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Solution of the delay differential equations via applying Rohit transform technique

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Abstract

In this paper, we consider the initial value problem for a linear 2nd order delay differential problem, we apply Rohit transformation technique to find the solution this problem. As well as, we introduce problems provided support the theoretical results.

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1. Introduction

Differential and delay differential equations (DDE') are very importance in many fields such as: physics, biosciences economics, neural, networks, engineering, sciences, population dynamics and bistable device [1, 3-10, 12-22] are modelled differential equations and delay differential equations. There are many researches who investigated oscillation, Hopf bifurcation, numerical aspect and stability analysis for delay differential equations [1, 3, 5]. Rohit integral transformation is applied to find the application problems in mathematics [2, 11], engineering, physics. Therefore, it is important tool t only for mathematicians but also for engineers and physicists. In this work, we present the following delay differential equation (intial value problem):

$$u''(x) + A\,u'(x) + B\,u'(x-r) + Cu(x) + Du(x-r) = f(x), \qquad (1.1)$$

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for $x > 0\ ;\ u(x) = \ell(x)\ ;\ -r \le x \le 0$ and $\acute{u}(0) = \Upsilon$. (1.2)

Where A, B, C and D are real numbers, f(x) and (x) are given real valued and sufficiently smooth functions, Υ is a constant and r is a positive real number large delay. As well as, the existence and uniqueness of the solution to delay differential equations is introduced and discussed in [1, 3, 5,]. The aim of this paper, we develop the Rohit integral transform technique to establish the exact solution a class of 2^{nd} order delay differential equation.

2. Basic concepts

Definition 2.1: [2] Assume that g is a real - valued function of the variable x >0 and v is a real parameter. The Rohit integral transform is defined as:

$\mathcal{R}\{g(x)\} = v^3 \int_0^\infty e^{-vx}\ g(x)\, dx = G(v)$, provided that the integral is convergent, where $\mathcal{R}$ is the "Rohit Transform " "($\mathcal{R}$T) operator.

Proposition 2.1: Suppose that f(x) is a real function with:

1. f(x) is a piecewise continuous in every $x < x_1$ $(x_1 > 0)$.
2. f(x) is of exponential order, that is there exists $a, N > 0, and (x_0 > 0)$

Such that $e^{ax}\,|f(x)| < N\ for\ x > x_0$,then Rohit integral technique exist for $s > a$.

3. The Rohit Transform for Some Important Functions, [2]:

In this part, we introduce some elementary functions in Rohit technique:

1. $\mathcal{R}\{k\} = kv^2$, where **k** is a constant and v >0.
2. $\mathcal{R}\{x^n\} = \frac{n!}{v^{n-2}}, n \in \mathbb{N}$.
3. $\mathcal{R}\{sin(bx)\} = \frac{bv^3}{v^2+b^2}, v > 0$.
4. $\mathcal{R}\{con(bx)\} = \frac{v^4}{v^2+b^2}, v > 0$.
5. $\mathcal{R}\{sinh(bx)\} = \frac{bv^3}{v^2-b^2}, v > |b|$.
6. $\mathcal{R}\{cosh(bx)\} = \frac{v^4}{v^2-b^2}, v > |b|$.
7. $\mathcal{R}\{e^{bx}\} = \frac{v^3}{v-b}, v > b$.

The inverse " Rohit Transform " ($\mathcal{R}$T) of the function $G(v)$ is denoted by $\mathcal{R}^{-1}\ \{G(v)\}\ or\ g(x)$.

If we write $\mathcal{R}${g(x)} = G(v) then $\mathcal{R}^{-1}\ \{G(v)\} = g(x)$, where $\mathcal{R}^{-1}$is said to be the "Inverse Rohit Transform " operator.

The "Inverse Rohit Transfor " of some elementary functions are given below.

1. $\mathcal{R}^{-1}\left\{\frac{1}{v^n}\right\} = \frac{x^{n-2}}{(n+2)!}$
2. $\mathcal{R}^{-1}\left\{\frac{v^3}{v-b}\right\} = e^{bx}$.
3. $\mathcal{R}^{-1}\left\{\frac{v^3}{v^2+b^2}\right\} = \frac{1}{b}sin(bx)$.
4. $\mathcal{R}^{-1}\left\{\frac{v^3}{v^2-b^2}\right\} = \frac{1}{b}sinh(bx)$.
5. $\mathcal{R}^{-1}\left\{\frac{v^4}{v^2+b^2}\right\} = cos(bx)$.
6. $\mathcal{R}^{-1}\left\{\frac{v^4}{v^2-b^2}\right\} = cosh(bx)$.

Where b $\neq$ 0 is a constant number and n $\in \mathbb{N}$.

4. Rohit Integral Transform of Derivatives,[2]

Let $g(x)$ is a continuous function and is apiecewise continuous on any interval, then the ($\mathcal{R}$T) of 1^{st} derivative of $g(x)$ that is

$\mathcal{R}\{g^{'}(\mathrm{x})\}$ is given as: $\mathcal{R}\left\{g^{'}(\mathrm{x}) = v\mathcal{R}\{(g(\mathrm{x})\}\right\} - v^3 g(\mathrm{o})$.And $\mathcal{R}\{g^{''}(\mathrm{x})\} = v^2\mathrm{G}(v)$-$v^4 g(o) - v^3 g^{'}(\mathrm{o})$. Similarly $\mathcal{R}\{g^{'''}(\mathrm{x})\} = v^3\mathrm{G}(v)$-$v^5 g(o) - v^4 g^{'}(0) - v^3 g^{''}(\mathrm{o})$.

Also $\mathcal{R}\{\mathrm{x}\, g(\mathrm{x})\} = \frac{3}{v}\,\mathcal{R}\{(g\mathrm{x})\} - \frac{\mathrm{d}}{\mathrm{d}v}\mathcal{R}\{(g\mathrm{x})\}$.

$$\mathcal{R}\{\mathrm{x}\, g^{'}(\mathrm{x})\} = 2\mathcal{R}\{g(\mathrm{x})\} - v\frac{\mathrm{d}}{\mathrm{dv}}\mathcal{R}\{g(\mathrm{x})\}. \mathcal{R}\{\mathrm{x}\, \mathrm{g}^{''}(\mathrm{x})\}$$
$$= v\mathcal{R}\{g(\mathrm{x})\} + v^3 g(\mathrm{o}) - v^2\frac{\mathrm{d}}{\mathrm{dv}}\mathcal{R}\{g(\mathrm{x})\}.$$

5. Solution of Delay Differential Problem by Rohit Transform Technique

Consider delay differential problem: $u^{''}(x) + Au'(x) + Bu'(x-r) + cu\,(x) + Du(x-r) = f(x), x > 0$ *...(5.1)*

$$u(x) = \ell(x)\,;\; -\mathrm{r} \leq \mathrm{x} \;\leq \mathrm{o}\;; u^{'}(0) = \Upsilon. \tag{5.2}$$

Where A, B, C, D are real constants, f(x) and $\ell(x)$ are given real valued and sufficiently smooth functions. Υ is a real constant and r is a positive number Large delay

Theorem 5.1: *Assume that* $\boldsymbol{\ell}(x)$; $\ell^{'}$ *(x) are continuous on the closed*

Interval [-r,o] G(v) is the Rohit transformation of f(x) in equation (5.1) then the exact solution of equations (5.1) and (5.4)

u(x) $= \mathcal{R}^{-1}\left\{\frac{\mathrm{G}(v)+\mathrm{T}(v)}{\mathrm{k}(v)}\right\}$ where

$$\mathrm{k}(v) = v^2 + \mathrm{A}v + \mathrm{C} + (\mathrm{B}v + \mathrm{D})\, \mathrm{e}^{-v\mathrm{r}},$$

$$\mathrm{T}(v) = v^3 \curlyvee + (v\ell(\mathrm{o}) + \mathrm{A}\ell(0))v^3 - B\bar{\bar{\ell}}(v)v^3 - D\ell^{\prime}(v),$$

$$\bar{\ell}(v) = v^3 \int_{t=-r}^{0} e^{-v(x+r)}\, \ell(x)\, dx, \bar{\bar{\ell}}(v) = v^3 \int_{t=-r}^{0} e^{-v(x+r)}\, \bar{\ell}(x)\, dx,$$

Proof: To proof the differential problem (5.1) - (5.2) applying the Rohit transform technique, it is known the Rohit transform of the derivatives of u(x) is:

$\mathcal{R}\{\mathrm{u}^{\prime}(\mathrm{x})\} = -v^3\mathrm{u}(\mathrm{o}) + v\mathcal{R}\{\mathrm{u}(\mathrm{x})\}$. And $\mathcal{R}\{\mathrm{u}^{\prime}(\mathrm{x})\} = -v^3\ell(\mathrm{o}) + v\mathcal{R}\{\mathrm{u}(\mathrm{x})\}$,

$\mathcal{R}\{\mathrm{u}^{\prime\prime}(\mathrm{x})\} = -\mathrm{u}^{\prime}(\mathrm{o})v^3 + v^4\mathrm{u}(\mathrm{o}) + v^2\mathcal{R}\{\mathrm{u}(\mathrm{x})\}$, $\mathcal{R}\{\mathrm{u}^{\prime\prime}(\mathrm{x})\} = -\curlyvee v^3 + v^4\ell(\mathrm{o}) + v^2\mathcal{R}\{\mathrm{u}(\mathrm{x})\}$.

The Rohit transform for $\mathrm{u}(\mathrm{x}-\mathrm{r})$,from the definition gets: $\mathcal{R}\{\mathrm{u}(\mathrm{x}-\mathrm{r})\} = v^3 \int_{\mathrm{t}=0}^{\infty} \mathrm{e}^{-v\mathrm{x}}\, \mathrm{u}(\mathrm{x}-\mathrm{r})\mathrm{dx}$.

After replacing integral variable by $\mathrm{x} = \mathrm{t} + \mathrm{r}$ then $\mathrm{t} = \mathrm{x} - \mathrm{r}$. We find that:

$$\mathcal{R}\{\mathrm{u}(\mathrm{x}-\mathrm{r})\} = v^3 \int_{\mathrm{t}=-\mathrm{r}}^{\infty} \mathrm{e}^{-v(\mathrm{x}-\mathrm{r})}\mathrm{u}(\mathrm{t})\mathrm{dx},$$

$$= v^3[\int_{\mathrm{t}=-\mathrm{r}}^{0} \mathrm{e}^{-v(\mathrm{t}+\mathrm{r})}\, \ell(\mathrm{t})\mathrm{dt} + \int_{0}^{\infty} \mathrm{e}^{-v(\mathrm{t}+\mathrm{r})}\, \mathrm{u}(\mathrm{t})\mathrm{dt}\,],$$

$$= v^3[\int_{-\mathrm{r}}^{0} \mathrm{e}^{-v(\mathrm{t}+\mathrm{r})}\, \ell(\mathrm{t})\mathrm{dt} + v^3\mathrm{e}^{-v\mathrm{r}} \int_{0}^{\infty} \mathrm{e}^{-v\mathrm{t}}\, \mathrm{u}(\mathrm{t})\mathrm{dt}\,].$$

Thus, we have

$\mathcal{R}\{\mathrm{u}(\mathrm{x}-\mathrm{r})\} = -\bar{\ell}(v) + \mathrm{e}^{-\mathrm{vr}}\mathcal{R}\{\mathrm{u}(\mathrm{x})\}$.

Similariy, the Rohit transform $\mathcal{R}(\mathrm{x})$ for $u^{\prime}(x-r)$, we can write $\mathcal{R}\{\mathrm{u}^{\prime}(\mathrm{x}-\mathrm{r})\} = v^3 \int_{\mathrm{x}=0}^{\infty} \mathrm{e}^{-v\mathrm{x}}\, \mathrm{u}^{\prime}(\mathrm{x}-\mathrm{r})\mathrm{dx}$,

$$= v^3[\int_{\mathrm{t}=-\mathrm{r}}^{0} \mathrm{e}^{-v(\mathrm{t}+\mathrm{r})}\, \mathrm{u}^{\prime}(\mathrm{t})\, \mathrm{dt},$$

$$= v^3[\int_{-\mathrm{r}}^{0} \mathrm{e}^{-v(\mathrm{t}+\mathrm{r})}\, \ell^{\prime}(\mathrm{t})\mathrm{dt} + \mathrm{e}^{-\mathrm{vr}} \int_{0}^{\infty} \mathrm{e}^{-v\mathrm{t}}\, \mathrm{u}^{\prime}(\mathrm{t})\, \mathrm{dt}\,].$$

Here, we get:$\mathcal{R}\{\mathrm{u}^{\prime}(\mathrm{x}-\mathrm{r})\} = \bar{\bar{\ell}}(v) + \mathrm{e}^{-\mathrm{vr}}\mathcal{R}\{\mathrm{u}^{\prime}(\mathrm{x})\}$.

Using the Rohit integral transform to equation (5.1) gives:

$$\mathcal{R}\{\mathrm{u}^{\prime\prime}(\mathrm{x})\} + \mathrm{A}\mathcal{R}\{\mathrm{u}^{\prime}(\mathrm{x})\} + \mathrm{B}\mathcal{R}\{\mathrm{u}^{\prime}(\mathrm{x}-\mathrm{r})\} + \mathrm{C}\mathcal{R}\,\{\mathrm{u}(\mathrm{x})\} + \mathrm{D}\mathcal{R}\{\mathrm{u}(\mathrm{x}-\mathrm{r})\} = \mathcal{R}\{\mathrm{f}(\mathrm{x})\}.$$

And Applying above equations, we get:

$$\begin{aligned} -\curlyvee v^3 - \ell(0)\, v^4 &+ v^2\mathcal{R}\{\mathrm{u}(\mathrm{x})\} + \mathrm{A}v\mathcal{R}\{\mathrm{u}(\mathrm{x})\} - \mathrm{A}\ell(0)\, v^3 + \mathrm{B}\bar{\bar{\ell}}(v) \\ &+ \mathrm{Be}^{-v\mathrm{r}}\mathcal{R}\{\mathrm{u}(\mathrm{x})\} - \mathrm{Be}^{-\mathrm{r}}\ell(0)v^3 + \mathrm{C}\mathcal{R}\{\mathrm{u}(\mathrm{x})\} + \mathrm{D}\bar{\ell}(v) \\ &+ \mathrm{De}^{-v\mathrm{r}}\mathcal{R}\{\mathrm{u}(\mathrm{x})\} \\ &= \mathcal{R}\{\mathrm{f}(\mathrm{x})\}, (v^2 + \mathrm{A}v + \mathrm{Be}^{-v\mathrm{r}} + \mathrm{C} + \mathrm{De}^{-v\mathrm{r}})\mathcal{R}\{\mathrm{u}(\mathrm{x})\} \\ &= \mathrm{G}(v) + \curlyvee v^3 + \ell(0)v^4 + \mathrm{A}\ell(0)v^3 - \mathrm{B}\bar{\bar{\ell}}(v) \\ &+ \mathrm{Be}^{-v\mathrm{r}}\ell v^3(0) - \mathrm{D}\bar{\ell}(v). \end{aligned}$$

It can be reduced to:$\mathcal{R}\{\mathrm{u}(\mathrm{x})\} = \frac{\mathrm{G}(v)+\mathrm{T}(v)}{\mathrm{K}(v)}$.

Example 5.1: We consider the following delay differential problem.

$$u''(x) - 3u'(x) + u'(x-1) + 2u(x) - u(x-1) = 0, x > 0.$$

Subject to the initial condition (I.C.),

$u(x) = e^x, -1 \le x \le 0, u'(0) := 1.$ If We take into consideration

$G(v) = 0, T(v) = v^3[v - 2 + e^{-v}]\ k(v) = v^2 - 3v + 2 + (v-1)e^{-v}$

So $\mathcal{R}\{u(x)\} = \frac{v^3(v-2+e^{-v}) + T(v)}{v^2-3v+2+(v-1)e^{-v}} = v^3\left(\frac{v-2+e^{-v}}{(v-1)(v-2+e^{-v})}\right)$,And

$\mathcal{R}\{u(x)\} = v^3\,(v-1)^{-1}$, then $u(x) = e^x$.

7. Conclusion

In this article, the Rohit Transform technique $\mathcal{R}(x)$ is applied to find the exact solution of a linear 2^{nd}order delay differential problems, demonstrating its efficacy as a technique to evaluate the exact solutions to abroad class for differential problems. This means that the technique given here can be used to issues of the neutral delay kind also the volterra delay integro differential kind.

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