Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 28 (2025), No. 1, pp. 245–251
DOI : 10.47974/JIM-1854

Solving single variable functions using a new secant method

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Abstract

The quadratically convergent Newton method is a fundamental and significant approach for solving one-variable functions. In order to solve a single minimization issue, we deduce a novel secant type approach in this study that is based on estimating the second derivative information. The convergence of the novel secant type iterative approach is of order

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two. The provided application examples show that the suggested technique outperforms the Newton method in terms of numerical properties.

Subject Classification: *90C30, 65K05, 49M37.*

Keywords: *Newton's methods, Secant method, Test functions.*

1. Introduction

Single variable minimization may appear academic because most practical optimization problems require many variables [12, 13]. The classical technique determines the υ values at the inflection points of a function μ(υ) of a single variable as the answers to:

$$\mu'(\upsilon) = 0. \tag{1}$$

For details see [1]. In [2], the most effective way to apply these kinds of techniques extensively is Newton's standard method, which is as follows:

$$\upsilon_{k+1} = \upsilon_k - \frac{\mu'(\upsilon_k)}{\mu''(\upsilon_k)}. \tag{2}$$

A variation of the Newton-Raphson Method is the Secant method [3, 4]. It can occasionally be rather laborious and time-consuming to evaluate the function's second derivative. Using an approximation of the derivative, the following estimate is provided in order to get around this:

$$\upsilon_{k+1} = \upsilon_k - \frac{\mu'(\upsilon_k)}{\mu'(\upsilon_k)-\mu''(\upsilon_{k-1})}(\upsilon_k - \upsilon_{k-1}). \tag{3}$$

More efficient iterative methods than Newton's have been developed by certain writers [2, 3, 5-10].

In an attempt to approach the secant update, the new update deriving makes use of the second derivative $f^{//}(c_k)$information.

2. New Secant Method

As is well known, Taylor's theorem is one of the ideas that plays a significant part in iterative approaches. Consider the following Three-order Taylor-expansion around point υ_k [11]:

$$\mu(\upsilon) \cong \mu(\upsilon_k) + (\upsilon - \upsilon_k)\mu'(\upsilon_k) + \frac{1}{2}(\upsilon - \upsilon_k)^2\mu''(\upsilon_k) + \frac{1}{6}(\upsilon - \upsilon_k)^3\mu'''(\upsilon_k) \tag{4}$$

Finding the minimum Point, where:

$$\mu'(\upsilon) \cong \mu''(\upsilon_k) + (\upsilon - \upsilon_k)\mu''(\upsilon_k) + \frac{1}{2}(\upsilon - \upsilon_k)^2\mu'''(\upsilon_k) = 0 \tag{5}$$

Leads to:

$$\mu'(\upsilon_k) + (\upsilon - \upsilon_k)\mu''(\upsilon_k) = -\frac{1}{2}(\upsilon - \upsilon_k)^2\mu'''(\upsilon_k) \tag{6}$$

$$(\upsilon - \upsilon_k)^2\mu'''(\upsilon_k) = -2(\mu'(\upsilon_k) + (\upsilon - \upsilon_k)\mu''(\upsilon_k))$$

As we plug Eq. (6) into Eq. (4), we obtain:

$$(v - v_k)^2 \mu''(v_k) = 6(\mu(v) - \mu(v_k)) - 4((v - v_k)\mu'(v_k)) \tag{7}$$

This results in:

$$\mu''(v_k) = \frac{6(\mu(v)-\mu(v_k))-4(\mu'(v_k)(v-v_k))}{(v-v_k)^2} \tag{8}$$

Substituting v with v_{k-1}in (7), we get:

$$\mu''(v_k) = \frac{6(\mu(v_{k-1})-\mu(v_k))-4((v_{k-1}-v_k)\mu'(v_k))}{(v-v_k)^2} \tag{9}$$

Doing some algebraic operations on 6 results:

$$\mu''(v_k) = \frac{\frac{6(\mu(v_{k-1})-\mu(v_k))}{(v_{k-1}-v_k)}-4(\mu'(v_k))}{(v_{k-1}-v_k)} \tag{10}$$

The result of plugging Eq. (9) into Eq. (1) is:

$$v_{k+1} = v_k - \frac{(v_{k-1}-v_k)\mu'(v_k)}{\frac{6(\mu(v_{k-1})-\mu(v_k))}{(v_{k-1}-v_k)}-4(\mu'(v_k))} \tag{11}$$

This gives Newton's method a new modification. Based on the aforementioned preparation, we are now ready to declare the new algorithm

Stage 1. Assume c_0 approximation of a min $\mu(v_k)$.

Stage 2. Put $k = k + 1$.

Stage 3. Set $v_k = v_{k-1} + 0.1$

Stage 4. Compute $v_{k+1} = v_k - \frac{(v_{k-1}-v_k)\mu'(v_k)}{\frac{6(\mu(v_{k-1})-\mu(v_k))}{(v_{k-1}-v_k)}-4(\mu'(v_k))}$.

Stage 5. If the stopping criterion is met, go to step 2; if not, stop.

3. Convergence Analysis

The following theorem will be used to analyze the convergence of the suggested method.

Theorem 3.1: *Be a zero of a sufficiently smooth function $\mu: I \rightarrow R$for an open interval I for a given $v^* \in I$. The iterative approach offers second order convergence if v_0 is near enough to v^*.*

Proof: The Newton technique has been modified as follows:

$$. v_{k+1} = v_k - \frac{(v_{k-1}-v_k)\mu'(v_k)}{\frac{6(\mu(v_{k-1})-\mu(v_k))}{(v_{k-1}-v_k)}-4(\mu'(v_k))} \tag{11}$$

Equation (11) may be solved by removing v^* from both sides to get:

$$e_{k+1} = e_k - \frac{\mu'(v_k)(e_{k-1}-e_k)}{\frac{6(\mu(v_{k-1})-\mu(v_k))}{(e_{k-1}-e_k)}-4(\mu'(v_k))} \tag{12}$$

where $e_k = v_k - v^*$. Taylor's series is extended to give us:

$$\mu(v^*) = \mu(v_k) + (v^* - v_k)\mu^1(v_k) + \frac{1}{2!}(v^* - v_k)^2\mu^2(v_k) + \frac{1}{3!}(v^* - v_k)^3\mu^3(v_k) + \frac{1}{4!}(v^* - v_k)^4\mu^4(z). \quad (13)$$

The lies between v^* and v_k is z, Equation (13) derivative, which equals zero, gave us:

$$\mu'(v^*) = \mu^1(v_k) + (v^* - v_k)\mu^2(v_k) + \frac{1}{2}(v^* - v_k)^2\mu^3(v_k) + \frac{1}{6}(v^* - v_k)^3\mu^4(z) = 0 \quad (14)$$

and making use of:

$$\cdot -\mu^1(v_k) = -e_k\mu^2(v_k) + \frac{e_k^2}{2}\mu^3(v_k) - \frac{e_k^3}{6}\mu^4(z) \quad (15)$$

Equation (12) gives us:

$$\cdot\ e_{k+1} = \frac{e_k^2}{2}\frac{\mu^3(c_k)}{\mu^2(c_k)} - \frac{e_k^3}{6}\frac{\mu^4(z)}{\mu^2(c_k)} \quad (16)$$

The order of convergence is quadratic, as shown.

4. Application Examples

It is built in Matlab by applying the iterative approaches to solve seven unimodal functions. For the multidimensional situation, there are additional test functions like in [14-25]. Comparing the various iterative approaches using the number of iterations and execution time for each function. We refer to accuracy in computer programming as $\varepsilon = 10^{-10}$. For the sake of simplicity, we refer to the Newton method (WM), the number of iterations (TN), new secant method by (NSM) and the execution time (TE).

Issue 1

Function $(\boldsymbol{v}) = \boldsymbol{cos}(\boldsymbol{v}) + (\boldsymbol{v} - \mathbf{2})^2$, Starting Point $\boldsymbol{v_0} = \mathbf{2}$.

Methods	**TN**	$\boldsymbol{v_k}$	**TE**
WM	4.	2.354.	0.579.
NSM	4.	2.133.	0.282.

Issue 2

Function $(\boldsymbol{v}) = \boldsymbol{v}/\boldsymbol{log}(\boldsymbol{v})$, Starting Point $\boldsymbol{v_0} = \mathbf{0.1}$.

Methods	**TN**	$\boldsymbol{v_k}$	**TE**
WM	Fail	Fail	Fail
NSM	5.	0.364.	0.250.

Issue 3
Function $(v) = -ve^{-v}$, Starting Point $v_0 = 0$.

Methods	TN	v_k	TE
WM	7.	1	0.390.
NSM	4.	0.139.	0.266.

ISSUE 4
Function $(v) = 0.65 - 0.75/(1+v^2) - 0.65v\, tan^{-1}(1/v)$,
Starting Point $v_0 = 0.1$.

Methods	TN	v_k	TE
WM	6.	0.480.	0.797.
NSM	4.	0.229.	0.375.

ISSUE 5
Function $(v) = 0.5v^2 - sin(v)$, Starting Point $v_0 = 2$.

Methods	TN	v_k	TE
WM	5.	0.7391	0.421.
NSM	5.	2.1608	0.250.

ISSUE 6
Function $(v) = v^4 + 2v^2 - v - 3$, Starting Point $v_0 = 1$.

Methods	TN	v_k	TE
WM	7.	0.236.	0.687.
NSM	5.	1.180.	0.234.

Conclusion

It is possible that the approaches will not always converge to zero. With respect to the computational results, it is clear that the new method has an advantage for the execution times and iterations of all the functions, and it also demonstrates order two convergence. Getting the worldwide minimum, though, will undoubtedly be challenging.

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Received June, 2024