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Fibrewise minimal and maximal regular spaces and fibrewise minimal and maximal normal spaces

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Abstract

The aim of this paper is to introduce and study some of the Fibrewise minimal regular, Fibrewise maximal regular, Fibrewise minimal completely regular, Fibrewise maximal completely regular, Fibrewise minimal normal, Fibrewise maximal normal, Fibrewise minimal functionally normal, and Fibrewise maximal functionally normal. This is done by providing some definitions of the concepts and examples related to them, as well as discussing some properties and mentioning some explanatory diagrams for those concepts.

Subject Classification: *55R70, 54C08, 54C10.*

Keywords: *Fibrewise minimal regular, Fibrewise maximal regular, Fibrewise minimal normal, Fibrewis maximal normal.*

1. Introduction

Minimal open sets and mappings in topological spaces were presented and investigated by S. S. Benchalli, Basavaraj M. Ittanagi, and R. S. Walli in source [23] in 2011. Additionally, in 2001 and 2003, F. Nakaoka and N. Oda discuss various applications of minimal open sets in source [6], while F. Nakaoka and N. Oda investigate some features of maximal open sets in source [7]. This work aims to present the Fibrewise minimal regular, in addition to the fiberwise minimal normal, Fibrewise minimal completely regular, Fibrewise maximal completely regular, Fibrewise minimal functionally normal, and Fibrewise maximal functionally normal. The base set is indicated by S then a Fibrewise set on S comprise a set R In addition to a function $\mathfrak{f}: R \to S$ Is the name of the

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project. For any point s of S of $R_S = P^{-1}(s)$ of$R$. Our investigation will be based on a few sources, including as [1-9], [10-20] and [21-24].

Definition 1.1: [13] Let R and Q be fib-wis sets over S, with proje. $£_R: R \rightarrow S$ and $£_Q : Q \rightarrow S$, ∀ s ∈ S and it known S base set. The function $\lambda : R \rightarrow Q$ is called fib-wis, if $£_R = £_Q o\lambda$, in the altemative idea if $ƒ(R_S) \subset Q_S$.

Definition 1.2: [13] If S a topool. sp. and any topology on R for which the proje. $ƒ : R \rightarrow S$ Continuous is called the Fibrewise topology. (Symbolizes it fibwise topool.).

Definition 1.3: [13] Let $\lambda : R \rightarrow Q$ be a fib-wis function, as R and Q are fib-wis topool. sp. over S is called Fibrewise continuous (symbolizes it fib-wis continue.) if ∀ r∈ R_S, where s∈ SThe converse image of any open of $\boldsymbol{\lambda}$(r) is an open of r.

Definition 1.4: [13] Let $\lambda : R \rightarrow Q$ be a fib-wis function, as R and Q are fib-wis topool. sp. over S is called Fibrewise open (symbolizes it fib-wis ope.) if ∀ r ∈ R_S, where s ∈ S the direct image of any open of R is an open of $\boldsymbol{\lambda}$(r) contain r.

Definition 1.5: [29] Let R and Q be a topool. sp.'s and $\lambda : R \rightarrow Q$ is called minimal (resp., maximal, minimal maximal, maximal minimal) continuous [symbolizes it mini (resp., maxi, mini maxi, maxi mini)-continue.) if $\lambda^{-1}(B)$ is an open (resp., open, maxi open, mini open) in R for each mini (resp., maxi, mini, maxi) ope. B in Q.

2. Fibrewise minimal and maximal Regular Spaces and Fibrewise minimal and maximal Normal Spaces

Definitions 2.1: If S is topool. sp., the Fibrewise mini (resp., maxi, mini maxi, maxi mini) topool. [symbolizes it fib-wis mini, (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini)-topool.] over fib-wis set R on$S$If any tool. over R for which the proje. $ƒ$ Is mini (resp., maxi, mini maxi, maxi mini)-continue.

Definition 2.2: Let R be a fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp. over S, R be known as fib-wis mini regular (resp., maxi regular, mini maxi regular, maxi mini regular), which is indicated via the symbol [fib-wis mini (resp., maxi, mini maxi, maxi mini) Reg.], when every r ∈ R_S; s ∈ S, and any open set ϑ of r in R, there is mini (resp., maxi, mini maxi, maxi mini) open ʍ of s in S, and open set Z of r in $R_ʍ$; ϑ is includes the closure of Z in $R_ʍ$(viz., $R_ʍ \cap$ clo.$(Z) \subset \vartheta$).

Definition 2.3: If £ : R → Q is a Fibrewise function, as R and Q are fib-wis topool. sp. over S is called Fibrewise bi-mini continuous (symbolizes it fib-wis bi-mini continue.) if ∀ r ∈ R_S, where s ∈ S, The inverse image of every mini open set of £(r) is a mini open set of r.

Definition 2.4: If £ : R → Q is a Fibrewise function, as R and Q are fib-wis topool. sp. over S is called Fibrewise bi-maxi continuous (symbolizes it fib-wis bi-maxi continue.) if ∀ r ∈ R_S, where s ∈ S, The inverse image of every maxi open set of £(r) is a maxi open set of r.

Proposition 2.5: Let λ: $R \rightarrow R^*$ be embedding fib-wis function, when R and R^* are fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp. over S. If R^* is a fib-wis mini (resp., maxi, mini maxi, maxi mini) Reg., then R be as well.

Proof: The proofs of four facts are similar, so we will only prove the fact (mini): Suppose that ϑ be open of r in$R$, when r ∈ R_S; s ∈ S. Then $\vartheta = \lambda^{-1}(\vartheta^*)$, when ϑ^* is open set of $r^* = \lambda(r)$ in R_S^*. Have a mini open ʍ of s in S, and an open set Z^* of r^* in $R_ʍ^*$, where $R_ʍ^* \cap$ clo.$\{Z^*\} \subset \vartheta^*$, because R^* is a fib-wis mini Reg. Then $Z = \lambda^{-1}(Z^*)$ is open set of r in $R_ʍ$; $R_ʍ \cap$ clo$\{Z\} \subset \vartheta$. Therefore, R is fib-wis mini Reg.

A fib-wis mini (resp., maxi, mini maxi, maxi mini) Reg. sp.'s are fib-wis multiplicative, as demonsted via the accompanying Proposition.

Proposition 2.6: If R_i is finite family of fib-wis mini (resp., maxi, mini maxi, maxi mini) Reg. sp.'s over$S$, following that, a fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. Product $R = \prod_S(R_i)$ is fib-wis mini (resp., maxi, mini maxi, maxi mini) Reg.

For infinite fib-wis products, the same result is valid as long as each component is fib-wis non empty.

Definition 2.7: Let λ : R → Q be a fib-wis function, as R and Q are fib-wis topool. sp. over S is called Fibrewise minimal (resp., maximal, minimal maximal, maximal minimal) open [symbolizes it fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) ope.] if ∀ r ∈ R_S, where s ∈ S the direct image of every open (resp., open, mini open, maxi open) of R is a mini (resp., maxi, maxi, mini) open of $\boldsymbol{\lambda}$(r) contain r.

Proposition 2.8: Consider $\lambda : R \rightarrow Q$ be mini (resp., maxi, mini maxi, maxi mini) open, closed, and continuous fib-wis a surjection function, when$R$, and Q are fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp.'s over S, if R is fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) Reg., then Q be as well.

Proof: The proofs of four facts are similar, so we will only prove the fact (mini): Suppose that ϑ be an open set of q in$Q$, when q ∈ Q_s such that s ∈ S, select r ∈ λ^{-1}(q) , then $Z = \lambda^{-1}(\vartheta)$ is open set r in R. Have a mini open ʍ of s in S, and open set Z^* of r; $R_ʍ \cap$ clo.$\{Z^*\} \subset Z$, because R is a fib-wis mini Reg., thus $Q_ʍ \cap$ λ(clo.(Z^*)) $\subset \lambda(Z) = \vartheta$, since λ is closed, λ(clo.(Z^*)) = clo.(λ (Z^*)), as well as λ is open, then λ (Z^*) is open set of q. Therefor, Q is fib-wis mini Reg.

Definition 2.9: Let R be a fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp. over S, be known as fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) completely regular which be indicated via the symbol [fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) C. Reg.], if at any r $\in R_s$, where s $\in S$, and for any open ϑ of r, there exists mini (resp., maxi, maxi , mini) open ʍ of s in S, and open set Z of r in $R_{ʍ}$, and continuous function $\text{ɣ} : R_{ʍ} \rightarrow \mathbb{I} = [0, 1]$; $R_s \cap Z \subset \text{ɣ}^{-1}(0)$, and $R_s \cap (R_s - \vartheta) \subset \text{ɣ}^{-1}(1)$

Proposition 2.10: Let $\lambda : R \rightarrow R^*$ embedding fib-wis function contin (resp., contin, bi-maxi contin, bi-mini contin), when$R$, and R^* are fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp.'s over$S$, If R^* are fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) C. Reg., then R be as well.

Next, demonstrate that the class of fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) C. Reg. sp.'s is finitely multiplicative.

Proposition 2.11: Suppose that R_i be family of fib-wis mini C. (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) Reg. sp.'s over$S$. The fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) topool. product $R = \prod_S(R_i)$ is a fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) C. Reg.

If all of the factors are infinite fib-wis products and not empty, an analogous inference applies.

Proposition 2.12: Suppose that $\lambda : R \rightarrow Q$ be open (resp., open, bi-maxi open, bi-mini open), closed, and continuous fib-wis surjection function, when$R$, and Q are fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp.'s over$S$, if R is fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) C. Reg., then Q be as well.

Proof: The proofs of four facts are similar, so we will only prove the fact (mini): Suppose that ϑ_q be a open set of q in$Q$, when q $\in Q_b$ such that s$\in S$, select r$\in \lambda^{-1}(q)$; $\vartheta_r = \lambda^{-1}(\vartheta_q)$ is open set of r in$R$. Have a mini open ʍ of s in S, and open set Z_r of r in $R_{ʍ}$ and continuous function $\text{ɣ} : R_{ʍ} \rightarrow \mathbb{I} = [0, 1]$; $R_s \cap Z_r \subset \text{ɣ}^{-1}(0)$, and $R_{ʍ} \cap (R_{ʍ} \_ \vartheta_r) \subset \text{ɣ}^{-1}(1)$, since R is a fib-wis mini C. Reg. Now procure on continuous function $\text{Ɏ} = \text{ɣ}\, o\, \text{ƒ}$; $\text{Ɏ} : R_{ʍ} \rightarrow \mathbb{I} = [0, 1]$, and $R_s \cap \text{Z} \subset \text{Ɏ}^{-1}(0)$, and $R_s \cap (R_s \_ \vartheta_q) \subset \text{Ɏ}^{-1}(1)$. The version of fib-wis mini normal space is then defined.

Definition 2.13: Let R be a fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp. over S, R is known as fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) normal which be indicated via the symbol [fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) $\mathcal{N}$], whenever $r \in R_s$; s $\in S$, as well as any disjoint pair of closed sets X and Y of R, there is mini (resp., maxi, mini, maxi) open ʍ of s in S, and disjoint pair of open (resp., open, mini open, maxi open) sets Z and ϑ of $R_{ʍ} \cap X$, $R_{ʍ} \cap Y$ in $R_{ʍ}$.

Remark 2.14: If R is fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) $\mathcal{N}$ sp. over S, then for any subspace S^* of S, leads to R_S^* is a fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) $\mathcal{N}$ sp. over S^*.

Proposition 2.15: Let $\lambda : R \rightarrow R^*$ be an embedding fib-wis function and closed (resp., closed, bi-maxi closed, bi-mini closed), when$R$, and R^* are fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp.'s over S , If R^* are fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) $\mathcal{N}$, then R be as well.

Proof: The proofs of four facts are similar, so we will only prove the fact (min): Suppose that$X$, and Y are a pair that are disjoint of mini closed and mini closed sets of R; $s \in S$. Then $\lambda(X)$, and $\lambda(Y)$ are a pair that are disjoint of mini closed set ofR^*. Because, $R^*$is fib-wis mini $\mathcal{N}$, then have a mini open ɱ of s in S, and mini open Z^*, and mini open ϑ^* of $R_{ʍ}^* \cap \lambda(X)$, $R_{ʍ}^* \cap \lambda(Y)$ in $R_{ʍ}^*$. Because λ is mini continuous, then $Z = \lambda^{-1}(Z^*)$, $\vartheta = \lambda^{-1}(\vartheta^*)$ are a pair that are disjoint of mini open sets and mini open sets of $R_{ʍ} \cap X$, $R_{ʍ} \cap Y$ in $R_{ʍ}$.

Definition 2.16: If £ : R → Q is a Fibrewise function, as R and Q are fib-wis topool.. sp. over S is called Fibrewise bi-mini (resp., bi-maxi) open [symbolizes it fib-wis bi-mini (resp., bi-maxi) ope.] if ∀ r ∈ R_S, where s ∈ S, The direct image of every mini (resp., maxi) open set of r is a mini (resp., maxi) open set of £(r).

Proposition 2.17: Suppose that $\lambda : R \rightarrow Q$ be open (resp., open, bi-maxi opeb, bi-mini open) closed, and continuous fib-wis surjection function, when R, and Q are fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp.'s over S, if R is fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) $\mathcal{N}$, if and only if Q be as well.

Definition 2.18: Let R be a fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp.over S, is known as fib-wis minimal (resp., maximal, minimal maximal, maximal minimal) functionally normal which be indicated via the symbol [fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) F. $\mathcal{N}$], if at any s ∈ S, and any disjoint pair of closed sets X and Y of R , there is a mini (resp., maxi, mini, maxi) open ʍ of s in S, and disjoint pair of open set Z, and open set ϑ, and continuer's function $ɣ : R_{ʍ} \rightarrow \mathbb{I} = [0, 1]$; $R_{ʍ} \cap X \cap Z \subset ɣ^{-1}(0)$, and $R_{ʍ} \cap Y \cap \vartheta \subset ɣ^{-1}(1)$ in $R_{ʍ}$.

Remark 2.19: Let R be a fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) F. $\mathcal{N}$ sp. over S, then R_S^* is a fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) F. $\mathcal{N}$ sp. over S^* for any subspace S^* of S.

Definition 2.20: If £ : R → Q is a Fibrewise function, as R and Q are fib-wis topool. sp. over S is called Fibrewise bi-mini (resp., bi-maxi) closed [symbolizes it fib-wis bi-mini (resp., bi-maxi) close] if ∀ r ∈ R_S, where s ∈ S, The direct image of every mini (resp.,maxi) closed set of r is a mini (resp.,maxi) closed set of £(r).

Proposition 2.21: Suppose that $\lambda : R \rightarrow R^*$ be an embedding fib-wis function and closed (resp., closed, bi-mini, bimaxi), when R and R^* are fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp.'s over$S$. If R^* is fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) F. $\mathcal{N}$, then R be as well.

Proof: The proofs of four facts are similar, so we will only prove the fact (mini):

Suppose that X and Y are a pair that are disjunct of closed set and closed sets of$R$; s $\in S$. Then $\lambda(X)$ and $\lambda(Y)$ are a pair that are disjunct of mini closed set ofR^*. Because R^* is fib-wis mini F. $\mathcal{N}$, then have a mini open ʍ of s in S, and a pair disjunct of n open Z and open ϑ, and continuous function ɣ : $R^*_{ʍ} \rightarrow \mathbb{I}$ = [0, 1]; $R^*_{ʍ} \cap \lambda(X) \cap Z \subset$ ɣ$^{-1}(0)$, and $R^*_{ʍ} \cap \lambda(Y) \cap \vartheta \subset$ ɣ$^{-1}(1)$ in $R^*_{ʍ}$. Because, λ is continuous, then $\lambda^{-1}(Z)$, $\lambda^{-1}(\vartheta)$, are mini open, and a function Ꞙ = ɣ o ⨍ is a continuous, Ꞙ : $R_{ʍ} \rightarrow \mathbb{I} = [0,1]$; $R_{ʍ} \cap X \cap £^{-1}(Z) \subset$ Ꞙ$^{-1}(0)$, and $R_{ʍ} \cap Y \cap £^{-1}(\vartheta) \subset$ Ꞙ$^{-1}(1)$ in $R_{ʍ}$.

Proposition 2.22: Suppose that $\lambda : R \rightarrow Q$ be mini (resp., maxi, bi-mini, bimaxi) open, closed, and continuous fib-wis surjection function, when R, and Q are fib-wis mini (resp., maxi, mini maxi, maxi mini) topool. sp.'s over S, if R is fib-wis mini (resp., fib-wis maxi, fib-wis mini maxi, fib-wis maxi mini) F. $\mathcal{N}$, then Q be as well.

Proof: The proofs of four facts are similar, so we will only prove the fact (mini): Suppose that$X$, and Y are a pair that are disjunct of closed set and closed sets of R; s $\in S$. Then$\lambda^{-1}(X)$, and $\lambda^{-1}(Y)$ are a pair that are disjunct of closed set of R. Since R is fib-wis mini F. $\mathcal{N}$, then now have a mini open ʍ of s in S, and pair disjunct of open Z and open ϑ, and continuous function ɣ : $R_{ʍ} \rightarrow \mathbb{I} = [0,1]$; $R_{ʍ} \cap \lambda^{-1}(X) \cap Z \subset$ ɣ$^{-1}(0)$, and $R_{ʍ} \cap \lambda^{-1}(Y) \cap \vartheta \subset$ Ꞙ$^{-1}(1)$ in $R_{ʍ}$. Thus, Ꞙ : $R_{ʍ} \rightarrow \mathbb{I} = [0,1]$, provided via Ꞙ$(q)$= $\sup_{r \in \lambda^{-1}(q)}$ ɣ(r); q ∈ $R_{ʍ}$. Since λ is mini open, closed, additionally to continuous, It eventually leads to it Ꞙ is continuous. Thus, $R_{ʍ} \cap X \cap \lambda(Z) \subset$ Ꞙ$^{-1}(0)$, and $R_{ʍ} \cap Y \cap \lambda(\vartheta) \subset$ Ꞙ$^{-1}(1)$ in $R_{ʍ}$.

3. Discussion and Conclusion

Through my use of two well-known topics in topological space, which are regular space and normal space, and employing them as Fibrewise minimal space and Fibrewise maximal space is clear from the presentation of the research, the result is that I obtained new topics as follows: Fibrewise minimal regular space, Fibrewise maximal regular space, Fibrewise minimal normal space, Fibrewise maximal normal space and some of the propositions show it.

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