

Fractal image compression using digital cone metric space

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Abstract

This work presents Fixed Point Theorems (FPT) on Digital Images (DI) based on the Banach contraction concept developed. The study aims to develop the application of Banach Contraction Mapping Theory (BCMT) for DI, which was presented. The following finding generates a single Fixed Point (FP) for DI, exploring the core idea of DI and implementing the FPT in the context of Digital Image Compression (DIC). Fractal Image Compression (FIC) is a standard method for DIC. It is founded on a search for an object that is accurately in the image. However, a significant problem with DIC is its computational weight. The study proposed a method to reduce data transmission time by applying Image Compression (IC). It highlights the challenges of maximizing Image Quality (IQ) or reducing transmission time

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for definite IQ. The classic FIC method, which uses non-linear contractive mapping as a constant contractive factor, can attain this.

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1. Introduction

Digital network topology is a rapidly emerging domain that studies how an object's topological features communicate with 2D and 3D Digital Images (DI) features. This study was required to find optimal outcomes about Image Processing (IP) and the Fixed Point Theorem (FPT) in this field. FPT includes many mathematical values, standard topology, and statistical and functional studies. The Banach Contraction Mapping Theory (BCMT) in Metric Spaces (MS) is the basis of this theory. Several research fields, including mathematics, computer science, engineering, game theory, and IP, are helpful for FPT. The Banach FPT is the key test for determining several mathematical and practical problems. The BCMT was primarily invented [1]. Its pattern is simple, valuable, and used to solve problems in several mathematical analytic fields. In recent years, significant studies have been conducted applying the BCMT [2]. Some have referred to the 21st century as the information and technological age. These days, traditional spoken communication has been replaced by Image Processing (IP). By their very nature, some applications are essential to the visual section of their messaging. These uses include digital communications, broadcasting, teleconferencing, desktop multimedia publishing, video calling, and image transfer. The proof aim for using digital IP is that the user can helpfully change IP because of the DI depiction of the data. As a result, this work assumes that our current CPU will receive a few feature additions to enable successful IP. The FPT is vital to computer graphics and IP. Important FPT for real-time DI applications is this study's primary objective. [3] This research formed Lefschetz FPT for DI and was experiential in the FPT for DI. [4] advanced relative and basic Lefschetz FPT for DI [5].

2. Preliminaries

Proof 1: [6] Let Z as all integers and n as Natural Numbers (NN). For the set Z^n , EQU (1).

$$Z^n = \{(a_1, a_2, a_3, \dots, a_n) / a_i \in Z, 1 \leq i \leq n\} \quad (1)$$

$Z^n \rightarrow$ is the array of all n -dimensional Linear vector values

Proof 2: [7] Let $x = (x_1, x_2, x_3, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ be any 2-unique values in Z_n . A NN ' m ' should exist such that $1 \leq m < n$. If there exist at most m states i such that $|x_i - y_i| = 1$, and for all other signifies ' j '

such that $|x_i - y_i| \neq 1$, this study has $x_j = y_j$, then the 2-points x and y are k_m adjacent in Z_n

2- x and y in Z are 2-adjacent if $|x - y| = 1$ (Figure 1).

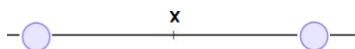


Figure 1
2-Point-Adjacent

If 2-points x and y in Z^2 are 8 neighboring and differ by precisely 1 coordinate; they are 4 adjacent (Figure 2).

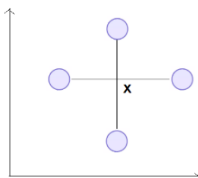


Figure 2
4-Point-Adjacent

If x and y in Z^2 are unique and differ by no more than 1 in each coordinate, then they are 8-adjacent (Figure 3).

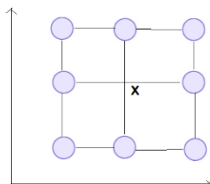


Figure 3
8-Point-Adjacent

Proof 3: An ordered pair (X, κ) representing a DI is any NN n for which X is a finite subset of Z_n and κ is a BCTM for the elements of X .

Proof 4: A point $p \in (X, d, \kappa)$ is called a FPT mapping $T: X \rightarrow X$ if $Tp = p$. Let (X, κ) be a DI. The DI as (X, κ) has the FPT. If every (κ, κ) constant map $T: (X, \kappa) \rightarrow (X, \kappa)$ has a FP.

Proof 2.5: Given a DCMS E and a sub-set P and is measured a shape of the circle if and only if it meets the following criteria.

- (i) P is locked, and $P \neq \emptyset$;

- (ii) $ax + by \in P \forall x, y \in P$, a, b are non-negative NN
- (iii) $P \cap (-P) = \emptyset$

Iff $y - x \in P$, define a partial ordering $\leq$ about the shape of circle P implicit a shape of circle $P \subset E$. When $y - x \in \text{inside of } P$, and $x \leq y$. If $K > 0$ in this degree for any $x, y \in E$, $0 \leq x \leq y$ specifies $\|x\| \leq K\|y\|$, then the shape of the circle P is said to be expected. If any falling order that is limited below and every ascending order bounded above converges, then the shape of circle P is said to be regular.

Proof 6: X is a $\emptyset$ and accept the mapping $d: X \times X \rightarrow E$ fill the following, EQU (2) to EQU (4).

$$(i) \quad 0 \leq d(x, y) \forall x, y \in X \text{ and } d(x, y) = 0 \text{ If and Only If } x = y \quad (2)$$

$$(ii) \quad d(x, y) = d(y, x) \text{ For All } x, y \in X \quad (3)$$

$$(iii) \quad d(x, y) = d(x, z) + d(z, y) \text{ For All } x, y, z \in X \quad (4)$$

A Digital Cone Metric Space (DCMS) with κ -adjacency is defined as the DI (X, κ) composed of the function ' d '. The notation for it is (X, d, κ) .

Proof 7: (X, d, κ) as DCMS, $x \in X$ and $\{x_n\} \rightarrow X$

- (i) $\{x_n\}$ joins to x when $\forall c \in E$ and $0 \ll c \rightarrow \text{NN } 'N'$ in this degree of $d(x_n, x) \ll c$ for all $n \geq N$
- (ii) $\{x_n\}$ is a Cauchy Order (CO) when $\forall c \in E$ with $0 \ll c$ there is an NN ' N ' in this degree of $d(x_n, x_m) \ll c$ for all $n, m \geq N$.

Proof 8: (X, d, κ) a complete DCMS if all CO is definite in X

Proof 9: (X, d) is a DCMS. This idea signifies $CB(X)$. The private of a $\emptyset$ closed limited $\subseteq$ of X . Let $H(., .)$ be the Hausdorff distance on $CB(X)$, EQU (5).

That is, for $A, B \in CB(X)$

$$H(A, B) = \text{Max}\{\text{Sup}_{a \in A} d(a, B), \text{Sup}_{b \in B} d(A, b)\} \quad (5)$$

where $d(a, B) = \inf\{d(a, b); b \in B\}$ is the range from the value of ' a ' to the $\subseteq B$. An element $x \in X$ is an FP of a multi-valued mapping $T: X \rightarrow 2^X$ if $x \in T(x)$. The BCMT attained this for DI, as stated below.

Proof 10: (X, d, κ) is complete DCMS. Let $T: (X, d, \kappa) \rightarrow (X, d, \kappa)$ is digital BCMT. T is unique FP; that is, there be present an $p \in X$ in this degree of $Tp = p$.

3. Result and Discussion

Proof 1: (X, d, κ) be a DCMS and the mapping, EQU (6).

$$T: (X, d, \kappa) \rightarrow (X, d, \kappa) \text{ and}$$

$$H(T\xi, T\zeta) \leq \theta_1[d(\xi, T\xi) + d(\zeta, T\zeta)] + b[d(\xi, T\zeta) + d(T\xi, \zeta)] \quad (6)$$

For All $\xi, \zeta \in X$ and $\theta_1 + \theta_2 < \frac{1}{2}$, $\theta_1, \theta_2 \in \left[0, \frac{1}{2}\right)$. Then T has an FP in (X, d, κ)

Definition: $\forall \xi_0 \in X$ and $n \geq 1$, $\xi_1 \in T\xi_0$ and $\xi_{n+1} \in T\xi_n$, EQU (7) to EQU (17).

$$d(\xi_{n+1}, \xi_n) \leq H(T\xi_n, T\xi_{n-1}) \quad (7)$$

$$\leq \theta_1[d(\xi_n, T\xi_n) + d(\xi_{n-1}, T\xi_{n-1})] + \theta_2[d(\xi_n, T\xi_{n-1}) + d(T\xi_n, \xi_{n-1})] \quad (8)$$

$$\leq \theta_1[d(\xi_n, \xi_{n+1}) + d(\xi_{n-1}, \xi_n)] + \theta_2[d(\xi_n, x_n) + d(\xi_{n+1}, \xi_{n-1})] \quad (9)$$

$$\leq \theta_1[d(x_n, x_{n+1}) + d(x_{n-1}, x_n)] + \theta_2[d(x_{n+1}, x_{n-1})] \quad (10)$$

$$\leq \theta_1[d(\xi_n, \xi_{n+1}) + d(\xi_{n-1}, \xi_n)] + \theta_2[d(\xi_{n+1}, \xi_n) + d(\xi_n, \xi_{n-1})] \quad (11)$$

$$\leq (\theta_1 + \theta_2)[d(\xi_n, \xi_{n+1}) + d(\xi_{n-1}, \xi_n)] \quad (12)$$

$$d(\xi_{n+1}, \xi_n) \leq Ad(\xi_{n-1}, \xi_n) \quad (13)$$

$$\text{where } A = \frac{\theta_1 + \theta_2}{1 - (\theta_1 + \theta_2)}$$

$$d(\xi_{n+1}, \xi_n) \leq A^n d(\xi_1, \xi_0) \quad (14)$$

For $n > m$,

$$d(\xi_n, \xi_m) \leq d(\xi_n, \xi_{n-1}) + d(\xi_{n-1}, \xi_{n-2}) + \cdots + d(\xi_{m+1}, \xi_m) \quad (15)$$

$$\leq [A^{n-1} + A^{n-2} + \cdots + A^m]d(\xi_1, \xi_0) \quad (16)$$

$$\leq \frac{A^m}{(1-A)} d(\xi_1, \xi_0) \quad (17)$$

Let $0 < c$ be implicit, and select a NN ' N_1 ' in this degree of $\frac{A^m}{(1-A)} d(\xi_1, \xi_0) < c$ for all $m \geq N_1$ this implies $d(\xi_n, \xi_m) < c$.

For $n > m$, $\{\xi_n\}$ is a CO in (X, d, κ) is a complete DCMS, there exists $\Delta_1 \in X$ in this degree of $\xi_n \rightarrow \Delta_1$.

Select a NN ' N_2 ' such that $d(\xi_n, P) < \frac{c(1-A)}{3}$, for all $n \geq N_2$, EQU (18) to EQU (27).

$\therefore$ for $n \geq N_2$ and have $d(\xi_n, \Delta_1) < \frac{c(1-k)}{3}$ where

$$k = \theta_1 + \theta_2$$

$$d(T\Delta_1, \Delta_1) \leq H(T\xi_n, T\Delta_1) + d(T\xi_n, \Delta_1) \quad (18)$$

$$\leq \theta_1[d(\xi_n, T\xi_n) + d(\Delta_1, T\Delta_1)] + \theta_2[d(\xi_n, T\Delta_1) + d(T\xi_n, \Delta_1)] + d(\xi_{n+1}, \Delta_1) \quad (19)$$

$$\leq \theta_1[d(\xi_n, \xi_{n+1}) + d(\Delta_1, T\Delta_1)] + \theta_2[d(\xi_n, T\Delta_1) + d(\xi_{n+1}, \Delta_1)] + d(\xi_{n+1}, \Delta_1) \quad (20)$$

$$\leq \theta_1[d(\xi_n, \xi_{n+1}) + d(\Delta_1, T\Delta_1)]$$

$$+ \theta_2[d(\xi_n, T\Delta_1) + d(\Delta_1, T\Delta_1) + d(\xi_{n+1}, \Delta_1)] + d(\xi_{n+1}, \Delta_1) \quad (21)$$

$$\leq \theta_1 d(\xi_n, \xi_{n+1}) + \theta_1 d(\Delta_1, T\Delta_1)$$

$$+ \theta_2 d(\xi_n, T\Delta_1) + \theta_2 d(\Delta_1, T\Delta_1) + \theta_2 d(\xi_{n+1}, \Delta_1) + d(\xi_{n+1}, \Delta_1) \quad (22)$$

$$\begin{aligned} &\leq \theta_1 d(\xi_n, \xi_{n+1}) + \theta_1 d(\Delta_1, T\Delta_1) + \theta_2 d(\xi_n, T\Delta_1) \\ &\quad + \theta_2 d(\Delta_1, T\Delta_1) + \theta_2 d(\xi_{n+1}, \Delta_1) + d(\xi_{n+1}, \Delta_1) \end{aligned} \quad (23)$$

$$(1-k)d(T\Delta_1, \Delta_1) \leq kd(\xi_n, T\Delta_1) + kd(\xi_{n+1}, \Delta_1) + d(\xi_{n+1}, \Delta_1) \quad (24)$$

$$\leq d(\xi_n, T\Delta_1) + d(\xi_{n+1}, \Delta_1) + d(\xi_{n+1}, \Delta_1) \quad (25)$$

$$d(T\Delta_1, \Delta_1) \leq \frac{[d(\xi_n, T\Delta_1) + d(\xi_{n+1}, \Delta_1) + d(\xi_{n+1}, \Delta_1)]}{(1-k)} \quad (26)$$

$$d(T\Delta_1, \Delta_1) \ll \frac{c}{3} + \frac{c}{3} + \frac{c}{3} \quad (27)$$

$$d(T\Delta_1, \Delta_1) \ll c$$

For all $n \geq N_2$, $d(T\Delta_1, \Delta_1) \ll \frac{c}{m}$ for all $m \geq 1$, find $\frac{c}{m} - d(T\Delta_1, \Delta_1) \in P$ and $m \rightarrow \infty$ accept that $\frac{c}{m} \rightarrow 0$ and P is closed $d(T\Delta_1, \Delta_1) \in P$ but $d(T\Delta_1, \Delta_1) \in P$

$\therefore d(T\Delta_1, \Delta_1) = 0$ and so $\Delta_1 \in T\Delta_1$

Proof 2: (X, d, κ) is a DCMS, and the mapping $T: (X, d, \kappa) \rightarrow (X, d, \kappa)$ and, EQU (28) to EQU (35).

$$H(T\xi, T\zeta) \leq r \text{Max}\{d(\xi, \zeta), d(\xi, T\xi), d(\zeta, T\zeta)\} \quad (28)$$

For All $\xi, \zeta \in X$ and $r \in [0, 1]$. Then T has an FP in (X, d, κ)

Definition: $\forall \xi_0 \in X$ and $n \geq 1$, $\xi_1 \in T\xi_0$ and $\xi_{n+1} \in T\xi_n$

$$d(\xi_{n+1}, \xi_n) \leq H(T\xi_n, T\xi_{n-1}) \leq r \text{Max}\{d(\xi_n, \xi_{n-1}), d(\xi_n, T\xi_n), d(\xi_{n-1}, T\xi_{n-1})\} \quad (29)$$

$$\leq r \text{Max}\{d(\xi_n, \xi_{n-1}), d(\xi_n, \xi_{n+1}), d(\xi_{n-1}, \xi_n)\} \quad (30)$$

$$\leq rd(\xi_n, \xi_{n-1}) \quad (31)$$

$$\leq r^n d(\xi_1, \xi_0) \quad (32)$$

For $n > m$,

$$d(\xi_n, \xi_m) \leq d(\xi_n, \xi_{n-1}) + d(\xi_{n-1}, \xi_{n-2}) + \dots + d(\xi_{m+1}, \xi_m) \quad (33)$$

$$\leq [r^{n-1} + r^{n-2} + \dots + r^m]d(\xi_1, \xi_0) \quad (34)$$

$$\leq \frac{r^m}{(1-r)} d(\xi_1, \xi_0) \quad (35)$$

$0 \ll c$ implicit, select a NN ' N_1 ' such that $\frac{r^m}{(1-r)} d(\xi_1, \xi_0) \ll c$ For All $m \geq N_1$ this implies $d(\xi_n, \xi_m) \ll c$, EQU(36) to EQU (39).

For $n > m$, $\{\xi_n\}$ is a CO in (X, d, κ) is a complete DCMS, $\exists \Delta_1 \in X$ such that $\xi_n \rightarrow \Delta_1$.

Select a NN ' N_2 ' such that $d(\xi_n, \Delta_1) \ll \frac{c}{3}$, for all $n \geq N_2$. $\therefore$ for $n \geq N_2$, and $d(\xi_n, \Delta_1) \ll \frac{c}{3}$

$$d(T\Delta_1, \Delta_1) \leq H(T\xi_n, T\Delta_1) + d(T\xi_n, \Delta_1) \quad (36)$$

$$\leq r \text{Max}\{d(\xi_n, \Delta_1), d(\xi_n, T\xi_n), d(\Delta_1, T\Delta_1)\} + d(\xi_{n+1}, \Delta_1) \quad (37)$$

$$\leq r \text{Max}\{d(\xi_n, \Delta_1), d(\xi_n, \xi_{n+1}), d(\Delta_1, T\Delta_1)\} + d(\xi_{n+1}, \Delta_1) \quad (38)$$

$$\leq r \text{Max}\{d(\xi_n, \Delta_1), d(\xi_n, \Delta_1) + d(\Delta_1, \xi_{n+1}), d(\Delta_1, T\Delta_1)\} + d(\xi_{n+1}, \Delta_1) \quad (39)$$

$$d(T\Delta_1, \Delta_1) \ll c$$

For all $n \geq N_2$, $d(T\Delta_1, \Delta_1) \ll \frac{c}{m} \exists m \geq 1$ this study get $\frac{c}{m} - d(T\Delta_1, \Delta_1) \in P$ and $m \rightarrow \infty$, found $\frac{c}{m} \rightarrow 0$ and P is closed $-d(T\Delta_1, \Delta_1) \in P$, $d(T\Delta_1, \Delta_1) \in P$
 $\therefore d(T\Delta_1, \Delta_1) = 0$ and so $\Delta_1 \in T\Delta_1$.

Proof 3.3: (X, d, κ) be a DCMS, and the mapping $T: (X, d, \kappa) \rightarrow (X, d, \kappa)$ and P a standard shape of a circle with fixed integer 'K'. The mapping [8-10] meets the following conditions: EQU (40) to EQU (46).

$$H(T\xi, T\zeta) \leq r \text{Max}\{d(\xi, \zeta), d(\xi, T\xi), d(\zeta, T\zeta), d(\xi, T\zeta), d(T\xi, \zeta)\} \quad (40)$$

For All $\xi, \zeta \in X$ and $r \in [0, 1)$. Then T has an FP in (X, d, κ)

Definition: $\forall \xi_0 \in X$ and $n \geq 1$, $\xi_1 \in T\xi_0$ and $\xi_{n+1} \in T\xi_n$

$$d(\xi_{n+1}, \xi_n) \leq H(T\xi_n, T\xi_{n-1}) \quad (41)$$

$$\leq r \text{Max}\{d(\xi_n, \xi_{n-1}), d(\xi_n, T\xi_n), d(\xi_{n-1}, T\xi_{n-1}), d(\xi_n, T\xi_{n-1}), d(T\xi_n, \xi_{n-1})\} \quad (42)$$

$$\leq r \text{Max}\{d(\xi_n, \xi_{n-1}), d(\xi_n, \xi_{n+1}), d(\xi_{n-1}, \xi_n), d(\xi_n, \xi_n), d(\xi_{n+1}, \xi_{n-1})\} \quad (43)$$

$$\leq r \text{Max}\{d(\xi_n, \xi_{n-1}), d(\xi_n, \xi_{n+1}), d(\xi_{n+1}, \xi_{n-1})\} \quad (44)$$

$$\leq r \text{Max}\{d(\xi_n, \xi_{n-1}), d(\xi_{n+1}, \xi_{n-1})\} \quad (45)$$

$$\leq r \text{Max}\{d(\xi_n, \xi_{n-1}), d(\xi_{n+1}, \xi_{n-1})\} \quad (46)$$

Case (i) If $d(\xi_{n+1}, \xi_n) \leq rd(\xi_n, \xi_{n-1})$ Then find EQU (47) to EQU (50)

$$d(\xi_{n+1}, \xi_n) \leq r^n d(\xi_1, \xi_0) \text{ for } n > m \quad (47)$$

$$d(\xi_n, \xi_m) \leq d(\xi_n, \xi_{n-1}) + d(\xi_{n-1}, \xi_{n-2}) + \dots + d(\xi_{m+1}, \xi_m) \quad (48)$$

$$\leq [r^{n-1} + r^{n-2} + \dots + r^m] d(\xi_1, \xi_0) \quad (49)$$

$$\leq \frac{r^m}{(1-r)} d(\xi_1, \xi_0) \quad (50)$$

And find $\|d(\xi_n, \xi_m)\| \leq K \frac{r^m}{(1-r)} \|d(\xi_1, \xi_0)\|$. $d(\xi_n, \xi_m) \rightarrow 0$ as $n, m \rightarrow \infty$.

$\therefore \{\xi_n\}$ is a CO. By the integrality of X , there is $\Delta_1 \in X$, EQU (51) to EQU (54).

Such that $\xi_n \rightarrow \Delta_1$ as $n \rightarrow \infty$

$$d(T\Delta_1, \Delta_1) \leq H(T\xi_n, T\Delta_1) + d(T\xi_n, \Delta_1) \quad (51)$$

$$\leq r \text{Max}\{d(\xi_n, \Delta_1), d(\xi_n, T\xi_n), d(\Delta_1, T\Delta_1), d(\xi_n, T\Delta_1), d(T\xi_n, \Delta_1)\} + d(\xi_{n+1}, \Delta_1) \quad (52)$$

$$\leq r \text{Max}\{d(\xi_n, \Delta_1), d(\xi_n, \xi_{n+1}), d(\Delta_1, T\Delta_1), d(\xi_n, T\Delta_1), d(\xi_{n+1}, \Delta_1)\} + d(\xi_{n+1}, \Delta_1) \quad (53)$$

$$\leq rd(\Delta_1, T\Delta_1) \quad (54)$$

$$d(T\Delta_1, \Delta_1) = 0. \therefore \Delta_1 \in T\Delta_1$$

Case (ii) $d(\xi_{n+1}, \xi_n) \leq rd(\xi_{n+1}, \xi_{n-1})$ then find EQU (55) to EQU (60)

$$d(\xi_{n+1}, \xi_n) \leq r[d(\xi_{n+1}, \xi_n) + d(\xi_n, \xi_{n-1})] \quad (55)$$

$$\leq \frac{r}{1-r} [d(\xi_n, \xi_{n-1})] \quad (56)$$

$$\leq B[d(\xi_n, \xi_{n-1})] \quad (57)$$

where $B = \frac{r}{1-r} < 1$, For $n > m$

$$d(\xi_n, \xi_m) \leq d(\xi_n, \xi_{n-1}) + d(\xi_{n-1}, \xi_{n-2}) + \dots + d(\xi_{m+1}, \xi_m) \quad (58)$$

$$\leq [B^{n-1} + B^{n-2} + \dots + B^m]d(\xi_1, \xi_0) \quad (59)$$

$$\leq \frac{B^m}{(1-B)} d(\xi_1, \xi_0) \quad (60)$$

This proof finds $\|d(\xi_n, \xi_m)\| \leq K \frac{h^m}{(1-h)} \|d(\xi_1, \xi_0)\|$. $d(\xi_n, \xi_m) \rightarrow 0$ as $n, m \rightarrow \infty$.

$\{\xi_n\}$ is a CO. By the integrality of X , there is $\Delta_1 \in X$. such that $\xi_n \rightarrow \Delta_1$ as $n \rightarrow \infty$, EQU (61) to EQU (68).

$$d(T\Delta_1, \Delta_1) \leq H(T\xi_n, T\Delta_1) + d(T\xi_n, \Delta_1) \quad (61)$$

$$\leq r \text{Max}\{d(\xi_n, \Delta_1), d(\xi_n, T\xi_n), d(\Delta_1, T\Delta_1), d(\xi_n, T\Delta_1), d(T\xi_n, \Delta_1)\} + d(\xi_{n+1}, \Delta_1) \quad (62)$$

$$\leq r \text{Max}\{d(\xi_n, \Delta_1), d(\xi_n, \xi_{n+1}), d(\Delta_1, T\Delta_1), d(\xi_n, T\Delta_1), d(\xi_{n+1}, \Delta_1)\} + d(\xi_{n+1}, \Delta_1) \quad (63)$$

$$\leq rd(\Delta_1, T\Delta_1) \quad (64)$$

$$d(T\Delta_1, \Delta_1) = 0. \therefore \Delta_1 \in T\Delta_1$$

$$d(\Delta_1, \Delta_2) = H(T\Delta_1, T\Delta_2) \quad (65)$$

$$\leq r \text{Max}\{d(\xi, \zeta), d(\Delta_1, T\Delta_1), d(\Delta_2, T\Delta_2), d(\Delta_1, T\Delta_2), d(T\Delta_1, \Delta_2)\} \quad (66)$$

$$\leq r \text{Max}\{d(\Delta_1, \Delta_2), d(\Delta_1, \Delta_1), d(\Delta_2, \Delta_2), d(\Delta_1, \Delta_2), d(\Delta_1, \Delta_2)\} \quad (67)$$

$$\leq r[d(\Delta_1, \Delta_2)] \quad (68)$$

This is a paradox, and $\therefore T$ has a unique FP in X

4. Application: A DIP Using FPT

Fractal geometry is of interest to a Max. Mandelbrot was the first to describe fractals. A fractal is a geometric shape with minor versions of each component that formulate the total. Figure 4 shows a few examples of fractals.

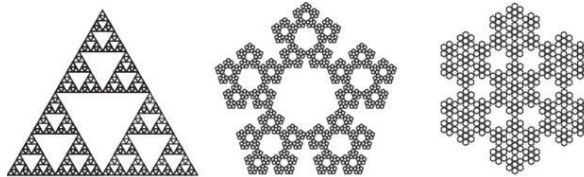


Figure 4
Examples of fractal

Each segment that formulates the original image is nearly equal to a minor replica of the actual image. By then, the total of all the portions accurately derived close to the actual image [11-15]. Thus, contractive mappings and the FPT are the basis of FIC.

The summary of the operation of FIC and the role played by the FPT:

- (a) **Partitioning the Image:** Dividing the image into smaller sections is the initial point in the FIC. Typically, these blocks have a square form and don't overlap.
- (b) **Finding Similarity:** The algorithm searches the image for blocks equal to each block it encounters. Because of self-similarity, this similarity is based on a computation of matches rather than actual matches.
- (c) **Contractive Transformations:** After identifying related blocks, the algorithm uses a contractive transformation to translate one block into another. Usually, these changes are affine (rotation, scaling, translation).
- (d) **FPT:** When determining the model transformation limits, the FPT is relevant. The theorem claims that for certain types of functions (contractive mappings), a unique FP exists, which is a point that remains constant after the transformation. This FP signifies the finest calculation of one block by another in the context of FIC.
- (e) **Iterative Process:** FIC iteratively improves the functional transformations of each block to reduce the error between the original block and computation. The FPT is frequently used in this iterative method to converge toward the optimal transformations.
- (f) **Encoding:** The algorithm encodes the modifications for each block and the data required to rebuild the image after recognition.
- (g) **Decoding:** The encoded transformations are completed repeatedly to recreate the image from the compressed data to decompress the image.

5. Conclusion

Digital Cone Metric Spaces (DCMS) are used in Fractal Image Compression (FIC), an advanced method that expands on traditional FIC. DCMS provides a more adaptable model for defining transformations and distances, making it possible to represent complicated structures and self-similar patterns in images more effectively. In general, the Fixed-Point Theorem (FTP) aids in identifying the best transformations to reduce compression error. FIC achieves large compression ratios with positive IQ preservation by iteratively fine-tuning these modifications. This work aims to present some FPT in DCMS with a digital implementation. This paper applied BCMT to Digital Images (DI) using a $f(x)$. This method attempted to use this proposed FPT to FIC to improve the apparent quality of the transformed DI. Using the image's object is exactly FIC, which is a method for reducing the size of DI. Its basis is the idea of fractals, which are geometric forms with self-similarity at numerous scales. In particular, the FPT helps determine the BCMT that results in self-similar transformations, which is vital for FIC methods.

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