1} - gv_n, s(1-r)\big) \\
&\quad * \mu_i(gw_{n+1} - gw_{n+2}\ , s(1-r)r) \\
&\quad * \mu_i(gv_{n+1} - gv_{n+2}, s(1-r)r) * \ldots \\
&\quad * \mu_i(gw_{m-1} - gw_m\ , s(1-r)r^{m-n-1}) \\
&\quad * \mu_i(gv_{m-1} - gv_m, s(1-r)r^{m-n-1}) \\
&\geq \mu_i\left(gw_0 - gw_1\ , (1-r)\frac{s}{r^n}\right) * \mu_i\left(gv_0 - gv_1\ , (1-r)\frac{s}{r^n}\right) * \ldots * \\
&\quad \mu_i\left(gw_0 - gw_1\ , (1-r)\frac{s}{r^n}\right) * \mu_i\left(gv_0 - gv_1\ , (1-r)\frac{s}{r^n}\right) \\
&\geq \left[\mu_i\left(gw_0 - gw_1\ , (1-r)\frac{s}{r^n}\right)\right]^{m-n} * \left[\mu_i\left(gv_0 - gv_1\ , (1-r)\frac{s}{r^n}\right)\right]^{m-n} \\
&\geq \left[\mu_i\left(gw_0 - gw_1\ , (1-r)\frac{s}{r^n}\right)\right]^{m} * \left[\mu_i\left(gv_0 - gv_1\ , (1-r)\frac{s}{r^n}\right)\right]^{m} \\
&\geq \left[\mu_i\left(gw_0 - gw_1\ , (1-r)\frac{s}{r^n}\right)\right]^{n^q} * \left[\mu_i\left(gv_0 - gv_1\ , (1-r)\frac{s}{r^n}\right)\right]^{n^q} \to 1,
\end{aligned}$$

$\forall \mu_i \in \mathfrak{S}, p > 0$ and $m < n^q$. Hence, the sequences $\{gw_n\}$ and $\{gv_n\}$ are Cauchy sequences in Z. then there exist $w, v \in$ Z such that

$$\lim_{n\to\infty} \mathcal{g}w_n = w, \quad \lim_{n\to\infty} \mathcal{g}v_n = v. \tag{2.17}$$

Since $\mathcal{g}$ is $FF-$ sequentially continuous, then

$$\lim_{n\to\infty} \mathcal{g}\mathcal{g}w_n = \mathcal{g}w, \quad \lim_{n\to\infty} \mathcal{g}\mathcal{g}v_n = \mathcal{g}v. \tag{2.18}$$

Since $\mathcal{g}$ commuting with $\mathcal{S}$, then

$$\mathcal{g}\mathcal{g}w_{n+1} = \mathcal{g}\mathcal{S}(w_n, v_n) = \mathcal{S}(\mathcal{g}w_n, \mathcal{g}v_n) \tag{2.19}$$

And
$$\mathcal{g}\mathcal{g}v_{n+1} = \mathcal{g}\mathcal{S}(v_n, w_n) = \mathcal{S}(\mathcal{g}v_n, \mathcal{g}w_n) \tag{2.20}$$

Now, we prove that $\mathcal{g}w = \mathcal{S}(w, v)$ and $\mathcal{g}v = \mathcal{S}(v, w)$. Suppose that $\mathcal{S}$ is $FF-$ sequentially continuous, then from (2.19) and (2.20), we have

$$\mathcal{g}w = \lim_{n\to\infty} \mathcal{g}\mathcal{g}w_{n+1} = \lim_{n\to\infty} \mathcal{S}(\mathcal{g}w_n, \mathcal{g}v_n) = \mathcal{S}(\lim_{n\to\infty} \mathcal{g}w_n, \lim_{n\to\infty} \mathcal{g}v_n) = \mathcal{S}(w, v),$$
$$\mathcal{g}v = \lim_{n\to\infty} \mathcal{g}\mathcal{g}v_{n+1} = \lim_{n\to\infty} \mathcal{S}(\mathcal{g}v_n, \mathcal{g}w_n) = \mathcal{S}(\lim_{n\to\infty} \mathcal{g}v_n, \lim_{n\to\infty} \mathcal{g}w_n) = \mathcal{S}(v, w).$$

Therefore we get $\mathcal{g}w = \mathcal{S}(w, v)$ and $\mathcal{g}v = \mathcal{S}(v, w)$. Suppose that Z possesses (i) and (ii), since $\{\mathcal{g}w_n\}$ is non-decreasing and $\mathcal{g}w_n \to w$, then, from (2.2), we get $\mathcal{g}\mathcal{g}w_n \sqsubseteq \mathcal{g}w$. In the same way, since $\{\mathcal{g}v_n\}$ is non-increasing and $\mathcal{g}v_n \to v$, then, from (2.3), we get $\mathcal{g}\mathcal{g}v_n \sqsupseteq \mathcal{g}v$. Then, by (2.19), (2.17) and (2.1), we get

$$\begin{aligned}\mu_i(\mathcal{g}\mathcal{g}w_{n+1} - \mathcal{S}(w, v), rs) &= \mu_i(\mathcal{g}\mathcal{S}(w_n, v_n) - \mathcal{S}(w, v), rs)\\ &= \mu_i(\mathcal{S}(\mathcal{g}w_n, \mathcal{g}v_n) - \mathcal{S}(w, v), rs)\\ &\geq \mu_i(\mathcal{g}\mathcal{g}w_n - \mathcal{g}w, s) * \mu_i(\mathcal{g}\mathcal{g}v_n - \mathcal{g}v, s), \forall \mu_i \in \mathfrak{S}\end{aligned}$$

Assume that in $n \to \infty$, yields $\mu_i(\mathcal{g}w - \mathcal{S}(w, v), rs) \geq 1, \forall \mu_i \in \mathfrak{S}$. Hence $\mathcal{g}w = \mathcal{S}(w, v)$ and similarly $\mathcal{g}v = \mathcal{S}(v, w)$. Therefore, (w, v) is a coupled coincidence point of $\mathcal{g}$ and $\mathcal{S}$.

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