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# Micro-generalized regular-minimal closed sets in micro-topological spaces

Nabila I. Aziz *
*Department of Physics*
*College of Education-Tuzkhurmatu*
*Tikrit University*
*Tikrit 34001*
*Iraq*

Siham I. Aziz †
*Ministry of Education General Directorate*
*Kirkuk Province Education*
*Kirkuk 36001*
*Iraq*

**Abstract**

The goal of this research is to create and investigate a new class of sets in micro-topological-spaces called micro-minimaḷ-closed-sets and micro-generalized regular-minimaḷ-closed sets are examined for their basic features.

***Subject Classification (2020):*** *18F60, 28C15, 54A05.*

***Keywords:*** *Micro-min-open sets, Micro-min-closed sets, Micro-generalized-regular-min-closed sets.*

## 1. Introduction

Beginning Ṇ. Levine [5] inserts some things in semi-in 1963, open sets. The term generalized semiclosed sets was coined in1990 the first developed by Arya et al. [8]. [3] Some properties of maximal closed and minimal closed sets,

* *ORCID-ID:* 0000-0003-2686-8580
† *ORCID-ID:* 0000-0002-6828-6418

* *E-mail:* nabila.be@tu.edu.iq (Corresponding Author)
† *E-mail:* suham_70100@yahoo.com

published in 2006, were discovered N. Oda and F. Nakaoka. S. S. Benchalli [2] In the year 2010, generalized minimal closed sets in topological spaces were developed. S. Bhattacharya [9] wrote a paper in 2011 on generalized minimal closed sets. Thivagar M. [10] was the first to demonstrate nano topology in 2013. In 2017, Bhuvaneswari and colleagues [1] proposed and looked into nano-generalized semi-closed sets S. Chandrasekar [7] published. On micro topological spaces In 2019, it was announced that the notion of Nano topology would be extended to Micro topology. In the year 2020, Micro-open sets were added to Micro topology by H. Z. Ibrahim [4]. A novel class of se𝑡s in micro-topological-spaces called micro-minimaḷ-closed-se𝑡s and micro-generalized-regular-minimaḷ-closed se𝑡s., as well as some of their properties, are introduced in these papers [11-20].

## 2. Preliminaries

Let's go over the **Definition** s below, which will come in handy later.

**Definition [7]:** If (ȹ, ȚŖ(X)) If a Ṇano-topological-space, ϺŖ(X) = Đᑌ(Ǥ ∩Ϻ):Đ, Đ∈ ȚŖ(X) is referred to as the Micro topoḷogy of ȚŖ(X) by means of ϺŖ(X)∉ ȚŖ(X).

**Definition [7]:** The following axioms apply to the Micro topology ϺŖ(X)

1. ȹ and ∅ ∈ ϺŖ(X)
2. ϺŖ(X) brings together any sub-items collection's in ϺŖ(X)
3. If ϺŖ(X) is the intersection of the elements of any finite sub collection of ϺŖ(ẋ), then ϺŖ(X) is Micro-topoḷogy on X.

The trinity (ȹ, ȚR(X), ϺŖ(X)) known as Micro-topoḷogy-space, as well as the elements ϺŖ(X) known as Micro-open se𝑡s, whereas a Micro-closed se𝑡s is known as the inverse of a Micro-open se𝑡s.

**Definition [6]:** (ȹ, ȚR(X), ϺŖ(X)) is Micro-topoḷogy-space. If ᶈ = Wic. int.(Wic. cl (ᶈ)), ᶈ is a subset of ȹ, then ᶈ is Micro-regular open. The complement of a micro regular open set of a space ȹ is called micro regular closed set in ȹ.

**Definition [3]:** Wic.R.cl.(ᶈ) is the intersection of all micro regular-closed se𝑡s contains ᶈ.

**Definition [5]:** Minimaḷ-open (resp. minimaḷ-closed) se𝑡 Ȩ is either empty or a non-empty subset of a topological space; that is, any open (resp. closed) subset of X included in Ȩ.

**Definition [2, 9]:** If ÇĮ(ŵ)⊆U, where ŵ ⊆ û and û is a minimal open set in X, then subset ŵ of the topological space X is a generalized minimal closed set.

## 3. Micro generalized regular minimal closed sets in Micro topological spaces.

**Definitions:** (Micro minimal, Micro generalized regular minimal) closed se$t$s, as well as some of their features, are covered in this section.

[micro-topological-space, micro-open-micro-closed, Micro-minimal-closed,Micro-minimal regular-closed, micro generalized-regular-closed, micro generalized-minimal-regular closed ]se$t$s is referred $t$o as [(ф, ȚR(X), MȒ($\mathcal{X}$)), Ϻ-o (X), Ϻ-c(X),Ϻmin − o($\mathcal{X}$), Ϻmin − c(X), Ϻmin − R − c(X), ɱic-g-R - C(X), ɱic-g-min-R-C(X)]

**Definition:** Micro minimal Open set (in contrast to Micro minimal Closed) is Micro-open (as opposed to Micro-closed)↔ subse$t$ of ф included in ꝑ either ꝑ or ∅, it is an element in a micro topological space that is not empty (ф, ȚR(X), MȒ(X))**.**

**Example:** Assume that Ħ ={ϖ, Ҳ, Ч,z}, Ħ /Ʀ={x},{Ч},{ϖ,z}, X ={ϖ,Ҳ}.,. ӎ ={ϖ} Ț R($\mathcal{X}$)={∅, Ħ,{x},{ϖ,z},{x,ϖ,z}}, MȒ(X)={∅, Ħ,{ϖ},{x},{ϖ, x},{ϖ,z},{x,ϖ,z}}

Ϻmin − o(X) ={{ϖ},{x}}
Ϻmin − c(X) ={{Ч}}
ϺR − c(X) ={∅, Ħ,{x},{ϖ,z}}

**Example:** If we takeϺ={Ч} for instance (3.2) the micro-open se$t$s is

ϺR(X)={∅, Ħ,{Ч},{x},{Ч, x},{ϖ,z},{Ч, ϖ,z}, {x, ϖ, z}}
Ϻmin − o(X) ={{Ч},{x},{ϖ,z}}
ϺR-c(x)= {∅, Ħ,{Ч},{x},{Ч, x},{ϖ,z},{Ч, ϖ,z}, {x, ϖ, z}}

**Definition:** If Mic. Rcl (Ҝ) ⊆Ƕ, Ҝ⊆Ƕ, Ƕ is micro-open-se$t$s, Ҝ ⊆ (ф, ȚR(X), MȒ(X)) is a ɱic-g-R -c(X).

**Example**: The Micro generalized Regular closed sets (3.2) is a good example.

ϺR(X)={∅, Ħ,{Ч},{x},{Ч, x},{ϖ,z},{Ч, ϖ,z}, {x,ϖ,z}}
ɱic-g-R-C(X)={∅, Ħ, {x},{ϖ},{z},{ϖ,z},{Ч, x,z}}

**Definition:** If Mic. Rcl (Ҝ) ⊆Ƕ, Ҝ⊆Ƕ, Ƕ is Ϻmin − o(ẋ), Ҝ ⊆ (ф, ȚR(X), MȒ(X)),Ҝ is a ɱic-g-min-R-c(X). Recall example3.2: ɱic-g-min-R-C(X)={∅, {x},{ϖ},{z}}

**Theorem:**

1. *Assume that X is a ɱic-g-min-R c(X) and that Y is a ɱic-g-s c(X).Then either X ∩Y =∅ or X ⊂ Y.*

2. *Allow for a ɱic-g-min-R c(X) X and W. Then either X ∩W =∅ or X =W.*

***Proof:***

1. Assume Y is ɱic-g-R c(X), X ∩Y ≠ ∅. Because X is ɱic-g-min-R c(X), and Since X ∩Y Subset from X, X ∩ Y is equal X holds. Therefore X subset from Y. It follows that each and every ɱic-g-min-R-c(X) is also a ɱic-g-R-c(X)
2. X,Y are ɱic-g-min-R c(X), X ∩W ≠ ∅, X ⊂ W, W ⊂ X by (1). Therefore X=W.

**Theorem:** *Ɱ ⊂ (Ü, ƬR(ẋ), ⱮṚ(ẋ)), Ɱ is ɱic-g-min-R c(X), if it is* $\text{Ɱ}min - c(X)$ *and Micro-regular-closed set.*

***Proof:*** Assume that the collection Ʊ is $\text{Ɱ}min - o(X)$ with Ɱ and ɱic-R c(X). So ɱic. Rcl (Ɱ) ⊆ ɱic. Rcl (Ʊ) = Ʊ ( Because Ʊ is a ɱic-R c(X).). ⇒. ɱic. Rcl (Ɱ) ⊆ (Ʊ) for Ɱ⊆Ʊ, Ʊ is $\text{Ɱ}min - o(X)$.Then, according to the definition ɱic-g-min-R c(X), Ɱ is ɱic-g-min-R c(X).

**Proposition**: (ȹ, ƬR(X), ⱮṚ(X)) Micro-topological space, W ⊂ ȹ and W is ɱic-g-min-R c(X) ⇒W is ɱic-g-R c(X).

***Proof:*** W is the closed-set object. that is ɱic-g-min-R c(X). We have a ɱic-g-min-R c(X), by definition . A subset ᵹ of Mic.Rcl (W), W inside ᵹ and ᵹ is $\text{Ɱ}min - o(X)$. ᵹ, a tiny open set, on the other hand, because each $\text{Ɱ}min - o(X)$ is a $\text{Ɱ} - o(X)$. As a result, Mic. Rcl ( W) ⊆ ᵹ, W ⊆ ᵹ, ᵹ stands for $\text{Ɱ} - o(X)$ As a result, W is ɱic-g-min-R-c(X).

**Remark:** Proposition 3.9's inverse does not have to be true. As shown in example 3.5. {x,Ч,z}is a ɱic-g-R-c(X), but not ɱic-g-min-R-c(X).

ɱic-g-R-C(X)={∅, Ħ, {x},{ϖ},{z},{ϖ,z},{Ч, x,z}}
ɱic-g-min-R-c(X)= {∅, Ħ, {x},{ϖ}}

**Theorem:** *Consider the (Ü, ƬR(ẋ), ⱮṚ(ẋ)) micro-topological space . Then ∅, despite the fact that is ɱic-g-min-R c(X), U isn't.*

***Proof:*** Since Mic. Rcl (∅) = ∅ ⊆ $\text{Ɱ}min - o(X)$ ⊆ ∅. ∅ is ɱic-g-min-R c(X). But U ⊈ $\text{Ɱ}min - o(X)$, ⇒ U isn't ɱic-g-min-R c(X).

**Theorem** *(p) and (q) be subsets in (ȹ, ƬR(X), ⱮṚ(X)). If (p) and (q) are ɱic-g-min-R c(X), then (p) ∩(q) is ɱic-g-min-R c(X) in (ȹ, ƬR(X), ⱮṚ(X)).*

***Proof***: Assume (p) and (q) are two ɱic-g-min-R c(X) in (ȸ, ŢR(X), ɱŖ(X)). Allow Ʊ to be a Ɱmin − o(X) in (Ü, ŢR(ẋ), ɱŖ(ẋ)) such that p ⊆ Ʊ and q ⊆ Ʊ, it suggests that (p) ∩(q) ⊆ Ʊ As (p) and (q) are is ɱic-g-min-R c(X) in (ȸ, ŢR(X), ɱŖ(X))., Mic. Rcl (p) ⊆ Ʊ and Mic. Rcl ( q)⊆ Ʊ. Now, Mic. Rcl ( p∩q) = Mic. Rcl (p) ∩ Mic. Rcl ( q) ⊆ Ʊ.⇒ Mic. Rcl ( p ∩q) ⊆ Ʊ, (p) ∩(q) ⊆ Ʊ and Ʊ is Ɱmin − o(X) in (ȸ, ŢR(X), ɱŖ(X)). ⇒ (p) ∩ (q) is ɱic-g-min-R c(X) .

**Remark:** A ɱic-g-min-R c(X) does not always follow from the union of two ɱic-g-min-R-c(X). Let's use the number 3.5 as an example. ῆ = {ϖ}, Б= {x}, ῆ ∪ Б = {ϖ}∪ {x}= {ϖ, x}.

**Theorem:** *W is ɱic-g-min-R c(X) subset of (ȸ, ŢR(X), ɱŖ(X)). If W⊆ Ḅ ⊆Mic.Rcl (W) is true, then Ḅ is an ɱic-g-min-R c(X) subset of (ȸ, ŢR(X), ɱŖ(X)).*

***Proof:*** Assume ̋Ω is a Ɱmin − o(X) subset of a ɱic-g-min-R c(X) subset of (ȸ, ŢR(X), ɱŖ(X))., Ꝁ ⊆ Ǥ., W ⊆ Ꝁ, ⇒ W ⊆Ǥ. Due to the fact that W is a ɱic-g-min-R c(X), Mic. scl (W) ⊆ Ǥ. ⇒ ꝀMic. Rcl (W), ⇒ Mic. Rcl (Ꝁ) ⊆ Mic. Rcl ( W). But Mic. Rcl (W) ⊆ Ǥ, ⇒ Mic. Rcl ( Ꝁ) ⊆ Ǥ, Ꝁ ⊆ Ǥ, Ǥ is Ɱmin − o(X). As a result, Ꝁ is a ɱic-g-min-R c(X), ⊆ (ȸ, ŢR(X), ɱŖ(X)).

**Theorem**: *If ̋Ω is ɱic-g-min-R c(X)in (ȸ, ŢR(X), ɱŖ(X)). Then for each X ∈ Mic Rcl (̋Ω), Mic. Rcl {X} ∩(̋Ω) ≠∅.*

***Proof:*** Choose any ̋Ω is ɱic-g-min-R c(X) in (ȸ, ŢR(X), ɱŖ(X)). in order that ∀ X ∈ Mic. Rcl (̋Ω). Let Mic. Rcl {X} ∩ (̋Ω) =∅.Then ̋Ω⊆[ Mic. Rcl $\{X\}]^c$, [Mic. Rcl $\{X\}]^c$ is a Ɱmin − o(X) in (Ü, ŢR(ẋ), ɱŖ(X)). By the Theorem مععه3.9, ̋Ω is ɱic-g-min-R c(X) ⇒ Mic. scl (̋Ω) ⊆ Mic. Rcl {X}, It is in direct opposition to the fact that X ∈ Mic. Rcl (Ā) ⇒ Mic. Rcl {X} ∩(̋Ω) ≠∅.

## 4. Conclusions

A new kind of open set, called (micro-minimaļ-closed-se𝑡s and micro-generalized regular-minimaļ-closed se𝑡s in micro topological spaces and study some of their properties with some important theorems. As well as the definition can generalized on (semi, pre, α, β and b) open sets in micro topological spaces.

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