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Bi-nano topological spaces with grill bi-open sets

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Abstract

The study introduces a new concept in Nano topological spaces is said Grill Bi-Nano topological spaces via studying three issues disjoint of each other rely on the number of equivalence relations for the universe set and its subset and also present definitions for the $GN_{1,2}$- interior and $GN_{1,2}$- closure of sets with this framework and we briefly discuss some characteristics of local functions.

Subject Classification: *18B30, 28C15.*

Keywords: *Grill bi-nano-topological space, Grill bi-nano interior, Grill bi-nano closure, Multi - Granular bi-nano topology new class of, Grill bi-nano closed sets.*

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1. Introduction

In (1947), Choquet [5] was the first who put forward the concept of Grills on a topological space. Since then, researchers have extensively studied grills and their applications in conjunction with separation axioms, compactness and connectedness all fundamental tools in general topology ([12, 13, etc.]). The year (2013) marked a significant development with the introduction of Nano topological spaces by Thivagar et al. [13] This concept, built upon the notion of Nano open sets, paved the way for Nano continuous function and its real-world applications [12,10,9,etc.] the exploration of Nano-related topologies continued with Bhanuvan Vari's definition of Nano Bi-topological spaces in (2016) [4], also saw the introduction of Multi-Granular Nano topological spaces by Thivagar [13] and Grill Nano generalized closed sets presented by Azzam [3]. In (2021), Abd El Fattah El Atik delved deeper into Grill Nano topological spaces, investigating of continuity functions [1]. The research remains active, as evidenced by Suliman and Esmaeel's work in (2022), which explored various topological spaces within the Grill setting and their applications([6]).Most recently, Ahmed et al presented a generalization of the semi-open set concept in Grill Nano spaces in (2023) [2], The concept's development progressed and it became linked to other concepts as evidenced by Esmaeel et al ([7]). In this paper devoted new styles of Grill Nano space is said to Grill Bi- $T_{\pi 1,2}(K)$ -spaces and the presented the concepts $\mathrm{GN}_{1,2}$-interior and $\mathrm{GN}_{1,2}$-closure and discussed their basic properties.

2. Preliminaries

Definition 2.1: [5] A collection G of non-empty subsets of a topological space $\mathbb{H}$ is said to be a Grill if it is satisfying: 1. B∈G, If A∈ G and A⊆ B⊆ $\mathbb{H}$. 2. A∈G or B∈G, if A, B ⊆ $\mathbb{H}$ & A∪B ∈G. Since $\mathbb{H} \neq \emptyset$ then the following families are Grill's on $\mathbb{H}$ [18]. So, $\emptyset$ and $P(\mathbb{H})\setminus\{\emptyset\}$ are Grills on $\mathbb{H}$,

- $G_{\mathrm{A}} = \{B: B \in P(\mathbb{H}), \mathrm{A} \cap B^c = \emptyset\}$ and $G_{\mathrm{e}} = \{B: B \in P(\mathbb{H}), \mathrm{e} \in \mathrm{B}\}$ is a specific pointe Grill on $\mathbb{H}$.(when e∈ $\mathbb{H}$) and G_{∞} all infinite subsets Grill of $\mathbb{H}$(when $\mathbb{H}$ is infinite set).
- $G = \{\mathrm{A} : \mathrm{int}(\mathrm{cl}(\mathrm{A}) \neq \emptyset\}$ is one kinds of Grill on $\mathbb{H}$ [5, 8].

A mapping ϕ: $\mathrm{P}(\mathbb{H}) \rightarrow \mathrm{P}(\mathbb{H})$ such that $\phi(\mathrm{A}) = \{\mathrm{w} \in \mathbb{H}: (\mathrm{A} \cap \mathrm{U}) \in \mathrm{G}, \forall\, \mathrm{U} \in \mathrm{T}, \mathrm{w} \in \mathrm{U}\}, \forall\, \mathrm{A} \in \mathrm{P}(\mathbb{H})$ and $\Psi(A): \mathrm{P}(\mathbb{H}) \rightarrow \mathrm{P}(\mathbb{H}) \ni \Psi(A) = \mathrm{A} \cup \phi(\mathrm{A})$ is satisfy Kuratowski closure axioms [4].

Definition 2.2: [6] A special topology $\mathrm{T_G}$ = {U⊆ $\mathbb{H}$, Ψ ($\mathbb{H}$-U) = ($\mathbb{H}$-U)}, when ∀A⊆ $\mathbb{H}$ that corresponds to a grill G on ($\mathbb{H}$,T), if u ∈ $\mathrm{T_G}$ then u is G-open set. As the complement is G-closed set. Thus, Ψ(A) is a closure of set (A) with respect to $\mathrm{T_G}$ and T⊆$\mathrm{T_G}$.

Remarks 2.3: 1. $\mathrm{T_G}$ = T only if G=P ($\mathbb{H}$) /{ϕ}and B ($\mathbb{H}$, $\mathrm{T_G}$)={$\mathrm{V} - \mathrm{A}; \mathrm{V} \in \mathrm{T}, \mathrm{A} \notin \mathrm{G}$ } is basie on $\mathrm{T_G}$ [6]. 2. Every open sets is Grill-open sets [13].

Definition 2.:4 [12] Suppose that $U \neq \emptyset$ is universe set and π is an equivalence relation on U. where $K \subseteq U$. The lower adducting of K for π is denoted by $\mathbb{L}_{\pi}(K)$, when $\mathbb{L}_{\pi}(K) = \cup_{x \in U}\{\pi(x)\colon \pi(x) \subseteq K\}$. The upper adducting of K for π is denoted by $\mathbb{U}_{\pi}(K)$, when $\mathbb{U}_{\pi}(K) = \cup_{x \in U}\{\pi(x)\colon \pi(x) \cap K \neq \emptyset\}$. Also $\mathbb{B}_{\pi}(K) = \mathbb{U}_{\pi}(K) - \mathbb{L}_{\pi}(K)$ is the boundary region of K for π.

Definition 2.5: [11] Let $U \neq \emptyset$ and and π be an equivalence relation onU, $T_{\pi}(K) = \{U, \emptyset, \mathbb{L}_{\pi}(K), \mathbb{U}_{\pi}(K), \mathbb{B}_{\pi}(K)\}$ is Nano topology for U and the pair $(U, T_{\pi}(K))$ is Nano topological space. So, the elements of $T_{\pi}(K)$ are Nano-open sets(N-O).

Definition 2.6: [13] A space $(U, T_{\pi}(K))$ with a Grill is called Grill Nano topological space $(U, T_{\pi}(K), G)$ denoted by $GT_{\pi}(K)$, if $v \in GT_{\pi}(K)$ then v is Grill Nano-open set (GN -open set) and $(U - v)$ is said Grill Nano-closed set (GN -closed set). The operator $\emptyset_G$ (A, T_{π})=$\{v \in U\colon (A \cap V) \in G, \text{for all } v \in T_{\pi}(K)\}$ is the local function associated with the Grill G and $T_{\pi}(K)$, so, $\Psi_G(A)$=A∪ ϕ_G (A)= $CL_{GN}(A)$, where $CL_{GN}(A)$ is closure of set (A) with respect to $GT_{\pi}(K)$. So, **Every** N-O set is GN -open set.

3. On Grill Bi-Nano Topological Spaces

We submit define of Grill bi-nano topology relying on Grill bi-nano open set, also will be investigated defined multi-Granular bi-nano open sets.

Definition 3.1: Let $U \neq \emptyset$ universe set together two different Nano topology $T_{\pi 1}(K), T_{\pi 2}(K)$ on U, with respect to K and $(U, T_{\pi 1,2}(K))$ is bi-nano topological spaces briefly $(T_{\pi 1,2})$. Elements of the $T_{\pi 1,2}$ are bi-nano open sets $(N_{1,2} - o)$ and $(T_{\pi 1,2}\ (K))^c$ are called bi-nano closed sets.

Example 3.2: Let $U = \{n, m, \ell, k\}$, with $U/\pi = \{\{n\}, \{m, k\}, \{\ell\}\}$, where $K_1 = \{m, \ell\}$ and $K_2 = \{n\}$. We get $T_{\pi_1}(K) = \{U, \emptyset, \{\ell\}, \{m, k\}, \{m, \ell, k\}\}$ and $T_{\pi_2}(K) = \{U, \emptyset, \{n\}\}$, then Nano open sets = $\{U, \emptyset, \{\ell\}, \{m, k\}, \{m, \ell, k\}, ,\{n\}\}$, but not from a Nano topology because$\{m, k\} \cup \{n\} = \{m, k, n\}$ not in N-O sets. It is called consisting of $N_{1,2}$ −o, fulfilling the conditions of a supremum space in Bi- Nano o topology ($T_{\pi 1,2}$ – topology) on with respect to K, as $(U, T_{\pi 1,2}(K))$ is $T_{\pi 1,2}$.

Remark 3.3: we will discuss the following three cases and for the account Bi-Nano topology.

1-Let U / π be identicalness class on U and two subsets of U. We will prove this

Example 3.4: Suppose that U =$\{1,2,3,4\}$ and U / π =$\{\{1,2\},\{3,4\}\}$ with

	$\mathbb{L}_{\pi}(\mathbf{K})$	$\mathbb{U}_{\pi}(\mathbf{K})$	$\text{Ƀ}_{\pi}(\mathbf{K})$	$\mathbf{T}_{\pi}(\mathbf{K})$
$\mathbf{K_1} = \{\mathbf{1}\}$	$\emptyset$	{1,2}	{1,2}	{U, $\emptyset$,{1, 2}}
$\mathbf{K_2} = \{\mathbf{3},\mathbf{4}\}$	{3,4}	{3,4}	$\emptyset$	{U, $\emptyset$, {3,4}}

Then $\mathrm{T}_{\pi1,2}$(K)= {U, $\emptyset$,{1, 2}, {3,4}} .

2-Using U / π two equivalences class defined on U and two subsets of U.

Example 3.5: U /π_1= {{1}, { 2,4}, {3}} and U / $\pi_2 = \{\{2\},\{1,4\},\{3\}\}$ on U,

	$\mathbf{K}_{B=1,2}$	$\mathbb{L}_{\pi}(\mathbf{K})$	$\mathbb{U}_{\pi}(\mathbf{K})$	$\text{Ƀ}_{\pi}(\mathbf{K})$	$\mathbf{T}_{\pi}(x)$
U /π_1	{2,3}	{3}	{2,3,4}	{2,4}	$\{U, \emptyset, \{3\}, \{2,4\}, \{2,3,4\}\}$
U / π_2	{2,4}	{2}	{1,2,4}	{1,4}	$\{U, \emptyset, \{2\}, \{1,4\}, \{1,2,4\}\}$

$\mathrm{T}_{\pi1,2}${U, $\emptyset$, {2,3}, {1,2,4}, {3,4}, {1,3,4}, {2,3,4}, {1,2,4} {3}, {2}, {4}, {1,4}, {2,4}}

3. Using U / π two equivalences class defined on U and one subsets of U.

Example 3.6: Let U ={1,2,3,4,5}, K ={3,4},so U/$\pi_1 = \{\{1,2\},\{3\},\{4,5\}\}$ and U/π_2={{1,3},{2,4},{5}},$N_{1,2}$ −o={U, $\emptyset$, {3}, {4}, {4, 5}, {3,4}, {3,4,5}, {1,2,3,4}}

	U /π_1	**U /π_2**
$\mathbb{L}_{\pi B}(\mathbf{K})$	{3}	$\emptyset$
$\mathbb{U}_{\pi B}(\mathbf{K})$	{3,4,5}	{1,2,3,4}
$\text{Ƀ}_{\pi B}(\mathbf{K})$	{4,5}	{1,2,3,4}

By adding condition for the adducting of the third issues by multiple equivalence relation in the universe set, is a obtained the multi -Granular bi-nano topological spaces that is named as $\mathscr{M}$G- π.

Definition 3.7: Let U $\neq \emptyset$ be universe set and any two equivalence relations on U, where K $\subseteq$ U. so, $\mathbb{L}\pi_{1,2}(k) = \cup_{x \in U} \{[x]: \pi_1(x) \subseteq k \; or \; \pi_2(k) \subseteq \mathrm{K}\}$ is Multi-lower adducting of K for $\pi_{1,2}$. And $\mathbb{U}\pi_{1,2}(\mathrm{K}) = \cup_{x \in U} \{[x]: \pi_1(\mathrm{K}) \cap \mathrm{K} \neq \emptyset \; and \; \pi_2(\mathrm{K}) \cap k \neq \emptyset\}$ is Multi- upper adducting of K for $\pi_{1,2}$ when .$\text{Ƀ}\pi_{1,2}(\mathrm{K}) = \mathbb{U}\pi_{1,2}(\mathrm{K}) - \mathbb{L}\pi_{1,2}(\mathrm{K})$ is Multi- bounds area of K for $\pi_{1,2}$. From example (3.6), hence $\mathrm{T}_{\pi1,2}$(K)= {U, $\emptyset$,{3},{3,4}, {4}}.

Remark 3.8: It is noted these issues are independent in terms of fundamental formation of $\mathrm{T}_{\pi1,2}$. Now Grill concept is used with the bi-nano topology concept to credit the (U,$\mathrm{T}_{\pi1,2}$(K), G)is called Grill bi-nano topological spaces. Then $\phi_G^{\#}$: P(U) $\rightarrow$ P(U) is defined as follows:$\phi_G^{\#}$ (A)=$\phi_G^{\#}(A, T_{\pi1,2}) = \{\mathrm{y} \in \mathrm{U}: (\mathrm{A} \cap \mathrm{V}) \in \mathrm{G}, \forall \mathrm{V} \in \mathrm{T}_{\pi1,2}(\mathrm{K}), \mathrm{A} \in \mathrm{P(U)}\}$ is "the local function associated with grill G "and $T_{\pi1,2}$. Also, the map $\Psi^{\#}{}_G$: P(U) $\rightarrow$ P(U) such that $\Psi^{\#}{}_G$(A)= A $\cup$ $\phi_G^{\#}$ (A), $\forall$ A$\subseteq$ U.

Properties 3.9: Let (U,$T_{\pi1,2}(K)$) be a bi-nano topological spaces. "then the satisfies: If G is any Grill on U ,then $\phi_G^{\#}$ is an growing function in the logic that $Q \subseteq S$ indicates $\phi_G^{\#}(Q, T_{\pi1,2}) \subseteq \phi_G^{\#}(S, T_{\pi1,2})$ and any Grill G on U, if $Q \notin G$ then $\phi_G^{\#}(Q, T_{\pi1,2}) = \emptyset$.

- " $\phi_G^{\#}$ (A) $\cup \phi_G^{\#}(\mathcal{H}) \subseteq \phi_G^{\#}(A \cup \mathcal{H})$".
- "$\phi_G^{\#}\left(\phi_G^{\#}(A)\right) \subseteq Cl_{T_{\pi1,2}}(\phi_G^{\#}(A)) = \phi_G^{\#}(A)$".

Definition 3.10: A special topology $GT_{\pi1,2}$={V⊆ U, $\Psi^{\#}$ (U-V)=(U-V)},when for any A⊆ U that corresponds to a grill on $(U, T_{\pi1,2})$, if v ∈ $GT_{\pi1,2}$then v is $GN_{1,2}$ −open set and the complement is said G $N_{1,2}$ −closed set, so the family of all(GN $_{1,2}$-O) (GN $_{1,2}$-C) subset of $T_{\pi1,2}(K)$, will be denoted by GN $_{1,2}$-O(U) (GN $_{1,2}$-C(U)).

Theorem 3.11: *Let (U,$T_{\pi1,2}(K), G$)be $GT_{\pi1,2}(K)$, then the collection β($G, T_{\pi1,2}$) = {$Y - A$:$Y \in T_{\pi1,2}$, $A \notin G$} is Nano open basis for $GT_{\pi1,2}$.*

Proof: Since $w \in GT_{\pi1,2}(K)$ and $\in w$, then $Y \setminus w$ is $GN_{1,2}$-closed, $\Psi^{\#}{}_G(Y \setminus w)$=($Y \setminus w$) and $\phi_G^{\#}(Y \setminus w) \subseteq Y \setminus w$, so, $x \in \phi_G^{\#}(Y \setminus w)$ and there exist $Y \in T_{\pi1,2}(K) \ni (Y \setminus w) \cap Y \notin G$. Let $A = (Y \setminus w) \cap Y \rightarrow x \notin A \wedge A \notin G$. Thus $x \in Y \setminus A = Y - (Y \setminus w) \cap Y)$ =Y-($Y \setminus w$) ⊆Y, where $Y - A \in \beta(G, T_{\pi1,2})$.

Definition 3.12: GN $_{1,2}$ − internal of Q of $GT_{\pi1,2}(K)$is the combination of entirely GN $_{1,2} - O$ subset contained in Q, denoted by IGN $_{1,2}$(Q) that is IGN $_{1,2}$(Q)is the largest GN $_{,2} - O$ sub set of Q . Also, ."GN $_{1,2}$ − closure of Q of $GT_{\pi1,2}(K)$ is the intersection of all GN $_{1,2} - C$" sets containing by(cloGN $_{1,2}$(Q)) is the smallest GN $_{1,2}$-closet set containing Q.

Theorem 3.13: *A space (U, $T_{\pi1,2}(K)$, G) and $T_{\pi1,2}$ ⊆ β(G, $T_{\pi1,2}$) ⊆ $GT_{\pi1,2}$ & $T_{\pi1,2}$ = β(G,$T_{\pi1,2}$) = $GT_{\pi1,2}$ only if G = P (U) /{ ∅}.*

Remark 3.14: We calculate a Grill bi-nano topological space, through Discussing three independent cases with Grill open sets.

1. From (3.4),we get $T_{\pi1,2}(K)$={U, ∅,{1, 2}, {3,4}} withG_{u_2} = { U ⊆ U , u_2 ∈ U }, Thus β(G,$T_{\pi1,2}$) = U, ∅, {2}, {3}, {4}{2,4}, {2,3}, {1,2}, {1,2,3}, {1,2,4} {2,3,4}, {3,4} $GT_{\pi1,2}(K)$ = {U, ∅, {2}, {3}, {4}, {1,2}, {2,3}, {2,4}, {3,4}, {1,2,3}, {2,3,4} {1,2,4}}
2. From the example (3.5), we get $GT_{\pi1,2}(K)$ = {U, ∅, {1}, {2}, {3}, {4}, {1,3}, {1,2}, {1,4}, {2,4}, {2,3}, {3,4}, {1,2,4}, {1,3,4}, {2,3,4}, {1,2,3}}.
3. From (3.6) ,we get $T_{\pi1,2}(K)$={U, ∅,{3},{3,4},{4}}, with $G_{u_2}$and let β(G,$T_{\pi1,2}$)={U, ∅, {3}, {2}, {4}, {1,2}, {2,3}, {2,4}, {1,2,3} , {1,2,4}, {2,3,4}},

we $GT_{\pi1,2}(K) = \{U, \emptyset, \{3\}, \{2\}, \{4\}, \{1,2\}, \{2,3\}, \{2,4\}, \{3,4\}\{1,2,3\}, \{1,2,4\}, \{2,3,4\}\}$.

Proposition 3.15: Let (U, $T_{\pi1,2}$(K), G) be $GT_{\pi1,2}$. If ɱ $\in GT_{\pi1,2}$, then ɱ $\cap \phi_G^{\#}$ (A) = ɱ $\cap$ $\phi_G^{\#}$ (ɱ $\cap$ A), any A $\subseteq$U.

Theorems 3.16: *A subset A of $GT_{\pi1,2}(K)$, the statements are correct;*

1. *If $T_{\pi1,2} - \{ \emptyset\} \subseteq G$, then $\forall$ $N_{1,2} - o$ is $GN_{1,2}$-open.*
2. *If A $\subseteq$U is $GN_{1,2}$-open set and $GN_{1,2}$-closed, then A is $GN_{1,2}$-open.*
3. *If A $\subseteq$U, is $GN_{1,2}$-Closed then $\phi_G^{\#}$ ($N_{1,2}$ − Ɨ(A)) $\subseteq$ $N_{1,2}$-C($N_{1,2}$- Ɨ (A)) $\subseteq$ A.*

Proof:

1. Let A be a $N_{1,2}$-open set and A = $GN_{1,2}$- Ɨ(A) $\subseteq$ $GN_{1,2}$-Ɨ ($\emptyset_G^*$ (A)).
2. Since A is $GN_{1,2}$- C , A = $GN_{1,2}$ -C(A) = A $\cup$ $\phi_G^{\#}$ (A) $\Rightarrow$ $\phi_G^{\#}$(A) $\subseteq$ A and since A is $GN_{1,2}$-o , A $\subseteq$ $GN_{1,2}$- Ɨ($\phi_G^{\#}$(A))$\subseteq$ $GN_{1,2}$- Ɨ (A).Thus A = $GN_{1,2}$- Ɨ(A).
3. Assume A is $GN_{1,2}$-closed, U − A is $GN_{1,2}$-o, U − A $\subseteq$ $GN_{1,2}$-Ɨ ($\phi_G^{\#}$ (U − A)) $\subseteq\phi_G^{\#}$ (U − A). By proposition (3.15), hence $\phi_G^{\#}$ (U − A) =$GN_{1,2}$-C (U − A) =U − $GN_{1,2}$- Ɨ(A), now U − A $\subseteq$ $GN_{1,2}$- Ɨ ($\phi_G^{\#}$ (U −A)) $\subseteq$ $GN_{1,2}$- Ɨ (U −$GN_{1,2}$- Ɨ (A)) $\subseteq$U − $GN_{1,2}$-C($GN_{1,2}$- Ɨ(A)). Thus clo$GN_{1,2}$($GN_{1,2}$-int Ɨ (A)) $\subseteq$ A, also $\phi_G^{\#}$ ($GN_{1,2}$- Ɨ (A)) $\subseteq$$GN_{1,2}$ –C($GN_{1,2}$- Ɨ (A)) $\subseteq$$GN_{1,2}$-C ($GN_{1,2}$- Ɨ (A)) $\subseteq$ A.

Remark 3.17: Notice that $GN_{1,2}$- open sets need not necessarily from a bi-nano topology. As the example (3.5) and every Grill – Nano open set is Grill Bi-Nano open set. But the convers is not true. As the example (3.5).

4. Discussion and Conclusion

Grill bi-nano topological present via three different Nano topology together with a Grill. It is noted this case are independent of terms of structural creation of $T_{\pi1,2}$. Also, by adding some condition approximations of Nano topology in the third issues. Hence new $GT_{\pi1,2}(K)$ and called $\mathscr{M}$G- π spaces. And G-o set is $GN_{1,2}$-open set, in particular if G = P (U) / $\{\emptyset\}$ then $T_{\pi1,2}$ = $\beta(G,T_{\pi1,2}) = GT_{\pi1,2}$.

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