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Solutions convergence of the Schrödinger equation in perdynamic model

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Abstract

In this paper, we develop a practical analytical framework for a one-dimensional linear peridynamic spring system model. Discuss the convergence of the Schrödinger equation's solution by extending the horizon while using the Schrödinger equation in the peridynamic model.

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1. Introduction

The wave function of a quantum mechanical system is governed by the Schrödinger equation, a linear partial differential equation. [11]: Its discovery was a crucial turning point in the growth of quantum physics. It bears the name of Erwin Schrödinger. He received the 1933 Nobel Prize in Physics for his 1925 equation postulation and 1926 publication of it. The quantum equivalent of Newton's second law of conventional physics is the Schrödinger equation [1]. Newton's second law applies mathematics to forecast the future behavior of a

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particular physical system under a given set of initial conditions. The Schrödinger equation describes a single physical system in terms of quantum mechanics and shows how a wave function changes over time. [2].

Applying the continuous theory of peridynamics with the nonlocal theory of force interaction [5,13]. The stress-strain relationship of classical elasticity is replaced with an integral operator that explicitly adds internal forces that are separated by a limited distance [3,7]. Peridynamic theory aims to provide a more extensive framework than the classical approach for addressing difficulties that lead to gaps or extra singularities in the deformation [8,10].

A nonlocal force model can be used to represent a long-range material interaction [12-14], which is represented by the integral equation, which has peridynamic model convergence to classical elasticity theory and a limit at a very narrow horizon δ [4]. These qualities make peridynamic theory a useful tool for simulating fracture and interface problems. [6]

By using mathematical analysis, we investigate the convergence of the Schrödinger equation solution in the peridynamic model, since the mathematical analysis provides a strong hint that the method utilized to solve the model was effective. Because of the Schrödinger equation's relevance to Newton's second law, we used it.

The partial integro-differential equation of second order in time t is the predynamic model [3,4]

$$\rho\ddot{\varphi} = \int_{R^0} f(y, \acute{y}, \varphi(y,t), \varphi(\acute{y},t))d\acute{x} + b(y,t) \tag{1}$$

where ρ denotes the mass density, φ displacement field of the body, f the pairwise force function that characterizes internal forces, and b for an in homogeneity that gathers all external forces per unit volume. The time under consideration is denoted by t>0, and the open ball with radius is denoted by R^0, where δ >0 is the interaction's so-called peridynamic horizon, as follows: $R^0 = \{y \in \Omega : |\acute{y} - y| \leq \delta\}$

Where Ω is sub region of R (the set of real number).

Mathematical Analysis for Peridynamic Model [9]

In section one we defined the displacement $\eta = \varphi(\acute{y}) - \varphi(y) = (\nabla\varphi)^T(\acute{y} - y)$

Since the deformation is elastic homogenous material $(\nabla\varphi)^T = (\nabla\varphi)$

$$\varphi(\acute{y}) - \varphi(y) = \frac{\big((y'-y)\nabla\big)\varphi(y)}{1!} + \frac{\big((y'-y)\nabla\big)^2\varphi(y)}{2!} + \frac{\big((y'-y)\nabla\big)^3\varphi(y)}{3!} + \cdots$$

$$= \sum_{n=1}^{m} \frac{\big((y'-y)\nabla\big)^n\varphi(y)}{k!} \tag{13}$$

Inserting Taylor expansion into the definition of L_δ in eq (12) yields

$$L_\delta\varphi(y) = \sum_{n=1}^{m} L_\delta^n\,\varphi(y). \tag{14}$$

$$L_\delta^n\varphi(y) = \frac{1}{n!\delta^2}\int_{y-\delta}^{y+\delta} \frac{1}{|\acute{y}-y|}\big((y'-y)\nabla\big)^n\varphi(y)d\acute{y} \tag{15}$$

We can write (15) form:

$$L_\delta^n \varphi(y) = \frac{1}{n!\delta^2}\int_{y-\delta}^{y+\delta} \frac{1}{|\acute{y}-y|}(y'-y)^n \varphi^n(y) d\acute{y} \tag{16}$$

$$L^1\varphi(y) = \frac{1}{\delta^2}\int_{y-\delta}^{y+\delta} \frac{(y'-y)}{|y'-y|} d\acute{y}\, \acute{\varphi}(y)$$

$$= \frac{1}{\delta^2}|y'-y|_{y-\delta}^{y+\delta}\, \acute{\varphi}(y) = 0 \tag{17}$$

$$L^2\varphi(y) = \frac{1}{\delta^2}\int_{y-\delta}^{y+\delta} \frac{(\acute{y}-y)^2}{|\acute{y}-y|} d\acute{y}\varphi''$$

$$\varphi(\acute{y}) = (\acute{y}-y)\ d\varphi = d\acute{y},\ \varphi = \varphi(y)$$

$$dv = \frac{(\acute{y}-y)}{|\acute{y}-y|}\ v = |\acute{y}-y| \tag{18}$$

$$= [(\acute{y}-y)|\acute{y}-y|]_{y-\delta}^{y+\delta} - \int_{y-\delta}^{y+\delta}|\acute{y}-y|\, d\acute{y}.$$

$$= \frac{1}{2}\varphi''(y).$$

Using eq (17),(18),(19) in eq(14) we have:

$$L_\delta\varphi(y) = \frac{1}{2}\varphi'' + \frac{1}{48}\delta^2\varphi''''(y) + \cdots \tag{20}$$

It follow $L_\delta^n\varphi(y) = 0$ if n is odd since then the integral is an odd function in $\acute{y} \to y$. Which we can use to determines the right-hand side of (20) for polynomial exact solutions $\varphi(y)$.

Solutions Convergence of Schrödinger Equation in the Model:

The Schrodinder equation $\varphi(y,t) = e^{i(ky-\omega t)}$ which represent the exact solution of Schrodinder equation $-\frac{\hbar^2}{2m}\nabla^2\varphi + V\varphi = i\hbar\frac{\partial\varphi}{\partial t}$ [1, 2]. Prove $\varphi(y,t)$ is solution of the model in eq(12)

Solve:

we prove $\varphi(y,t)$ is the solution of the model by mathematical analysis:

we apply $\varphi(y,t)$ in eq(12) we have

$$L_\delta = \frac{1}{\delta^2}\int_{y-\delta}^{y+\delta} \frac{e^{i(k\acute{y}-\omega t)} - e^{i(ky-\omega t)}}{|\acute{y}-y|} d\acute{y}$$

using eq(20) we have:

$$L_\delta = \frac{1}{2}(-e^{i(ky-\omega t)} + \frac{1}{48}\left(e^{i(ky-\omega t)}\right)\delta^2 - \cdots$$

we note the above solutions, the derivatives of $\varphi(y,t)$ is frequency, we cannot determine the convergence of the solutions, we take:

$$|\,L_\delta| = \left|\frac{1}{2}(-e^{i(ky-\omega t)} + \frac{1}{48}\left(e^{i(ky-\omega t)}\right)\delta^2 - \cdots\right| \le e^{i(ky-\omega t)}(\frac{1}{2} + \frac{1}{48}\delta^2 + \cdots)$$

since δ convergence to 0

then $(\frac{1}{2} + \frac{1}{48}\delta^2 + \cdots)$ is convergence of 0

then $e^{i(ky-\omega t)}(\frac{1}{2} + \frac{1}{48}\delta^2 + \cdots)$ is convergence

by convergence absolutely, if $|\,L_\delta|$ is convergence then L_δ is convergence.

The convergence of the solutions series dependent on the horizon δ, the horizon δ control on convergence of the solution.

Conclusion

Through the results above, we reached a convergence of solutions to the mathematical model represented by the Schrödinger equation, which is equivalent to Newton's second law of motion. Therefore, we can use any equation subject to Newton's second law of motion to solve the berydynamic mathematical model.

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