Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 28 (2025), No. 1, pp. 253–260
DOI : 10.47974/JIM-1855

New closure operators associated with digraphs

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Abstract

The Aim of this research is to apply a new definition of the $Çl_{\Gamma_n}$-closure operators to create the topology construction on the power set of edges of digrs using closure operator on an out-linked digr. Some of the aspects of topological structure are explored and examples are provided. Furthermore, we introduce the R-open subdigr, a new generalization subdigr, and look into its accuracy.

Subject Classification: *05c30.*

Keywords: *Digraph, Accuracy, Topology, Interior operator, Closure operator.*

1. Introduction

Graph theory is a key idea in multiple academic areas and a helpful instrument in many additional mathematical fields for two reasons. Let's start with the mathematical process for building graphs. Selected in an intellectual capacity. Graphs are straightforward relational combinations, but they can also be topological spaces, collection objects, and many other kinds of mathematical groups. A second reason is that graphs make some concepts simpler to

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comprehend. become more rational. In terms of the relationship between topology and graph theory, topological concepts, such as transforming a set of vertices or edges into topological space are one of the tools used to express the graph. Within the large field of thin king mathematicians, topology is the most popular and current study. A few earlier studies on the subject of topological graphs are listed here. The idea of topology on digr was first put up in 1967 by Evans and Harary [4]. One relationship between the set of all topologies with n vertices and the set of all topologies with n vertices was found by them. Bridge graphs are introduced, and the terms hyper-Zagreb index, first multiple Zagreb index, second multiple Zagreb index, Zagreb polynomials, and M-polynomial for bridge graph microstructures are defined by A.J.M. Khalaf and colleagues in [1]. See [13, 14] for other relationships between polynomial and topological indices. Kh. Sh. Al'Dzhabri and colleagues created a new idea in [5-11] pertaining to undigrs and digrs generated by a novel open set, and they used the closed relationship between topology and graph to study different aspects of this idea [15-19].

2. Preliminaries

Are the beginning stages of the procedure. We present a few basic descriptions of topology [2] and graph theory [3, 12] in this research. A graph (resp., directed graph or briefly digr) $\Gamma = (\mathcal{L}, \Xi)$ consists of a vertex set $\mathcal{L}$ and an edge set Ξ of unordered (resp., ordered) pairs of elements of $\mathcal{L}$. We presume that both the vertex and edge sets are disjoint in order to avert ambiguities. That is, two vertices ŏ and ő of a digraph Γ (briefly digr) are adjacent if there is an edge of the form ŏő joining them. A subdigraph(briefly subdigr) Ⱨ of a digrr Γ is a digr, each of whose vertices belong to $\mathcal{L}$ and each of whose edges belong to Ξ. The degree of a vertex ŏ of Γ is the number of edges incident with ŏ, and written de(ő). An isolated vertex is one whose degree is 0 that is there is no edges incident with it. Let Ӽ be a set. A topology on Ӽ is a set $\tau \subset \mathcal{P}(Ӽ)$ of a subsets of Ӽ, called open sets, such that: (1) $\emptyset$ and X are open. (2) The intersection of finitely many open sets is open. (3) Any union of open sets is open. A topological space is a set Ӽ together with a topology τ on Ӽ. A subset of a topological space is closed if its complement is open. In the topology on Ӽ, if $\tau = \{\emptyset, Ӽ\}$ is called the indiscrete topology, and only two subsets are open. If $\tau = \{A | A \subseteq Ӽ\}$ is called discrete topology, and all subsets are open.The interior of A is the union of all open sets contained in A, $\imath\mathrm{nt}\, Å = \cup \{U \subset Å |\ U \text{ is open}\}$.And the closure of Å is the intersection of all closed sets contained Å, $Çl\, Å = \cap \{F \supset Å |\ F \text{ is closed}\}$. A subset Å of a space Ӽ is called dense if $\overline{Å} = Ӽ$.

3. Closure Operators Associated with Digraphs

We present and examine the ideas of closure operators on digr, dependent on the power set of edges, in this section. We study a number of established topological features on the generated $(\Xi(Ⱨ), Çl_{\Gamma_n}))\ \Gamma_n$ −closure space.

Definition 3.1: Let $\Gamma = (\mathcal{L}(\Gamma), \Xi(\Gamma))$ be a digr, P($\mathcal{L}(\Gamma)$) its power set of all subdigr of Γand $Çl_\Gamma$:P(Ξ(Γ)) → P(Ξ(Γ)) is a mapping associating with each subdigr Ḥ =($\mathcal{L}$(Ḥ), Ξ(Ḥ)) a subdigr $Çl_\Gamma$ (Ξ(Ḥ))⊆ Ξ(Γ) called the closure subdigr of Ḥ such that: $Çl_\Gamma$ (Ξ(Ḥ))= Ξ(Ḥ)∪{ɕ′ ∈ Ξ(Γ)\ Ξ(Ḥ); for all ȩ∈ Ξ(Ḥ), ȩ incident with ɕ′ in the opposite direction}.

The operation $Çl_\Gamma$ is called digr closure operator and the pair ($\mathcal{L}(\Gamma)$,$Çl_\Gamma$) is called Γ-closure space, where $Çl_\Gamma$ (Ξ(Γ)) is the family of elemnts of $Çl_\Gamma$. The du*a*l of the digr closure operator $Çl_\Gamma$ is the digr interior operator m_Γ:P(Ξ(Γ))→P(Ξ(Γ)) defined by m_Γ(Ξ(Ḥ))= Ξ(Γ)\ $Çl_\Gamma$(Ξ(Γ)\ Ξ(Ḥ)) for all subdigr Ḥ ⊆ Γ. A family of elements of INT_Γ is called interior subdigr of Ḥ and denoteed by m_Γ(Ξ(Γ)). Clear that (Ξ(Γ), m_Γ) is a topological space. Then the domain of m_Γ is equal to the domain of m_Γ and also CLU_Γ (Ξ(Ḥ))= Ξ(Γ)\ m_Γ(Ξ(Γ))\ Ξ(Ḥ)). A subdigr Ḥ of Γ-closure space ($\mathcal{L}(\Gamma)$,$Çl_\Gamma$) is called closed subdigr if CLU_Γ (Ξ(Ḥ))= Ξ(Ḥ). It is called open subdigr if its complement is closed subdigr, i.e.,$Çl_\Gamma$ (Ξ(Γ))\ Ξ(Ḥ))= Ξ(Γ)\ Ξ(Ḥ), or equivalently m_Γ(Ξ(Ḥ))= Ξ(Ḥ).

Example 3.2: Let $\Gamma(\mathrm{V}, \mathrm{D})$ be a digr such that $\mathcal{L}(\Gamma) = \{ő_1, ő_2, ő_3, ő_4\}$, $\Xi(\Gamma) = \{ɕ_1, ɕ_2, ɕ_3, ɕ_4\}$ (see Figure 1 and Table 1).

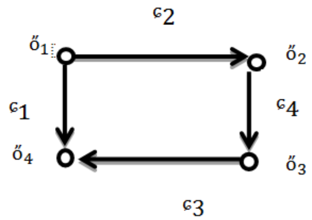

Figure 1
Digraph with 4 vertices and 4 arcs

Table 1
The closure operator for subdigraphs

Ξ(Ḥ)	**Çl$_\Gamma$**	Ξ(Ḥ)	**Çl$_\Gamma$**
Ξ(Γ)	Ξ(Γ)	{ɕ$_1$, ɕ$_4$}	Ξ(Γ)
∅	∅	{ɕ$_2$, ɕ$_3$}	{ɕ$_1$, ɕ$_2$, ɕ$_3$}
{ɕ$_1$}	{ɕ$_1$, ɕ$_2$, ɕ$_3$}	{ɕ$_2$, ɕ$_4$}	{ɕ$_1$, ɕ$_2$, ɕ$_4$}
{ɕ$_2$}	{ɕ$_1$, ɕ$_2$}	{ɕ$_3$, ɕ$_4$}	{ɕ$_1$, ɕ$_3$, ɕ$_4$}
{ɕ$_3$}	{ɕ$_1$, ɕ$_3$}	{ɕ$_1$, ɕ$_2$, ɕ$_3$}	Ξ(Γ)
{ɕ$_4$}	{ɕ$_4$}	{ɕ$_1$, ɕ$_3$, ɕ$_4$}	Ξ(Γ)
{ɕ$_1$, ɕ$_2$}	{ɕ$_1$, ɕ$_2$, ɕ$_3$}	{ɕ$_1$, ɕ$_2$, ɕ$_4$}	Ξ(Γ)
{ɕ$_1$, ɕ$_3$}	{ɕ$_1$, ɕ$_3$, ɕ$_2$}	{ɕ$_2$, ɕ$_3$, ɕ$_4$}	Ξ(Γ)

$Ç l_\Gamma = \{\Xi(\Gamma), \emptyset, \{ɕ_4\}, \{ɕ_1, ɕ_2\}, \{ɕ_{1,}\ ɕ_3\}, \{ɕ_1, ɕ_2, ɕ_3\}, \{ɕ_1, ɕ_2, ɕ_4\}, \{ɕ_1, ɕ_3, ɕ_4\}\}$,
$\imath n_\Gamma = \{\Xi(\Gamma), \emptyset, \{ɕ_1, ɕ_2, ɕ_3\}, \{ɕ_3, ɕ_4\}, \{ɕ_2, ɕ_4\}, \{ɕ_4\}, \{ɕ_3\}, \{ɕ_2\}\ \}\}$

Definition 3.3: Let $\Gamma = (\mathcal{L}(\Gamma), \Xi(\Gamma))$ be a digr and $Çl_\Gamma : P(\Xi(\Gamma)) \rightarrow P(\Xi(\Gamma))$ an operotor

a) It is called $\Gamma_n -$ closure operotor if $Çl_\Gamma\, \Xi(Ḩ) = Çl_\Gamma\,(Çl_\Gamma\,(Çl_\Gamma\,(\ldots Çl_\Gamma\, \Xi(Ḩ)$ n-times for every subdigraph. $Ḩ \subseteq \Gamma$

b) It is called $\Gamma -$ topological closure operotor if $Çl_{\Gamma\, n+1}(\Xi(Ḩ), Çl_{\Gamma\, n})$ for all subdigr $Ḩ \subseteq \Gamma$ the space $(\Xi(Ḩ), Çl_{\Gamma\, n})$ is called $\Gamma_n -$ topological closure space.

Example. 3.4: Let $\Gamma = (\mathcal{L}(\Gamma), \Xi(\Gamma))$ be a digr such that $\mathcal{L}(\Gamma) = \{ő_1, ő_2, ő_3, ő_4\}$, $\Xi(\Gamma) = \{ɕ_1, ɕ_2, ɕ_3, ɕ_4\}$. We note that $Çl_{\Gamma\, 1} = \{\Xi(\Gamma), \emptyset, \{ɕ_1, ɕ_2\}, \{ɕ_3, ɕ_4\},\ \}$, $Çl_{\Gamma\, 2} = \{\Xi(\Gamma), \emptyset, \{ɕ_1, ɕ_2\}, \{ɕ_3, ɕ_4\},\ \}$ (see Figure 2 and Table 2).

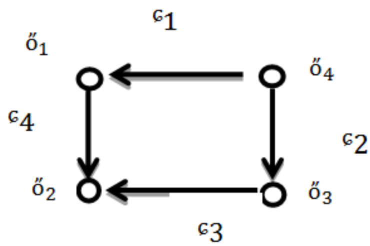

Figure 2
Digraph with 4 vertices and 4 arcs

Table 2
$Çl_{\Gamma\, 1}$ and $Çl_{\Gamma\, 2}$ for subdigrs $\Xi(Ḩ)$.

$\Xi(Ḩ)$	$Çl_{\Gamma\, 1}$	$Çl_{\Gamma\, 2}$	$\Xi(Ḩ)$	$Çl_{\Gamma\, 1}$	$Çl_{\Gamma\, 2}$
$\Xi(\Gamma)$	$\Xi(\Gamma)$	$\Xi(\Gamma)$	$\{ɕ_1, ɕ_4\}$	$\Xi(\Gamma)$	$\Xi(\Gamma)$
$\emptyset$	$\emptyset$	$\emptyset$	$\{ɕ_2, ɕ_3\}$	$\Xi(\Gamma)$	$\Xi(\Gamma)$
$\{ɕ_1\}$	$\{ɕ_1, ɕ_2\}$	$\{ɕ_1, ɕ_2\}$	$\{ɕ_2, ɕ_4\}$	$\Xi(\Gamma)$	$\Xi(\Gamma)$
$\{ɕ_2\}$	$\{ɕ_1, ɕ_2\}$	$\{ɕ_1, ɕ_2\}$	$\{ɕ_3, ɕ_4\}$	$\{ɕ_3, ɕ_4\}$	$\{ɕ_3, ɕ_4\}$
$\{ɕ_3\}$	$\{ɕ_3, ɕ_4\}$	$\{ɕ_3, ɕ_4\}$	$\{ɕ_1, ɕ_2, ɕ_3\}$	$\Xi(\Gamma)$	$\Xi(\Gamma)$
$\{ɕ_4\}$	$\{ɕ_3, ɕ_4\}$	$\{ɕ_3, ɕ_4\}$	$\{ɕ_1, ɕ_3, ɕ_4\}$	$\Xi(\Gamma)$	$\Xi(\Gamma)$
$\{ɕ_1, ɕ_2\}$	$\{ɕ_1, ɕ_2\}$	$\{ɕ_1, ɕ_2\}$	$\{ɕ_1, ɕ_2, ɕ_4\}$	$\Xi(\Gamma)$	$\Xi(\Gamma)$
$\{ɕ_1, ɕ_3\}$	$\Xi(\Gamma)$	$\Xi(\Gamma)$	$\{ɕ_2, ɕ_3, ɕ_4\}$	$\Xi(\Gamma)$	$\Xi(\Gamma)$

Proposition 3.5: Let $\Gamma = (\mathcal{L}(\Gamma), \Xi(\Gamma))$ be a digr and $(\Xi(Ḩ), Çl_{\Gamma_n}))$ be Γ_n $-$closure space if $Ḩ = (\mathcal{L}(\Gamma), \Xi(Ḩ))$, $Ž = (\mathcal{L}(Ž), \Xi(Ž))$ are two subdigr of Γ such that $Ḩ \subseteq Ž \subseteq \Gamma$ then $Çl_{\Gamma_n}(\Xi(Ḩ)) \subseteq Çl_{\Gamma_n}(\Xi(Ž))$ and$\imath n_\Gamma(\Xi(\Gamma)) \subseteq \imath n_\Gamma(\Xi(Ž))$.

Proof: Let $ȩ \in Çl_{\Gamma_n}(\Xi(Ḥ)) \Rightarrow ȩ \in \Xi(Ḥ) \cup \{ç \in \Xi(\Gamma) \setminus \Xi(Ḥ)$; for all $ȩ \in \Xi(Ḥ)$, incidend with ç in the oppsite direection$\} \Rightarrow ȩ \in \Xi(Ḥ)$or $\{ç \in \Xi(\Gamma) \setminus \Xi(Ḥ)$; for all $ȩ \in \Xi(Ḥ)$, incidend with ç in the oppsite direection$\} \Rightarrow ȩ \in \Xi(Ḥ)$ or $\exists h \in \Xi(Ḥ)$; for all $ȩ \in \Xi(Ḥ)$, ȩ incidend with ç in the oppsite direction since $Ḥ \subseteq Ž \Rightarrow ȩ \in \Xi(Ž)$ or $\exists h \in \Xi(Ž)$ or $\exists h \in \Xi(Ž)$; such that ȩ incidend with ç in the oppsite direction $\Rightarrow ȩ \in \Xi(Ž)$ or $\{ç \in \Xi(\Gamma) \setminus \Xi(Ž)$; ȩ incident with ç for all $ȩ \in \Xi(Ž) \Rightarrow ȩ \in \Xi(Ž) \cup \{ç \in \Xi(\Gamma) \setminus \Xi(Ž)$; ȩ incident with ç for all $ȩ \in \Xi(Ž) \Rightarrow ȩ \in Çl_{\Gamma_n}(\Xi(Ž))$. Hence $Çl_{\Gamma_n}(\Xi(Ḥ)) \subseteq Çl_{\Gamma_n}(\Xi(Ž))$. Now, let $y \in ın_{\Gamma_n}(\Xi(\Gamma) = \Xi(\Gamma) \setminus (Çl_{\Gamma_n}\Xi(Ḥ) \Rightarrow$ $y \notin Çl_{\Gamma_n}(\Xi(\Gamma) \setminus \Xi(Ḥ))$, since $\Xi(Ḥ) \subseteq \Xi(Ž) \Rightarrow \Xi(\Gamma) \setminus \Xi(Ž) \subseteq \Xi(\Gamma) \setminus \Xi(Ḥ) \Rightarrow Çl_{\Gamma_n}(\Xi(\Gamma) \setminus \Xi(Ž) \subseteq Çl_{\Gamma_n}(\Xi(\Gamma) \setminus \Xi(Ḥ))$. So $ȩ \notin Çl_{\Gamma_n}(\Xi(\Gamma) \setminus \Xi(Ž)) \Rightarrow y \in \Xi(\Gamma) \setminus Çl_{\Gamma_n}\Xi(\Gamma) \setminus \Xi(Ž)) \Rightarrow ȩ \in ın_{\Gamma_n}(\Xi(Ž)$.Hence $ın_{\Gamma_n}(\Xi(Ḥ)) \subseteq Çl_{\Gamma_n}(\Xi(Ž))$.

Propostion 3.6: Let $\Gamma = (\mathcal{L}(\Gamma), \Xi(\Gamma))$ be a digr $(\Xi(\Gamma), Çl_{\Gamma_n})$ be $\Gamma_n -$ closure space and $Ḥ = (\mathcal{L}(Ḥ), \Xi(Ḥ)), Ž = (\mathcal{L}(Ž), \Xi(Ž))$ be two subdigrs of Γ then

a) $Çl_{\Gamma_n}(\Xi(Ḥ)) \cap \Xi(Ž) \subseteq Çl_{\Gamma_n}(\Xi(Ḥ)) \cap Çl_{\Gamma_n}(\Xi(Ž))$ and

b) $ın_{\Gamma_n}(\Xi(Ḥ)) \cup \Xi(Ž) \subseteq ın_{\Gamma_n}(\Xi(Ḥ)) \cup ın_{\Gamma_n}(\Xi(Ž))$

Proof:

a) Since $\Xi(Ḥ)) \cap \Xi(Ž) \subseteq (\Xi(Ḥ)$ and $\Xi(Ḥ)) \cap \Xi(Ž) \subseteq (\Xi(Ž) \Rightarrow Çl_{\Gamma_n}(\Xi(Ḥ)) \cap \Xi(Ž) \subseteq Çl_{\Gamma_n}(\Xi(Ḥ))$ and $Çl_{\Gamma_n}(\Xi(Ḥ)) \cap \Xi(Ž) \subseteq Çl_{\Gamma_n}(\Xi(Ž))$ $Çl_{\Gamma_n}(\Xi(Ḥ)) \cap \Xi(Ž) \subseteq Çl_{\Gamma_n}(\Xi(Ḥ)) \cap Çl_{\Gamma_n}(\Xi(Ž))$

b) Since $\Xi(Ḥ)) \subseteq (\Xi(Ḥ) \cup (\Xi(Ž))$ and $\Xi(Ž)) \subseteq (\Xi(Ḥ) \cup (\Xi(Ž) \Rightarrow ın_{\Gamma_n}(\Xi(Ḥ)) \subseteq ın_{\Gamma_n}(\Xi(Ḥ) \cup (\Xi(Ž))$ and $ın_{\Gamma_n}(\Xi(Ž)) \subseteq ın_{\Gamma_n}(\Xi(Ḥ) \cup (\Xi(Ž)) \Rightarrow ın_{\Gamma_n}(\Xi(Ḥ)) \cup ın_{\Gamma_n})\Xi(Ž)) \subseteq ın_{\Gamma_n}(\Xi(Ḥ)) \cup (\Xi(Ž))$

Definition 3.7: Let $\Gamma = (\mathcal{L}(\Gamma), \Xi(\Gamma))$ be a digr and $(\Xi(Ḥ), Çl_{\Gamma_n})$ be $\Gamma_n -$ closure space. An open subdigr $Ḥ = (\mathcal{L}(Ḥ), \Xi(Ḥ))$ in $(\Xi(\Gamma), Çl_{\Gamma_n})$is called regular open subdigr (barfly Ʀ$-$ open subdigr) if $\Xi(Ḥ) = IN\Gamma_n(CL\Gamma_n(\Xi(Ḥ))$, the complement of Ʀ$-$ open subdigrr is called Ʀ$-$ closed subdigr of Ḥ is $Cl^{Ʀ}_{\Gamma n}(\Xi(Ḥ)) = \cap\{\Xi(Ħ); \Xi(Ħ)$ is Ʀ$-$ closed subdigr $\Xi(Ḥ) \subseteq \Xi(Ħ)$, and $ın^{R}_{\Gamma_{\Gamma n}}(\Xi(Ḥ)) = \{\Xi(\Gamma) \setminus Çl^{R}_{\Gamma n}(\Xi(\Gamma) \setminus \Xi(Ḥ)$ the family of all Ʀ$-$open subdigr is denoted by $O^{Ʀ}_{\Gamma n}(\mathcal{L}(\Gamma))$.

Example 3.8: Consider the digr in example (3.2) then we have $O^{Ʀ}_{\Gamma n}(\Xi(\Gamma)) = (\Xi(\Gamma), \emptyset, \{ç_3\}, \{ç_2, ç_3, ç_4\}, \{ç_4\}\}$

$$C^{Ʀ}_{\Gamma n}(\Xi(\Gamma)) = \{\Xi(\Gamma), \emptyset, \{ç_1, ç_2, ç_3\}, \{ç_3\}, \{ç_1, ç_2, ç_4\}\}$$

Example 3.9: Consider the digr in example (3.4) then we have $O^{R}_{\Gamma n}(\Xi(\Gamma)) = \{\Xi(\Gamma), \emptyset, \{ɕ_1, ɕ_3, ɕ_4\}\}$

$$Çl^{R}_{\Gamma n}\big(\Xi(\Gamma)\big) = \{\Xi(\Gamma), \emptyset, \{ɕ_2\}\}$$

Definition 3.10: Let $\Gamma = (\mathcal{L}(\Gamma), \Xi(\Gamma))$be a digr $(\mathcal{L}(\Gamma), Çl_{\Gamma_n})$ be, Γ_n −closure space and $Ḩ = (\mathcal{L}(Ḩ), \Xi(Ḩ))$ be a subdigrr $\Gamma = (\mathcal{L}(\Gamma), \Xi(Ḩ))$ then

The accuracy of Ḩ is denoted by $M_{\Gamma n}\Xi(Ḩ) = \frac{|m_{\Gamma n}(\Xi(Ḩ))|}{|Çl_{\Gamma n}(\Xi(Ḩ))|}$ the R −accuracy of Ḩ is denoted by $M^{R}_{\Gamma n}(\Xi(Ḩ) = \frac{|m^{R}_{\Gamma n}(\Xi(Ḩ))|}{|Çl^{R}_{\Gamma n}(\Xi(Ḩ))|}$

Example 3.11: Consider the digr in example (3.2) and example (3.8) the accuracy of any subdigrr is Table 3:

Table 3
Accuracy $M_{\Gamma n}\Xi(Ḩ)$ and $M^{R}_{\Gamma n}(\Xi(Ḩ)$.

$\Xi(H)$	$M_{\Gamma n}\Xi(Ḩ)$	$M^{R}_{\Gamma n}(\Xi(Ḩ)$	$\Xi(H)$	$M_{\Gamma n}\Xi(Ḩ)$	$M^{R}_{\Gamma n}(\Xi(Ḩ)$
$\Xi(\Gamma)$	1	1	$\{ɕ_1, ɕ_4\}$	0	1/4
$\emptyset$	0	0	$\{ɕ_2, ɕ_3\}$	1/4	0
$\{ɕ_1\}$	1/2	0	$\{ɕ_2, ɕ_4\}$	0	1/3
$\{ɕ_2\}$	0	0	$\{ɕ_3, ɕ_4\}$	0	0
$\{ɕ_3\}$	3/4	1	$\{ɕ_1, ɕ_2, ɕ_3\}$	1/3	1
$\{ɕ_4\}$	1	0	$\{ɕ_1, ɕ_3, ɕ_4\}$	0	0
$\{ɕ_1, ɕ_2\}$	1/2	1/4	$\{ɕ_1, ɕ_2, ɕ_4\}$	0	1/4
$\{ɕ_1, ɕ_3\}$	1/2	0	$\{ɕ_2, ɕ_3, ɕ_4\}$	1	0

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Received June, 2024