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Variational iteration method for solving generalized complex Ginzburg-Landau equation

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Abstract

This study investigates the application of the variational iterative method (VIM) to solve the nonlinear complex Ginzburg-Landau problem. This study investigates the use of VIM to solve the complex nonlinear Ginzburg-Landau problem. Using MATLAB, a powerful scientific computing program, the study develops a algorithm based on VIM principles to

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solve probable solutions that recur to convergence and tests this approach if in four cases it shows high accuracy and efficiency for solving complex equations.

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1. Introduction

Mathematics is a fundamental tool across scientific disciplines, aiding analysis and application [1-5]. Scientists use mathematical principles in physics and chemistry for data analysis, predictions, model development [6,7]. Differential equations play a crucial role in modeling physical phenomena like object movement, fluid flow, heat transfer, chemical reactions [8,9]. The complex Ginzburg-Landau equations, specifically the cubic and cubic-quintic variants, are extensively examined in physics, chemistry, and biology, enhancing the comprehension of superconductivity, superfluidity, nonlinear optics, and Bose-Einstein condensation. The cubic-quintic equation aims to determine two-dimensional solitary spiral solutions in active media defined by an excitation threshold. Research has concentrated on the development of soliton solutions in one dimension [10]. The quintic equation consistently represents the field dynamics of a passively mode-locked laser. The generalized quintic complex Ginzburg-Landau equation has efficiently modeled non-equilibrium processes, such as ultrashort pulse propagation in spectrally filtered optical transmission lines and erbium-doped fiber amplifiers, addressing issues overlooked by other models [11]. This work examines the generalized complex Ginzburg-Landau equation (GCGLE), expressed as

$$\frac{\partial\varphi(\chi,t)}{\partial t} = (1+\alpha i)\Delta\varphi(\chi,t) + \rho\varphi(\chi,t) - (1+\beta i)|\varphi(\chi,t)|^{2\mu}\varphi(\chi,t) - (1+\gamma i)|\varphi(\chi,t)|^{4\mu}\varphi(\chi,t), \quad (1)$$

and $\varphi(\chi,0) = h(\chi), i = \sqrt{-1}, \chi \in R^d, d = 1,2,3, t > 0, \mu \geq 1,$

in where α, ρ, β, and γ are undefined real constants, $\varphi(\chi,t)$ is an unknown complex function, t is a non-negative real number, χ is a real number, and Δ is the Laplacian operator [10,11].

Refer to [11,12] for an in-depth exploration of the (GCGLE). Researchers, including Isah and Yokuş [13], utilized a φ^6-model expansion for soliton solutions with dual power-law non-linearity. Zafar, Asim, et al. [14] analyzed optical solitons in nonlinear optics, while Arnous et al. [15] suggested a cubic-quartic optical soliton answer extraction method. Isah and Yokuş [16] investigated the complex Ginzburg-Landau equation with Kerr laws. Our study employed the VIM to obtain accurate solutions for Equation (1) in four examples, showcasing its efficacy [17,18].

2. Method Analysis

This section presents an introduction to the analysis of the VIM. The fundamental principle of the VIM involves approximating the solution of a differential equation by utilizing an iteration formula, which manifests as a corrective function. Through the application of variational theory, the Lagrange multiplier can be determined. To elucidate the fundamental principle of this technique, Equation (1) is posited as follows:

$$\frac{\partial\varphi(\chi,t)}{\partial t} = \alpha_0(1+\alpha i)\Delta\varphi(\chi,t) + \rho\varphi(\chi,t) - \beta_0(1+\beta i)\varphi(\chi,t)^{2\mu+1} - (1+\gamma i)\varphi(\chi,t)^{4\mu+1}, \quad (2)$$

The corrective function for Equation (2) is invariably defined iteratively as:

$$\varphi_{n+1} = \varphi_n + \int_0^t \lambda(t,\tau)\left(\frac{\partial\varphi}{\partial t} - \alpha_0(1+\alpha i)\Delta\varphi - \rho\varphi + \beta_0(1+\beta i)\varphi^{2\mu+1} + (1+\gamma i)\varphi^{4\mu+1}\right)d\tau, \quad (3)$$

where $\lambda(t,\tau) = -1$ is a Lagrange multiplier [18-20], iterations are carried out using a simple initial function φ_0 until convergence is achieved at a fixed point [21]. At this stage, $\varphi_{n+1}(X,t)$ may be regarded as an approximate solution to Equation (2). The procedure can be expressed in the form of an algorithm and implemented as follows.

2.1 *Algorithm*:

Let n represent the index of iteration.

Step 1: Set an appropriate value for the tolerance (δ) for numerical purposes.

Step 2: Choose an appropriate φ_0, set n=0.

Step 3: Use the calculated values of φ_n to compute φ_{n+1} from equation (3).

Step 4: If $Max\ |\varphi_n - \varphi_{n+1}| < \delta$ stop, otherwise continue.

Step 5: Set $n = n + 1$, define $\varphi_n = \varphi_{n+1}$ and return to Step 3.

3. Implemenration

In this section, the VIM is utilized to solve three instances of generalized complex Ginzburg-Landau equations. The programs have been provided by MATLAB package. A few first approximations for $\varphi(x,t)$ for each example are calculated and presented as below:

Example 3.1: The first example in one-dimension (1D), we have considered the following parameters:

$\alpha_0 = \beta_0 = \beta = 1, \alpha = 3, , \rho = 3 = 2, \gamma = -6, \mu = 2$ in equation (2), yield

$$\frac{\partial\varphi}{\partial t} = (1+3i)\frac{\partial^2\varphi}{\partial x^2} + 3\varphi - (1+i)\varphi^5 - (1-6i)\varphi^9,$$

with initial condition: $\varphi(x,0) = \exp(-ix)$, We derive the subsequent consecutive approximations:

$$\varphi_1 = exp(-xi) - t(-exp(-xi)(2-3i) + exp(-5xi)(1+i) + exp(-9xi)(1+6i))$$

$\varphi_2 = \varphi_1 - t^2(\exp(-xi)(5/2+6i) + \exp(-5xi)(53/2-97i/2) + \exp(-9xi)(690-711i/2) - \exp(-4xi)(-\exp(-xi)(2-3i) + \exp(-5xi)(1+i) + \exp(-9xi)(1+6i))(5/2+5i/2) - \exp(-8xi)(-\exp(-xi)(2-3i) + \exp(-5xi)(1+i) + \exp(-9xi)(1+6i))(9/2+27i)) + t^6(\exp(-4xi)(-exp(-xi)(2-3i) + exp(-5xi)(1+i)+\ldots$ See Figure 1.

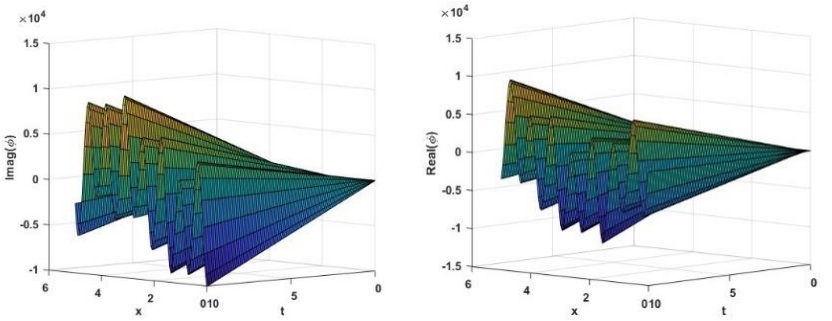


Figure 1
Imaginary and real parts of the estimation answer for Example 3.1.

Example 3.2: This example is in 2D [**10**], with which we have considered the following parameters: $\alpha_0 = 0.1$, $\alpha = 1$, $\beta_0 = 2.5$, $\rho = -0.5$ $\beta = 0.4$, $\gamma = -0.1$, $\mu = 1$, in equation (2), yield

$$\frac{\partial \varphi}{\partial t} = 0.5(1+i)\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right) - 0.5\varphi + (2.5+i)\varphi^3 - (1-0.1i)\varphi^5,$$

with initial condition:$\varphi(x,0) = 1.6\exp\left(-\frac{x^2+y^2}{25}\right)(x+iy)/\sqrt{x^2+y^2}$. We get the following sequential approximations:

$\varphi_1 = t((\exp(-x^2/5 - y^2/5)(x+yi)^5(-32768/3125 + 16384i/15625))/(x^2+y^2)^{\frac{5}{2}} + (\exp(-(3x^2)/25 - (3y^2)/25)(x+yi)^3(256/25 + 512i/125))/(x^2+y^2)^{\frac{3}{2}} + (xexp(-x^2/25 - y^2/25)(-16/125 - 16i/125))/(x^2+y^2)^{\frac{1}{2}} + (xexp(-x^2/25 - y^2/25)(-8/5 - 8i/5))/(x^2 + y^2)\text{^}(3/2) + (yexp(-x^2/25 - y^2/25)(16/125 - 16i/125))/(x^2+y^2)^{\frac{1}{2}} + (yexp(-x^2/25 - y^2/25)(8/5 - 8i/5))/(x^2+y^2)^{\frac{3}{2}} + (\exp(-x^2/25 - y^2/25)(x+yi)(-\frac{116}{125} - 16i/125)) + \cdots$

$\varphi_2 = \varphi_1 + -t^4((exp(-(3x^2)/5 - (3y^2)/5)(x+yi)^6(xexp((4x^2)/25 + (4y^2)/25)(3125 + 3125i) + yexp((4x^2)/25 + (4y^2)/25)(3125 - 3125i) - x^3exp((2x^2)/25 + (2y^2)/25)(40000 + 16000i) + x^3exp(4x^2/25 + (4y^2)/25)(3625 + 500i) - x^5exp((4x^2)/25 + (4y^2)/25)(20 + 20i) + y^3exp((2x^2)/25 + (2y^2)/25)(16000 - 40000i) + y^3exp((4x^2)/25 + (4y^2)/25)(500 - 3625i) - y^5exp((4x^2)/25 + (4y^2)/25)(20 - 20i) + \cdots$. See Figure 2.

Example 3.3: The last example is in 3D [**10**], here we consider $\alpha_0 = 0.5, \alpha = 1, \rho = -0.4, \beta_0 = 2.3, \beta = 10/23, \gamma = -0.1, \mu = 1$, in equation (2), yield $\frac{\partial \varphi}{\partial t} = 0.5(1+i)\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}\right) - 0.4\varphi + (2.3+i)\varphi^3 - (1-0.1i)\varphi^5$, with initial condition:$\varphi(x,0) = \exp(-0.5(x^2+y^2+z^2))$, we obtain the following successive approximations:

$\varphi_1 = exp(-x^2/2 - y^2/2 - z^2/2) + t(-exp(-x^2/2 - y^2/2 - z^2/2)(19/10 + 3i/2) + exp(-(3x^2)/2 - (3y^2)/2 - (3z^2)/2)(23/10 + i) - exp(-(5x^2)/2 - (5y^2)/2 - (5z^2)/2)(1 - i/10) + x^2exp(-x^2/2 - y^2/2 - z^2/2)(1/2 + i/2) + y^2exp(-x^2/2 - y^2/2 - z^2/2)(1/2 + i/2) + z^2exp(-x^2/2 - y^2/2 - z^2/2)(1/2 + i/2))$.

$\varphi_2 = \varphi_1 + t^2(-(exp(-(5x^2)/2 - (5y^2)/2 - (5z^2)/2)(exp(x^2+y^2+z^2)(23+10i) - exp(2(x^2+y^2+z^2))(19+15i) + x^2exp(2(x^2+y^2+z^2))(5+5i) + y^2exp(2(x^2+y^2+z^2))(5+5i) + z^2exp(2(x^2+y^2+z^2))(5+5i) - 10 + i))/50 - exp(-(9x^2)/2 - (9y^2)/2 - (9z^2)/2)(exp(x^2+y^2+z^2)(6+77i/40) - exp(2(x^2+y^2+z^2))(41/8 + 131i/40) + x^2exp(2(x^2+y^2+z^2))(11/8 + 9i/8) + y^2exp(2(x^2+y^2+z^2)) + \cdots$. See Figure 3.

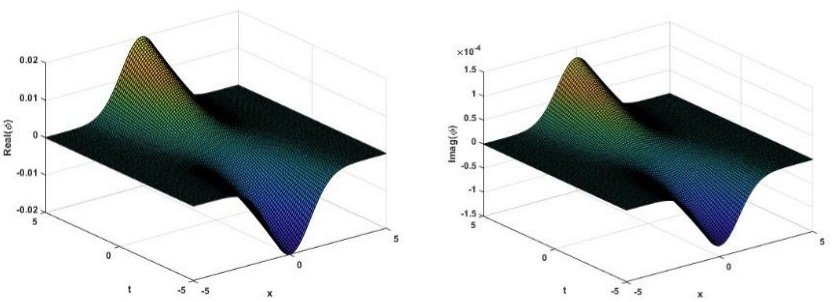


Figure 2
Imaginary and real parts of the estimation answer for Example 3.2 at $y = 1$.

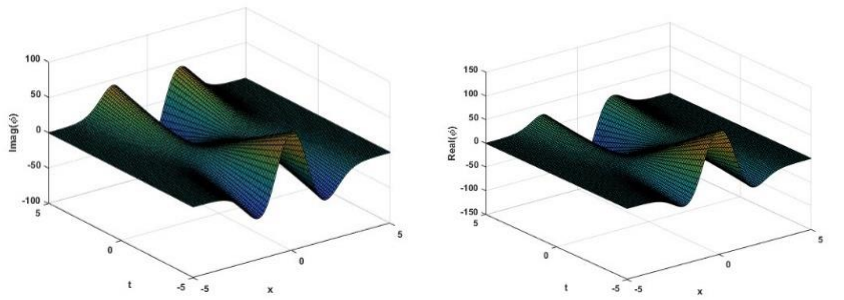


Figure 3
Imaginary and real parts of the estimation answer for Example 3.3 at $y = 1, z = 1$.

Discussion and Conclusion

This study explores the effectiveness of the Variational Iteration Method (VIM) in solving GCGLE. Through experiments in three dimensions and four examples, we confirm VIM's reliability for nonlinear differential equations. The results demonstrate the precision of VIM in providing approximate solutions. Implemented in MATLAB, the VIM algorithm's outcomes are presented. This research serves as a valuable reference for scholars in nonlinear differential equations, particularly the complex Ginzburg-Landau equation, offering a dependable method for further exploration in this domain.

References

[1] M. R. Farahani, S. Jafari, S. A. Mohiuddine, and M. Cancan, "Intuitionistic fuzzy stability of generalized additive set-valued functional equation via fixed point method," *Mathematical Statistician and Engineering Applications*, vol. 71, no. 3s3, pp. 142–154 (2022). [Online]. Available: https://doi.org/10.17762/msea.v71i3s3.355.

[2] W. Gao and M. R. Farahani, "The hyper-Zagreb index for an infinite family of nanostar dendrimer," *Journal of Discrete Mathematics, Sciences and Cryptography*, vol. 20, no. 2, pp. 515–523 (2017). [Online]. Available: https://doi.org/10.1080/09720529.2016.1220088.

[3] W. Gao and M. R. Farahani, "The Zagreb topological indices for a type of Benzenoid systems jagged-rectangle," *Journal of Interdisciplinary Mathematics*, vol. 20, no. 5, pp. 1341–1348 (2017). [Online]. Available: https://doi.org/10.1080/09720502.2016.1232037.

[4] W. Gao, L. Shi, and M. R. Farahani, "Szeged related indices of TUAC6[p, q]," *Journal of Discrete Mathematics, Sciences and Cryptog-*

raphy, vol. 20, no. 2, pp. 553–563 (2017). [Online]. Available: https://doi.org/10.1080/09720529.2016.1228312.

[5] X. Zhang, A. Razzaq, K. Ali, S. T. R. Rizvi, and M. R. Farahani, "A new approach to find eccentric indices of some graphs," *Journal of Informational and Optimization Sciences,* vol. 41, no. 4, pp. 865–877 (2020). [Online]. Available: https://doi.org/10.1080/02522667.2020.1744304.

[6] A. Al-Haboobi and A. H. Al-Muslimawi, "Novel algorithm for compressible Newtonian axisymmetric thermal flow," *International Journal of Modern Physics C,* vol. 2450025, pp. 17 (2024). [Online]. Available: https://doi.org/10.1142/S0129183124500256.

[7] A. Al-Haboobi, G. A. Al-Juaifri, and A. H. Al-Muslimawi, "Numerical study of Newtonian laminar flow around circular and square cylinders," *Results in Control and Optimization*, vol. 100328 (2023). [Online]. Available: https://doi.org/10.1016/j.rico.2023.100328.

[8] Mechee, M. S., Al-Shaher, O. I., & Al-Juaifri, G. A. Haar wavelet technique for solving fractional differential equations with an application. In AIP Conference Proceedings (Vol. 2086, No. 1). AIP Publishing (2019, April).

[9] A. K. Joohy, G. A. Al-Juaifri, and M. S. Mechee, "An investigation of solving third-order nonlinear ordinary differential equation in complex domain by generalizing Prelle-Singer method," *International Journal of Differential Equations*, vol. 2020, pp. 1–9 (2020). [Online]. Available: https://doi.org/10.1155/2020/5276024.

[10] S. Wang and L. Z., "An efficient split-step compact finite difference method for cubic-quintic complex Ginzburg-Landau equations," C*omputer Physics Communications,* vol. 184, no. 6, pp. 1511–1521 (2013). [Online]. Available: https://doi.org/10.1016/j.cpc.2013.01.019.

[11] J. M. Soto-Crespo, N. N. Akhmediev, and V. V. Afanasjev, "Stability of the pulselike solutions of the quintic complex Ginzburg-Landau equation," *Journal of the Optical Society of America B*, vol. 13, no. 7, pp. 1439–1449 (1996). [Online]. Available: https://doi.org/10.1364/JOSAB.13.001439.

[12] W. Liu, X. Zhang, Y. Li, L. Yan, and A. Q. Baig, "Analytic solutions for the generalized complex Ginzburg-Landau equation in fiber lasers," *Nonlinear Dynamics*, vol. 89, pp. 2933–2939 (2017). [Online]. Available: https://doi.org/10.1007/s11071-017-3636-5.

[13] M. A. Isah and A. Yokuş, "Optical solitons of the complex Ginzburg-Landau equation having dual power nonlinear form using φ^6-

model expansion approach," *Mathematical Modelling and Numerical Simulation with Applications*, vol. 3, no. 3, pp. 188–215 (2023). [Online]. Available: https://doi.org/10.53391/mmnsa.1337648.

[14] A. Zafar, M. M. R. Laghari, A. T. M. Nazari, M. H. Shakir, and S. Zubair, "Analytical study of complex Ginzburg-Landau equation arising in nonlinear optics," *Journal of Nonlinear Optical Physics & Materials*, vol. 32, no. 01, pp. 2350010 (2023). [Online]. Available: https://doi.org/10.1142/S0218863523500108.

[15] A. H. Arnous, N. S. Malik, M. S. Anwar, and Z. Y. Zhang, "Cubic-quartic optical solitons of the complex Ginzburg-Landau equation: A novel approach," *Nonlinear Dynamics*, vol. 2023, pp. 1–16. [Online]. Available: https://doi.org/10.1007/s11071-023-08854-4.

[16] M. A. Isah and A. Yokuş, "The investigation of several soliton solutions to the complex Ginzburg-Landau model with Kerr law nonlinearity," *Mathematical Modelling and Numerical Simulation with Applications*, vol. 2, no. 3, pp. 147–163 (2022). [Online]. Available: https://doi.org/10.53391/mmnsa.2022.012.

[17] J. H. He, "Variational iteration method—a kind of non-linear analytical technique: Some examples," *International Journal of Nonlinear Mechanics*, vol. 34, no. 4, pp. 699–708 (1999). [Online]. Available: https://doi.org/10.1016/S0020-7462(98)00048-1.

[18] G. A. Al-Juaifri, A. Al-Haboobi, and J. M. Al-Ghabban, "Programming variational iteration method via Wolfram Mathematica for solving multi-order differential equations," *International Journal of Applied Mathematics*, vol. 32, no. 5, pp. 785 (2019). [Online]. Available: https://doi.org/10.12732/ijam.v32i5.6.

[19] M. A. Abdou and A. A. Soliman, "New applications of variational iteration method," Physica D: Nonlinear Phenomena, vol. 211, no. 1-2, pp. 1–8 (2005). [Online]. Available: https://doi.org/10.1016/j.physd.2005.08.002.

[20] X. Zhang, F. A. Shah, Y. Li, L. Yan, A. Q. Baig, and M. R. Farahani, "A family of fifth-order convergent methods for solving nonlinear equations using variational iteration technique," *Journal of Informational and Optimization Sciences*, vol. 39, no. 3, pp. 673–694 (2018). [Online]. Available: https://doi.org/10.1080/02522667.2018.1443628.

[21] A.M. Darweesh, R.H. Hasan, and S.M. Kadham, "Results of Fourth-Order Differential Superordination and Subordination for Univalent Functions Defined by Integral Operator," *Mathematical Modelling and Engineering Problems*, vol. 11, no. 2, pp. 385–392 (Feb. 2024), doi: https://doi.org/10.18280/mmep.110210.

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