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On new secant-method for minimum functions of one variable

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Abstract

In this article, we developed the Newton method by utilizing the Taylor series to estimate derivatives based on the function's minimum value. The aim was to reduce the number of iterations required to obtain the optimal solution of the function. We compare the execution time and number of iterations between the proposed approach and the classical

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Newton method. After applying the suggested approach to a set of unimodal functions, the numerical results demonstrate its efficacy and efficiency.

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1. Introduction

One of the main functions of iteration approaches is to solve nonlinear problems involving only one variable, see [1, 10-13]. Finding the minimal value of non-linear problems of the following kind is one of the most renowned and ancient iteration problems:

$$Minimize\{\Phi(\lambda), \lambda \in R, \Phi: R \to R\}. \tag{1}$$

It might not be simple to tackle these problems. As a result, we quickly look at a numerical approach to solving it. The traditional Newton's approach, which converges quadratically, is the most popular and well-known technique for determining the value minimum of optimization techniques. Further information is available in [2]. Newton's technique of using a quadratic function that corresponds to the function at a location to locally approximate the goal function is the basis for this strategy. Next, we enhance the exact approximate function, see [3]. Thus, Newton's technique iteration is provided by:

$$\lambda_{k+1} = \lambda_k - \frac{\phi'(\lambda_k)}{\phi''(\lambda_k)}. \tag{2}$$

In [4], the secant approach, which avoids computing the second derivation, may be thought of as a simplification of Newton's method and yields the following results:

$$\lambda_{k+1} = x_k - \frac{\phi'(\lambda_k)(\lambda_k - \lambda_{k-1})}{\phi'(\lambda_k) - \phi'(\lambda_{k-1})}. \tag{3}$$

Many investigations of Secant-type techniques have been conducted recently. has shifted its attention to addressing nonlinear optimization problems where $\boldsymbol{\phi}''(\lambda_k)$ is not necessary, as in [4-9] [14-16].

The idea is to develop a new version of Secant that, by eliminating issues based on the $\Phi''(x_k)$ evaluation, will improve the Newton's method iteration.

2. On New Secant-Method

By use of the Taylor expansion, an essential step in the derivation of the new secant-method, [4], [9] as:

$$\phi(\lambda) \cong \phi(\lambda_k) + \phi'(\lambda_k)(\lambda - \lambda_k) + \frac{1}{2}\phi''(\lambda_k)(\lambda - \lambda_k)^2 \tag{4}$$

This yields the gradient of $\phi(\lambda)$:

$$\phi'(\lambda) \cong \phi'(\lambda_k) + \phi''(\lambda_k)(\lambda - \lambda_k) = 0 \tag{5}$$

By taking advantage (5) and (4) we get :

$$\phi''(\lambda_k)(\lambda - \lambda_k)^2 \cong \phi(\lambda) - \phi(\lambda_k) - 3/2\phi'(\lambda_k)(\lambda - \lambda_k) \quad (6)$$

This results in:

$$\phi''(\lambda_k) = \frac{\phi(\lambda)-\phi(\lambda_k)-3/2\phi'(\lambda_k)(\lambda-\lambda_k)}{(\lambda-\lambda_k)^2} \quad (7)$$

By using (7) into (2), to yield:

$$\lambda_{k+1} = \lambda_k - \frac{\phi'(\lambda_k)(\lambda_{k-1}-\lambda_k)^2}{\phi(\lambda_{k-1})-\phi(\lambda_k)-3/2\phi'(\lambda_k)(\lambda_{k-1}-\lambda_k)} \quad (8)$$

This is an adaptation of Newton's technique. Based on the derivation above the adaptation of Newton's algorithm as:

Stage 1: To state it ε, λ_0, set $k = 0$.

Stage 2: It will be $k = k + 1$.

Stage 3: Use $\lambda_k = \lambda_{k-1} - \frac{\phi'(\lambda_{k-1})}{\phi''(\lambda_{k-1})}$.

Stage 4: $\lambda_{k+1} = \lambda_k - \frac{\phi'(\lambda_k)(\lambda_{k-1}-\lambda_k)^2}{\phi(\lambda_{k-1})-\phi(\lambda_k)-3/2\phi'(\lambda_k)(\lambda_{k-1}-\lambda_k)}$

Stage 5: Stop, if $|\phi'(\lambda_k)| \leq \varepsilon$.

3. Analysis of convergence

Investigating the global convergence of New-secant will be our next section focus.

Theorem 3.1: *An interval I, let $\lambda^* \in I$ represent the zero of a suitably differentiable function $\phi: I \to R$. The suggested approach achieves quadratic convergence if λ_0 is near enough to λ^*.*

Proof : Our New-secant method as:

$$\lambda_{k+1} = \lambda_k - \frac{\phi'(\lambda_k)(\lambda_{k-1}-\lambda_k)^2}{\phi(\lambda_{k-1})-\phi(\lambda_k)-3/2\phi'(\lambda_k)(\lambda_{k-1}-\lambda_k)}. \quad (9)$$

If x^* is subtracted from both sides of (10), then:

$$e_{k+1} = e_k - \frac{\phi'(\lambda_k)(e_{k-1}-e_k)^2}{\phi(\lambda_{k-1})-\phi(\lambda_k)-3/2\phi'(\lambda_k)(e_{k-1}-e_k)}. \quad (10)$$

where $e_k = \lambda_{k+1} - \lambda^*$. By making Taylor's series:

$$\phi(\lambda^*) = \phi(\lambda_k) + \phi'(\lambda_k)(\lambda^* - \lambda_k) + \frac{1}{2!}\phi''(\lambda_k)(\lambda^* - \lambda_k)^2 + \frac{1}{3!}\phi'''(c)(\lambda^* - \lambda_k)^2 \quad (11)$$

but c lies between λ_k and λ^*, the gradient of (12), we as:

$$\phi'(\lambda^*) = \phi'(\lambda_k) + \phi''(\lambda_k)(\lambda^* - \lambda_k) + \frac{1}{2}\phi'''(c)(\lambda^* - \lambda_k)^2 = 0 \quad (12)$$

Using the (13), we get:

$$-\phi'(\lambda_k) = -e_k\phi''(\lambda_k) + \frac{e_k^2}{2}\phi'''(c) \tag{13}$$

From (14) and (10) we obtain:

$$e_{k+1} = 1/2e_k^2 \frac{\phi'''(c)}{\phi''(x_k)} \tag{14}$$

The proof is finished.

4. Application Examples

The approach gave result methods for solving seven unimodal functions, the Matlab code was built to apply it. For computer applications, we utilize $\varepsilon = 10^{-10}$ accuracy. Analyzing the methods according to the number of iterations (OI) and execution-time (OT) of each function's execution (see Tables 1 & 2.).

Table 1
Test functions

No.	Function	Initial point
1	$\boldsymbol{\phi(\lambda) = cos(\lambda) + (\lambda - 2)^2}$	2
2	$\boldsymbol{\phi(\lambda) = e^{\lambda} - 3\lambda^2}$	0.25
3	$\boldsymbol{\phi(\lambda) = e^{-\lambda} + \lambda^2}$	1
4	$\boldsymbol{\phi(\lambda) = -\lambda e^{-\lambda}}$	0
5	$\boldsymbol{\phi(\lambda) = 0.65 - 0.75/(1 + \lambda^2) - 0.65\lambda\, tan^{-1}(1/\lambda)}$	0.1
6	$\boldsymbol{\phi(\lambda) = 0.5\lambda^2 - sin(\lambda)}$	2
7	$\boldsymbol{\phi(\lambda) = \lambda^4 + 2\lambda^2 - \lambda - 3}$	1

Table 2
The results

Fun. number	Newton			New-secant		
	OT	λ_k	OI	OT	λ_k	OI
1	4	2.3542	0.5790	3	2.3946	0.2030
2	4	0.2045	0.2660	2	0.2042	0.1410
3	5	0.3517	0.3130	3	0.3740	0.1560
4	7	1	0.3900	4	0.3413	0.2190
5	6	0.4809	0.7970	3	0.3466	0.3120
6	5	0.7391	0.4210	3	0.7448	0.1410
7	7	0.2367	0.6870	4	0.4442	0.2350

5. Conclusion

In certain situations, all techniques may struggle to reach the minimum value. Each method carries the risk of not reaching the global minimum under specific conditions. After analyzing various Newton methods, we have determined that the new approach is the most efficient. Obtaining the global minimum may be difficult and necessitate the evaluation of the first derivative throughout the process.

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