

Competing dynamics : Soft intense set in soft ideal topological spaces

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Abstract

Three important mathematical concepts incorporated into the ideal soft topological spaces. The relation between soft semi intense, soft Too intense and soft intense sets .We showed the relation between these concepts with soft boundary as well as the effect of soft codense on those concepts.

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1. Introduction

When a scientific problem arises and there is no ability to solve it in the usual ways within a specific space. Attention turns to expanding or contracting that space and constructing the algebraic and topological properties of these spaces in order to solve those problems or contribute to the solution process. Hence the successive ideas in building new forms of sets on the basis of existing and familiar mathematical concepts, as Zadeh built in 1965 [1] the fuzzy sets. In 1999, researcher Molodtsov [2] expanded fuzzy sets and called them soft sets, depended on the parameters E of members of the universal set x such that the soft set $F_A = \{(e, F(e)); F : E \rightarrow P(X), A \subseteq E\}$, soft empty $\varphi_E = \{(e, \varphi); \forall e \in E\}$, $X_E = \{(e, F(e)); \forall e \in E\}$, soft union and intersection between two soft sets $F_A \cup_E G_B = \{(e, F(e) \cup G(e)); \forall e \in E\}$, $F_A \cap_E G_B = \{(e, F(e) \cap G(e)); \forall e \in E\}$, the soft complement of F_A , $F_A^c = \{(e, X - F(e)); \forall e \in E\}$. In 2011 Shabir [3], defined the topology space on these sets in their canonical form but using soft sets. In 2014 [4], the local function was studied in its usual forms, but using soft on soft points in its three types by kandil and others as follows, $F_A^* = \{\text{soft point } x; \forall \text{ soft open containing the soft point } x, x \cap F_A \notin \hat{I}_E\}$, where $\hat{I}_E$ soft ideal on X_E . In 2014 [5], ψ - operation was studied in its usual forms, but using soft sets on soft points in its three types, by Rodynas [5] $\psi(F_A) = \{\text{soft point } x, \exists \text{ soft open } U_x \text{ containing the soft point } x, U_x \cap F_A^e \in \hat{I}_E\} = [(F_A^e)^*]^e$. It is worth noting that we mention some of the contributions in this field, where we knew two types of soft points that we called them soft turning and bench point [6], [7] also we defined the local function in fuzzy ideal topological space [8], where AL Razzaq [9], [10] conducted a process of merging between the two fuzzy and soft sets, and through this merging new and diverse points were identified.

Where that, $X_{\hat{I}_A}^{\hat{T}}$ be the soft ideal topological spaces and K_A be the soft set.

In these papers, all concepts will be depending on a certain type of soft points that is F_e where $F(\acute{e}) \neq \varphi$ for $e = \acute{e}$ and $F(\acute{e}) = \varphi$ for $e \neq \acute{e}$. As for other soft points, the concepts can simply be studied by simple tactics. An important result we need for this study is,

Theorem 1: [5] *If $\hat{I}_A$ is soft codense for any soft subset K_A of $X_{\hat{I}_A}^{\hat{T}}$.*

1. $\psi[K_A] \subset K_A^*$.
2. If K_A is soft closed, then $\psi[K_A] \subset K_A$.

Theorem: 2 [5] *For any soft subset K_A of X .*

1. If $K_A \in \hat{T}$, then $K_A \subset \psi[K_A]$
2. $K_A^{**} \subset K_A^*$
3. $X - K_A^* = \psi[X - K_A]$
4. $X - \psi[K_A] = [X - K_A]^*$
5. If $H_A \in \hat{T}$, then for any K_A in $X_{\hat{I}_A}^{\hat{T}}$, $H_A \cap K_A^* \subset [H_A \cap K_A]^*$

2. ψ – Soft Operator

There are some important conclusions about ψ – soft operator characteristics in addition to those about the soft ideal in case of being soft codense where $(\hat{T}_E \cap \hat{I}_E = \varphi_E$ iff $X_E = X_E^*$)[5].

Proposition 1: For any soft subset K_A of $X_{\hat{I}_A}^{\hat{T}}$ and $\hat{I}_A$ is soft codense then:

1. $\psi[[\psi[K_A^*]]^*] = \psi[K_A^*]$
2. $[\psi[[\psi[K_A]]^*]]^* = [\psi[K_A]]^*$

Proof 2: Since $[\psi[K_A]]^*$ is soft closed set by **(Theorem 1)**, $[\psi[(\psi[K_A])^*]]^* \subset [\psi[K_A]]^{**} \subset [\psi[K_A]]^*$. Conversely, let $F_e \in [\psi[K_A]]^*$ and if possible $F_e \notin [\psi[[\psi[K_A]]^*]]^*$, $\exists W_A \in \hat{T}(F_e)$, $W_A \cap \psi[[\psi[K_A]]^*] \in \hat{I}_A$. But by **(Theorem 1,2)**, we get $\psi[K_A] \subset [\psi[K_A]]^*$. So $W_A \cap \psi[K_A] \in \hat{I}_A$. That means, $F_e \notin [\psi[K_A]]^*$ which contradiction.

Proposition 2: Let $\hat{I}_A$ be soft codense. If $K_{A_1} \in \hat{T}$ or $K_{A_2} \in \hat{T}$, then $\psi[[K_{A_1} \cap K_{A_2}]^*] = \psi[[K_{A_1}]^*] \cap \psi[[K_{A_2}]^*]$.

Proof: Let $K_{A_1} \in \hat{T}$, by **(Theorem 2)**, $[K_{A_1} \cap [K_{A_2}]^*] \subset [K_{A_1} \cap K_{A_2}]^*$. So $[K_{A_1} \cap \psi[K_{A_2}^*]] \subset \psi[K_{A_1} \cap K_{A_2}] \subset \psi[[K_{A_1} \cap K_{A_2}]^*]$, then $\psi[[K_{A_1} \cap \psi[K_{A_2}^*]]^*] \subset \psi[[\psi[[K_{A_1} \cap K_{A_2}]^*]]^*] = \psi[[K_{A_1} \cap K_{A_2}]^*]$ by **(Proposition 1)**. But $[K_{A_1}^* \cap \psi[K_{A_2}^*]] \subset [K_{A_1} \cap \psi[K_{A_2}^*]]^*$, then $\psi[K_{A_1}^*] \cap \psi[\psi[K_{A_2}^*]] \subset \psi[K_{A_1} \cap \psi[K_{A_2}^*]]^*$ and by **(Theorem 2)**, $[\psi[K_{A_1}^*] \cap \psi[K_{A_2}^*]] \subset \psi[[K_{A_1} \cap \psi[K_{A_2}^*]]^*] \subset \psi[[K_{A_1} \cap K_{A_2}]^*]$. And the conversely part is directly.

Corollary 3: Let $\hat{I}_A$ be soft codense. For any K_{A_1}, K_{A_2} in $X_{\hat{I}_A}^{\hat{T}}$, the following are hold:

1. $\psi[\psi[K_{A_1}] \cap K_{A_2}]^* = \psi[\psi[[K_{A_1}]]^*] \cap \psi[K_{A_2}^*]$
2. $\psi(\psi[K_{A_1} \cap K_{A_2}]^*) = \psi[[\psi(K_{A_1})]^*] \cap \psi[[\psi(K_{A_2})]^*]$
3. $[\psi[K_{A_1}^*]]^* \cup [\psi[K_{A_2}^*]]^* = [\psi[[K_{A_1} \cup K_{A_2}]^*]]^*$

Definition 4: A soft subset K_{A_2} of $X_{\hat{I}_A}^{\hat{T}}$ is said to be,

1. Soft Too intense, if $\psi[H_A^*] = \psi[H_A]$.
2. Soft Semi intense, if $[\psi[H_A]]^* = H_A^*$.
3. Soft Intense, if $[\psi[H_A]]^* = H_A^*$ and $\psi[H_A^*] = \psi[H_A]$.

For $\hat{I}_A = 2^X$, then for any soft subset M_A of X is soft Intense set because $M_A^* = \varphi$ and $\psi[M_A] = X$. For any members of proper soft ideal $\hat{I}_A$ is not soft Semi intense, not soft Too intense and not soft Intense set. Also every $\hat{I}_A$ - soft dense set ($K_A^* = X$) is not soft Too intense, is soft Semi intense and not soft Intense. The soft empty set φ is soft Too intense, but not soft Semi intense and X is soft Semi intense, but not soft Too intense. For $\hat{I}_A$ is soft codense we get that φ and X are soft Intense sets. From the **(Definition 4)**, also K_A in $X_{\hat{I}_A}^{\hat{I}}$ is soft Intense if and only if K_A is soft Too intense and soft Semi intense set. Finally, we see that the soft complement of the soft Too intense set is soft Semi intense set and the soft complement of soft Semi intense set is soft Too intense set by using the **(Theorem 2)**, if H_A is $*$ perfect [5] and $\psi[H_A^*] \subset H_A$, then H_A is soft Too intense.

Proposition 5: Let $\hat{I}_A$ be soft codense. Then

1. Let H_A be soft closed set in $X_{\hat{I}_A}^{\hat{I}}$. H_A is soft Too intense iff $\psi[H_A^*] \subset H_A$.
2. Let K_A be soft open set in $X_{\hat{I}_A}^{\hat{I}}$. K_A is soft Semi intense iff $K_A \subset [\psi[K_A]]^*$ (K_A is ψ^* -soft set).
3. Let K_A be soft clopen set in $X_{\hat{I}_A}^{\hat{I}}$. K_A is soft Intense iff $\psi[K_A^*] \subset K_A \subset [\psi[K_A]]^*$.

Definition 6:

1. A soft set K_A in $X_{\hat{I}_A}^{\hat{I}}$ is said to be Θ^* – soft open set if $K_A = \psi[K_A^*]$.
2. A soft set H_A in $X_{\hat{I}_A}^{\hat{I}}$ is said to be Θ^* – soft closed set if $H_A = [\psi[H_A]]^*$.

The soft empty set φ and X in any $X_{\hat{I}_A}^{\hat{I}}$ are not Θ^* – soft open and not Θ^* – soft closed, but if $\hat{I}_A$ is soft codense, then φ and X are Θ^* – soft open and Θ^* – soft closed. For $\hat{I}_A = 2^X$, then X is Θ^* – soft open, but not Θ^* – soft closed and φ is Θ^* – soft closed, but not Θ^* – soft open. By using **(Proposition 2)**, for any soft set H_A in $X_{\hat{I}_A}^{\hat{I}}$, $\psi[H_A^*]$ is Θ^* – soft open set and $[\psi[H_A]]^*$ is Θ^* – soft closed. Also by using **(Theorem 1)**, the soft complement of Θ^* – soft open is Θ^* – soft closed and the conversely is also true.

Remark 7:

1. The soft intersection of two Θ^* – soft open is Θ^* – soft open.
2. The soft union of two Θ^* – soft closed is Θ^* – soft closed.

Proof: Let H_{A_1}, H_{A_2} are any soft sets in $X_{\hat{I}_A}^{\hat{I}}$. Since $\psi[K_{A_2}^*]$ is soft open, then by **(Theorem 2)** $K_{A_1}^* \cap \psi[K_{A_2}^*] \subset [K_{A_1} \cap \psi[K_{A_2}^*]]^*$, so $\psi[K_{A_1}^*] \cap \psi[K_{A_2}^*] \subset \psi[K_{A_1}^*] \cap \psi[\psi[K_{A_2}^*]] \subset \psi[[K_{A_1} \cap \psi[K_{A_2}^*]]^*] = \psi[[K_{A_1} \cap K_{A_2}]^*]$. Then $K_{A_1} \cap$

$K_{A_2} \subset \psi[[K_{A_1} \cap K_{A_2}]^*]$, and $\psi[[K_{A_1} \cap K_{A_2}]^*] \subset \psi[K_{A_1}^*] \cap \psi[K_{A_2}^*] = K_{A_1} \cap K_{A_2}$. Therefore $K_{A_1} \cap K_{A_2}$ is Θ^* – soft open.

Proposition 8: Let $\hat{I}_A$ be soft codense on X . then

1. K_A is Θ^* – soft open iff $K_A = \psi[K_A]$ and K_A is soft Too intense.
2. K_A is Θ^* – soft open iff $K_A = K_A^*$ and K_A is soft semi intense.

Proof 1: Let K_A is Θ^* – soft open, so $K_A = \psi[K_A^*]$. By **(Theorem 1)**, $\psi[K_A] \subset K_A^*$, so $\psi[K_A] \subset \psi[\psi[K_A^*]] \subset \psi[K_A^*] = K_A$. Also $K_A = \psi[K_A^*] \subset \psi[\psi[K_A^*]] = \psi[K_A]$, Therefore $K_A = \psi[K_A]$. Now, from above $\psi[K_A^*] \subset \psi[K_A]$ and **(Theorem 1)**, $\psi[K_A] \subset K_A^*$, imply that $\psi[K_A] \subset \psi[\psi[K_A^*]] \subset \psi[K_A^*]$. Therefore $\psi[K_A] = \psi[K_A^*]$. Conversely, let K_A is soft Too intense and $\psi[K_A] = K_A$, then $K_A = \psi[K_A] = \psi[K_A^*]$, that means K_A is Θ^* – soft open.

Corollary 9: Let $\hat{I}_A$ be soft codense in X , then

1. K_A is Θ^* – soft open iff K_A is soft Intense and $K_A = \psi[K_A]$.
2. K_A is Θ^* – soft closed iff K_A is soft Intense and $K_A = K_A^*$.

Theorem 10: Let $\hat{I}_A$ be soft codense in X . A set K_A is soft Intense iff the $*$ – soft boundary of K_A coincides with $\mathcal{F}_{Ar}^*[K_A] = [\psi[K_A]]^* \cap [\psi[X - K_A]]^*$.

Proof: Since $\mathcal{F}_{Ar}^*[K_A] = K_A^* \cap [X - K_A]^*$ and K_A is soft Intense set, so $K^* = [\psi[K_A]]^*$ and $\psi[K_A] = \psi[K_A^*]$ iff $X - \psi[K_A] = X - \psi[K_A^*]$ iff $[X - K_A]^* = [\psi[X - K_A]]^*$. Therefore $\mathcal{F}_{Ar}^*[K_A] = [\psi[K_A]]^* \cap [\psi[X - K_A]]^*$.

Conversely, let $\mathcal{F}_{Ar}^*[K_A] = [\psi[K_A]]^* \cap [\psi[X - K_A]]^*$, also $\mathcal{F}_{Ar}^*[K_A] = K_A^* \cap [X - K_A]^* = K_A^* \cap X - \psi[K_A]$ iff $K_A^* = \mathcal{F}_{Ar}^*[K_A] \cup \psi[K_A] = \psi[K_A] \cup [[\psi[K_A]]^* \cap \psi[X - K_A]]^* \subset \psi[K_A] \cup [\psi[K_A]]^*$. But $\hat{I}_A$ is soft codense and $\psi[K_A] \in \hat{T}$, so $\psi[K_A] \subset [\psi[K_A]]^*$, then $K_A^* \subset [\psi[K_A]]^*$. Again using soft codense, we get $\psi[K_A] \subset K_A^*$, so $[\psi[K_A]]^* \subset K_A^{**} \subset K_A^*$, then K_A is soft Semi intense. Since $\mathcal{F}_{Ar}^*[K_A] = \mathcal{F}_{Ar}^*[X - K_A]$ and $[X - K_A]^* = [\psi[X - K_A]]^*$ iff $X - \psi[K_A] = X - \psi[K_A^*]$ iff $\psi[K_A] = \psi[K_A^*]$. Therefore K_A is soft Too Intense.

Remark 11: Let K_A be soft subset of $X_{\hat{I}_A}^{\hat{T}}$. Then

1. If K_A be satisfy, $\psi[K_A^*] \subset [\psi[K_A]]^*$, then $X - K_A$ also satisfy $\psi[[X - K_A]^*] \subset [\psi[X - K_A]]^*$.
2. If K_A be satisfy, $[\psi[K_A]]^* \subset \psi[K_A^*]$, then $X - K_A$ also satisfy $[\psi[X - K_A]]^* \subset \psi[[X - K_A]^*]$.
3. If K_A be satisfy, $\psi[K_A^*] \not\subset [\psi[K_A]]^*$ and $[\psi[K_A]]^* \not\subset \psi[K_A^*]$, so $X - K_A$ is not satisfy these inequalities.

Proposition 12: Let $\hat{I}_A$ be soft codense in X . Then

1. If $\psi [K_A^*] \subset [\psi [K_A]]^*$, then $\psi [K_A^*] = \psi [[\psi [K_A]]^*]$ and $[\psi [K_A]]^* = [\psi [K_A^*]]^*$ are hold.
2. If $\psi [K_A^*] = \psi [[\psi [K_A]]^*]$ is hold or $[\psi [K_A]]^* = [\psi [K_A^*]]^*$ is hold, then $\psi [K_A^*] \subset [\psi [K_A]]^*$.

Proof: Let $\psi [K_A^*] \subset [\psi [K_A]]^*$ is hold, so $\psi [K_A^*] \subset \psi [\psi [K_A^*]] \subset \psi [[\psi [K_A]]^*] \dots (1)$. Since $\hat{I}_A$ is soft codense, $[\psi [K_A]]^* \subset K_A^{**} \subset K_A^*$, then $\psi [[\psi [K_A]]^*] \subset \psi [K_A^*] \dots (2)$. From (1) and (2) we get $\psi [K_A^*] = \psi [[\psi [K_A]]^*]$. Now, by using **(Remark 11(1))** $\psi [X - K_A]^* \subset [\psi [X - K_A]]^*$. Then $\psi [X - K_A]^* = \psi [[\psi [X - K_A]]^*]$ iff $X - [\psi [K_A]]^* = X - [\psi [K_A^*]]^*$ iff $[\psi [K_A]]^* = [\psi [K_A^*]]^*$.

Finally if $[\psi [K_A]]^* = [\psi [K_A^*]]^*$ is hold and $\psi [K_A^*] \in \hat{T}$, then by using **(Theorem 1)**, $\psi [K_A^*] \subset [\psi [K_A^*]]^* = [\psi [K_A]]^*$. Also if $\psi [K_A^*] = \psi [[\psi [K_A]]^*]$ is hold and $[\psi [K_A]]^*$ is soft closed, but I_A is soft codense and again using **(Theorem 1)**, we get $\psi [K_A^*] = \psi [\psi [K_A^*]] \subset [\psi [K_A]]^*$.

Proposition 13: Let $\hat{I}_A$ be soft codense in $\hat{X}$ and for any soft subsets K_{A_1} and K_{A_2} of $X_{\hat{I}_A}$. Satisfies $\psi [K_{A_i}^*] \subset [\psi [K_{A_i}]]^*$ for $i = 1, 2, \dots$. Then

1. $\psi [K_{A_1}^*] \cap \psi [K_{A_2}^*] = \psi [[K_{A_1} \cap K_{A_2}]]^*$
2. $[\psi [K_{A_1}]]^* \cup [\psi [K_{A_2}]]^* = [\psi [K_{A_1} \cup K_{A_2}]]^*$.

Proof: Clearly that $\psi [[K_{A_1} \cap K_{A_2}]]^* \subset \psi [K_{A_1}^*] \cap \psi [K_{A_2}^*]$. Now, by **(Proposition 12)** and **(Corollary 3)**, $\psi [K_{A_1}^*] \cap \psi [K_{A_2}^*] = \psi [[[\psi [K_{A_1}]]^*] \cap [\psi [K_{A_2}]]^*] = \psi [[\psi [K_{A_1} \cap K_{A_2}]]^*]$, but $\hat{I}_A$ is soft codense, so by **(Theorem 1)**, we get $\psi [K_{A_1} \cap K_{A_2}] \subset [K_{A_1} \cap K_{A_2}]^*$ and $[\psi [K_{A_1} \cap K_{A_2}]]^* \subset [K_{A_1} \cap K_{A_2}]^{**} \subset [K_{A_1} \cap K_{A_2}]^*$, so $\psi [K_{A_1}^*] \cap \psi [K_{A_2}^*] = \psi [[\psi [K_{A_1} \cap K_{A_2}]]^*] \subset \psi [[K_{A_1} \cap K_{A_2}]]^*$.

Conclusion

The sports concepts that we have studied here can be invested using w - open [10,14] after their modification in soft spaces. The concept of Para compact [11, 12, 13] can be adjusted on soft space and studying its relationship with the concepts that have been studied in these papers and we believe there are great results.

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