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JD-paracompact space

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Abstract

The concepts of paracompact and ideal are two important mathematical notions in topology. They have a significant role in applied science. In light of these two concepts, the idea of merging them into one concept, which we called ID-paracompact space has emerged. Its most important algebraic and topological properties and its relationships to I-paracompact space and paracompact space were studied. In addition, we discussed the effect of some different functions on preserving the ID-paracompact space property.

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1. Introduction

In 1944, Dieudonne [5] first defined the paracompact space based on the concepts of local finiteness and refinement. The ideal is considered one of the important mathematical concepts that Kuratwaski [7] defined in 1933. He also presented a new definition, the local function, and based on the concepts of topology and ideal, his idea crystallized as follows: $Ą^*(\mathcal{J}, \mathfrak{T}) = \{x : Ą \cap Ű \notin \mathcal{J},$ for every open set $Ű$ of $x\}$. By defining the local function, Jankovic and Hamlett [6] proposed a broader topology than the one known as local function, where base is the basis of this topology $\mathfrak{T}^*(\mathcal{J})$, which is defined as follows $\beta(\mathcal{J}, T) = \{V - Ą : V \in \mathfrak{T}, Ą \in \mathcal{J}\}$, the ideal $\mathcal{J}$ is called $\mathfrak{T}$ -simple, if $\beta = \mathfrak{T}^*$. There is a group of ideal families: $*\mathcal{J}_p = \{\mathfrak{N} \subseteq X : p \notin \mathfrak{N}\}$, $*\mathcal{J}_f$, the ideal of all finite subsets of X, $*\mathcal{J}_{co}$, the ideal of all countable subsets of X.

Zahid [15] was first researcher to generalize paracompact for ideal, whereas the concept of almost paracompact space was defined by Singia [12, 13] who studied its most important characteristics. Many researchers such as [1-4, 10, 11, 14] explored such a variety types of paracompact and compact as: Semiparacompactness and z-paracompactness, and Compactness with Gem set, s-paracompactness, $\beta - \mathcal{J}$-paracompact, α-S-paracompact (mod $\mathcal{J}$), perparacompactness.

The current study consists of two sections. The first section surveys the fundamental concepts where as in the second section we introduce a new definition of ideal, namely $\mathcal{J}$ -refinement, $\mathcal{J}$-locally finite and $\mathcal{J}D$-paracompact space. In addition, useful characterizations are given for $\mathcal{J}D$-paracompact space.

2. Preliminaries

Definition 2.1: [8]. Let $\mathcal{J}$ be ideal on set X, then $\mathcal{J}$ is called maximal ideal on X, if $\mathcal{J}$ is not contained in any other ideal on X.

Theorem 2.2: [8]. *Let $(X, \mathfrak{T}, \mathcal{J})$ be ideal topological space. Then $\mathcal{J}$ is maximal ideal if and only if for any subsets $\mathfrak{D}$ and $\mathfrak{E}$ of X such that $\mathfrak{D} \cap \mathfrak{E} \in \mathcal{J}$, we have either $\mathfrak{D} \in \mathcal{J}$ or $\mathfrak{E} \in \mathcal{J}$.*

Proposition 2.3: [8]. Let $(X, \mathfrak{T}, \mathcal{J})$ be ideal topological space. Then $\mathcal{J}$ is maximal ideal if and only if for each subset $\mathfrak{D}$ of X, then either $\mathfrak{D} \in \mathcal{J}$ or $X - \mathfrak{D} \in \mathcal{J}$.

Lemma 2.4: [9]. *Let $\mathfrak{F} : X \to Y$ be function then:*

i. *If $\mathfrak{F}$: $(X, \mathfrak{T}, \mathcal{J}) \to (Y, \ddot{v})$ is one to one function, then $\mathfrak{F}(\mathcal{J}) = \{\mathfrak{F}(Ą) : Ą \in \mathcal{J}\}$ is an ideal on Y.*

ii. *If $\mathfrak{F}$: $(X, \mathfrak{T}) \to (Y, \ddot{v}, ɟ)$ is one to function, then $\mathfrak{F}^{-1}(ɟ) = \{\mathfrak{F}^{-1}(B) : B \in ɟ\}$ is an ideal on X.*

A collection $\mathfrak{U} = \{\mathfrak{U}_\lambda : \lambda \in \Delta\}$ of subsets of topological space $(\mathfrak{X}, \mathfrak{T})$ is called Locally finite, if each point of $\mathfrak{X}$ has a neighborhood $\mathfrak{K}$ such that $\mathfrak{K} \cap \mathfrak{U}_\lambda \neq \emptyset$, for at most finitely many indices λ [5]. A space $(\mathfrak{X}, \mathfrak{T})$ is named paracompact, if each open cover $\mathcal{H}$ of $\mathfrak{X}$ has a Locally finite open refinement $\mathcal{V}$ [5]. A space $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$, we say $\mathcal{J}$ is $\mathfrak{T}$-boundary, if $\mathcal{J} \cap \mathfrak{T} = \{\emptyset\}$ [9]. A space $(\mathfrak{X}, \mathfrak{T})$ is named almost paracompact, if each open cover $\mathfrak{V}$ of $\mathfrak{X}$ has a Locally finite open refinement $\mathcal{W}$ with $\mathfrak{X} = \mathrm{cl} \bigcup\{\mathrm{W} : \mathrm{W} \in \mathcal{W}\}$ [12].

Definition 2.5: [15]. A ideal topological space $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$ is called $\mathcal{J}$-paracompact, if each open cover $\mathfrak{H}$ of $\mathfrak{X}$ has a Locally finite open refinement $\mathcal{W}$ (not necessary a cover of $\mathfrak{X}$) with $\mathfrak{X} - \bigcup\{W : W \in \mathcal{W}\} \in \mathcal{J}$.

3. $\mathcal{J}D$-Paracompact space

In this work, we introduce the definitions of called $\mathcal{J}$-refinement, $\mathcal{J}$-Locally finite and $\mathcal{J}D$-paracompact and show its most important properties, particularly union and intersection and relationship between them.

Definition 3.1: Let $\mathfrak{H} = \{\mathfrak{H}_\alpha : \alpha \in \Lambda\}$ and $\mathfrak{W} = \{\mathfrak{W}_\beta : \beta \in \varphi\}$ be two covering of ideal topological space $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$, $\mathfrak{W}$ is called $\mathcal{J}$-refinement of $\mathfrak{H}$, if for each $\mathfrak{W}_\beta$ of $\mathfrak{W}$ there exists $\mathfrak{H}_\alpha$ of $\mathfrak{H}$ such that $\mathfrak{W}_\beta - \mathfrak{H}_\alpha \in \mathcal{J}$.

Definition 3.2: Let $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$ be ideal space. A collection $\mathfrak{V} = \{\mathfrak{V}_\delta : \delta \in \Omega\}$ of subset of $\mathfrak{X}$ is said to be $\mathcal{J}$-Locally finite, if each point of $\mathfrak{X}$ has a neighborhood $\mathfrak{E}$ such that $\mathfrak{E} \cap \mathfrak{V}_\delta \notin \mathcal{J}$, for at most finitely many indices δ.

Remark 3.3:

i. Every refinement is $\mathcal{J}$-refinement for any ideal.

ii. Every $\mathcal{J}$-Locally finite is Locally finite for any ideal.

The opposite of Remark (3. 3, i, ii) is not generally true, as shown in the following examples.

Example 3.4: Suppose that $\mathcal{A} = \{\{x\} : x \in \mathfrak{X}\}$ and $\mathfrak{E} = \{\{x, y\} : x, y \in \mathfrak{X}\}$ be covers of discrete space $(\mathfrak{X}, \mathfrak{T})$, then $\mathfrak{E}$ is not refinement of $\mathcal{A}$, but $\mathfrak{E}$ is $\mathcal{J}_f$-refinement of $\mathcal{A}$.

Example 3.5: Let $(\mathcal{R}, \mathfrak{T})$ be discrete space and $\mathcal{J}_f$ be ideal on $\mathcal{R}$, where $\mathfrak{B} = \{\{x\} : x \in \mathcal{R}\}$, then $\mathfrak{B}$ is Locally finite in $\mathcal{R}$ but $\mathfrak{B}$ is not $\mathcal{J}_f$-Locally finite in $\mathcal{R}$.

Remark 3.6: If $\mathcal{J} = \{\emptyset\}$, then the concepts of refinement (resp. Locally finite) and $\mathcal{J}$-refinement (resp. $\mathcal{J}$-Locally finite) are equivalent.

Definition 3.7: A ideal topological space $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$ is called $\mathcal{J}D$-paracompact, if every open cover $\mathfrak{V} = \{\mathfrak{V}_\alpha : \alpha \in \Lambda\}$ of $\mathfrak{X}$ has $\mathcal{J}$-Locally finite open $\mathcal{J}$-refinement $\mathfrak{W} = \{\mathfrak{W}_\lambda : \lambda \in \Delta\}$ (not necessary a cover of $\mathfrak{X}$) such that $\mathfrak{X} - \bigcup_{\lambda\in\Delta} \mathfrak{W}_\lambda \in \mathcal{J}$.

Example 3.8: Let $(\mathcal{R}, \mathfrak{T})$ be usual space and $\mathcal{J}_p$ be ideal on $\mathcal{R}$, then $(\mathcal{R}, \mathfrak{T})$ is $\mathcal{J}_p D$-paracompact space, $p \in \mathcal{R}$.

Remark 3.9:

i. It clear that, $(\mathfrak{X}, \mathfrak{T})$ is paracompact (resp. $\mathcal{J}$-paracompact) if $(\mathfrak{X}, \mathfrak{T})$ is $\{\emptyset\}D -$ paracompact space such that $\mathcal{J} = \{\emptyset\}$.
ii. The topological space $(\mathfrak{X}, \mathfrak{T})$ is almost paracompact, if $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$ is $\mathcal{J}D$-paracompact with $\mathcal{J} = \{\emptyset\}$.

Proposition 3. 10: Let $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$ be $\mathcal{J}D$-paracompact space and $\mathcal{J}$ be a maximal ideal on $\mathfrak{X}$. If $\mathfrak{F}$ is closed subspace of $\mathfrak{X}$ such that $\mathfrak{F} \notin \mathcal{J}$, then $\mathfrak{F}$ is $\mathcal{J}D$-paracompact subspace.

Proof: Let $\mathfrak{F}$ be closed subspace of $\mathcal{J}D$-paracompact space $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$ and $\mathfrak{H} = \{\mathfrak{H}_\beta : \beta \in \Omega\}$ be open cover of $\mathfrak{F}$, there is open set $\mathfrak{K}_\beta$ of $\mathfrak{X}$ such that $\mathfrak{H}_\beta = \mathfrak{F} \cap \mathfrak{K}_\beta$, thus $\{\mathfrak{K}_\beta : \beta \in \Omega\}$ is open cover of $\mathfrak{B}$ and $\mathfrak{H}^* = \{\mathfrak{K}_\beta : \beta \in \Omega\} \cup (\mathfrak{X} - \mathfrak{F})$ is open cover of $\mathfrak{X}$. Given $\mathfrak{X}$ is $\mathcal{J}D$-paracompact space, there is $\mathcal{J}$-Locally finite open $\mathcal{J}$-refinement $\mathfrak{W} = \{\mathfrak{V}_\lambda : \lambda \in \Lambda\} \cup \{\mathfrak{V}\}$ $(\mathfrak{V}_\lambda - \mathfrak{K}_\beta \in \mathcal{J}$ and $\mathfrak{V} - (\mathfrak{X} - \mathfrak{F}) \in \mathcal{J})$ such that $\mathfrak{X} - (\bigcup_{\lambda\in\Omega} \mathfrak{V}_\lambda \cup \{\mathfrak{V}\}) \in \mathcal{J}$.

To prove that $\mathfrak{W}^* = \{\mathfrak{F} \cap \mathfrak{V}_\lambda : \mathfrak{V}_\lambda \in \mathfrak{V}, \lambda \in \Lambda\}$ is $\mathcal{J}$-Locally finite $\mathcal{J}$-refinement of $\mathfrak{H}$, let $\mathfrak{F} \cap \mathfrak{V}_\lambda \in \mathfrak{W}^* \ni \mathfrak{V}_\lambda \in \mathfrak{W}, \lambda \in \Lambda$, since $\mathfrak{W}$is $\mathcal{J}$-refinement of $\mathfrak{H}^*$, there is $\mathfrak{K}_\beta \in \mathfrak{H}^*$ such that $\mathfrak{V}_\lambda - \mathfrak{K}_\beta \in \mathcal{J}$. But $(\mathfrak{F} \cap \mathfrak{V}_\lambda) - \mathfrak{K}_\beta \subseteq \mathfrak{V}_\lambda - \mathfrak{K}_\beta$ and $(\mathfrak{F} \cap \mathfrak{V}_\lambda) - \mathfrak{H}_\beta = (\mathfrak{F} \cap \mathfrak{V}_\lambda) - \mathfrak{K}_\beta$, then $(\mathfrak{F} \cap \mathfrak{V}_\lambda) - \mathfrak{H}_\beta \in \mathcal{J}$, thus $\mathfrak{W}^*$ is $\mathcal{J}$-refinement of $\mathfrak{H}$. Suppose that $x \in \mathfrak{X}$, but $\mathfrak{W}$ is $\mathcal{J}$-Locally finite of $\mathfrak{X}$, there is neighborhood $\mathfrak{N}$ of x such that $\mathfrak{N} \cap (\mathfrak{V}_\lambda \cup \{\mathfrak{V}\}) \notin \mathcal{J}$, for at most finitely many indices λ. Since $\mathfrak{F} \notin \mathcal{J}$ and $\mathcal{J}$ be maximal ideal on $\mathfrak{X}$, then by Theorem (2. 2), $\mathfrak{N} \cap (\mathfrak{F} \cap \mathfrak{V}_\lambda) \notin \mathcal{J}$, for at most finitely many indices λ. But $\mathfrak{F} - \bigcup_{\lambda\in\Lambda}(\mathfrak{F} \cap \mathfrak{V}_\lambda) = \mathfrak{F} - \bigcup_{\lambda\in\Omega} \mathfrak{V}_\lambda$ and $\mathfrak{F} - \bigcup_{\lambda\in\Lambda} \mathfrak{V}_\lambda \subseteq \mathfrak{X} - \bigcup_{\lambda\in\Lambda} \mathfrak{V}_\lambda$, then $\mathfrak{F} - \bigcup_{\lambda\in\Lambda}(\mathfrak{F} \cap \mathfrak{V}_\lambda) \in \mathcal{J}$. Therefore $\mathfrak{F}$ is $\mathcal{J}D$-paracompact subspace.

Theorem 3.11: *A space $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$ is $\mathcal{J}D$-paracompact, if $(\mathfrak{X}, \mathfrak{T}^*, \mathcal{J})$ is $\mathcal{J}D$-paracompact space defined on maximal $\mathfrak{T}$ -simple ideal $\mathcal{J}$ on $\mathfrak{X}$.*

Proof: It is simple to prove by using the definitions of $\mathcal{J}D$-paracompact space and $\mathfrak{T}$ -simple, Theorem (2. 2), and Proposition (2. 3).

Proposition 3.12: If $\mathfrak{E}$ is closed subset of maximal ideal topological space $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$ and $\mathfrak{K}$ is $\mathcal{J}D$-paracompact subset of $\mathfrak{X}$ such that $\mathfrak{E} \notin \mathcal{J}$, then $\mathfrak{E} \cap \mathfrak{K}$ is $\mathcal{J}D$-paracompact set in $\mathfrak{X}$.

Proof: Let $\mathfrak{V} = \{\mathfrak{V}_\beta : \beta \in \Omega\}$ be open cover of $\mathfrak{E} \cap \mathfrak{K}$ in $\mathfrak{X}$, then $\mathfrak{K} \subseteq \bigcup_{\beta \in \Omega} \mathfrak{V}_\beta \cup (\mathfrak{X} - \mathfrak{E})$, thus $\mathfrak{V}^* = \bigcup_{\beta \in \Omega} \mathfrak{V}_\beta \cup (\mathfrak{X} - \mathfrak{E})$ is open cover of $\mathfrak{K}$. Given $\mathfrak{K}$ is $\mathcal{J}D$-paracompact set in $\mathfrak{X}$, there is $\mathcal{J}$-Locally finite open $\mathcal{J}$-refinement $\mathcal{U} = \{\mathcal{U}_\eta : \eta \in \Delta\} \cup (\mathfrak{X} - \mathfrak{E})$ of $\mathfrak{V}^*$ in $\mathfrak{X}$ such that $\mathfrak{K} - (\bigcup_{\eta \in \Delta} \mathcal{U}_\eta \cup (\mathfrak{X} - \mathfrak{E})) \in \mathcal{J}$. Let $\mathcal{U}^* = \{\mathcal{U}_\eta : \mathcal{U}_\eta \in \mathcal{U}, \eta \in \Delta\}$, to prove that $\mathcal{U}^*$ is $\mathcal{J}$-Locally finite $\mathcal{J}$-refinement of $\mathfrak{V}$. Let $\mathcal{U}_\eta \in \mathcal{U}^* \ni \mathcal{U}_\eta \in \mathcal{U}, \eta \in \Delta$, then $\mathcal{U}_\eta \cup (\mathfrak{X} - \mathfrak{E}) \in \mathcal{U}$, then there exists $\mathfrak{V}_\beta \cup (\mathfrak{X} - \mathfrak{E}) \in \mathfrak{V}^*$ such that $(\mathcal{U}_\eta \cup (\mathfrak{X} - \mathfrak{E})) - (\mathfrak{V}_\beta \cup (\mathfrak{X} - \mathfrak{E})) \in \mathcal{J}$. But $(\mathcal{U}_\eta \cup (\mathfrak{X} - \mathfrak{E})) - (\mathfrak{V}_\beta \cup (\mathfrak{X} - \mathfrak{E})) = \mathcal{U}_\eta \cap (\mathfrak{X} - \mathfrak{V}_\beta) \cap \mathfrak{E}$ and $\mathfrak{E} \notin \mathcal{J}$, then by Theorem (2. 2), $\mathcal{U}_\eta - \mathfrak{V}_\beta \in \mathcal{J}$. Hence $\mathcal{U}^*$ is $\mathcal{J}$-refinement of $\mathfrak{V}$.

Assume that $x \in \mathfrak{X}$, there is neighborhood $\mathfrak{N}$ of x such that $\mathfrak{N} \cap (\mathcal{U}_\eta \cup (\mathfrak{X} - \mathfrak{E})) \notin \mathcal{J}$, for at most finitely many indices η. Since $\mathfrak{E} \notin \mathcal{J}$, then by Proposition (2. 3), $\mathfrak{X} - \mathfrak{E} \in \mathcal{J}$, but $\mathfrak{N} \cap (\mathfrak{X} - \mathfrak{E}) \subseteq (\mathfrak{X} - \mathfrak{E})$, thus $\mathfrak{N} \cap (\mathfrak{X} - \mathfrak{E}) \in \mathcal{J}$, then $\mathfrak{N} \cap \mathcal{U}_\eta \notin \mathcal{J}$, for at most finitely many indices η, then $\mathcal{U}^*$ is $\mathcal{J}$-Locally finite of $\mathfrak{X}$. But $\mathfrak{K} - \bigcup_{\eta \in \Delta}(\mathcal{U}_\eta \cup (\mathfrak{X} - \mathfrak{E})) = (\mathfrak{E} \cap \mathfrak{K}) - \bigcup_{\eta \in \Delta} \mathcal{U}_\eta$, then $(\mathfrak{E} \cap \mathfrak{K}) - \bigcup_{\eta \in \Delta} \mathcal{U}_\eta \in \mathcal{J}$. Therefore $\mathfrak{E} \cap \mathfrak{K}$ is $\mathcal{J}D$-paracompact set in $\mathfrak{X}$.

Proposition 3.13: Let $\mathfrak{g}: (\mathfrak{X}, \mathfrak{T}, \mathcal{J}) \to (\mathcal{Y}, \Upsilon, \mathfrak{g}(\mathcal{J}))$ be homeomorphism function. If $(\mathfrak{X}, \mathfrak{T})$ is $\mathcal{J}D$-paracompact space, then $(\mathcal{Y}, \Upsilon)$ is $\mathfrak{g}(\mathcal{J}))$ D-paracompact space.

Proof: Assume that $\mathfrak{H} = \{\mathfrak{H}_\lambda : \lambda \in \Lambda\}$ be open cover of $\mathcal{Y}$, so $\mathfrak{H}^* = \{\mathfrak{g}^{-1}(\mathfrak{H}_\lambda) : \lambda \in \Lambda\}$ is open cover of $\mathfrak{X}$, given $\mathfrak{X}$ is $\mathcal{J}D$-paracompact space, then $\mathfrak{H}^*$ has $\mathcal{J}$-Locally finite open $\mathcal{J}$-refinement $\mathfrak{V} = \{\mathfrak{V}_\alpha : \alpha \in \Delta\}$ of $\mathfrak{H}^*$ in $\mathfrak{X}$ such that $\mathfrak{X} - \bigcup_{\alpha \in \Delta} \mathfrak{V}_\alpha \in \mathcal{J}$. Suppose that $\mathfrak{V}^* = \{\mathfrak{g}(\mathfrak{V}_\alpha) : \mathfrak{V}_\alpha \in \mathfrak{V}, \alpha \in \Delta\}$, to prove that $\mathfrak{V}^*$ is $\mathcal{J}$-Locally finite $\mathcal{J}$-refinement of $\mathfrak{H}$, let $y \in \mathcal{Y}$, there is $x \in \mathfrak{X}$ with $\mathfrak{g}(x) = y$, there is neighborhood $\mathfrak{D}$ of x such that $\mathfrak{D} \cap \mathfrak{V}_\alpha \notin \mathcal{J}$, for at most finitely many indices α, thus $\mathfrak{g}(\mathfrak{D} \cap \mathfrak{V}_\alpha) \notin \mathfrak{g}(\mathcal{J})$, then $\mathfrak{g}(\mathfrak{D}) \cap \mathfrak{g}(\mathfrak{V}_\alpha) \notin \mathfrak{g}(\mathcal{J})$, thus $\mathfrak{V}^*$ is $\mathfrak{g}(\mathcal{J})$-Locally finite of $\mathcal{Y}$. Let $\mathfrak{g}(\mathfrak{V}_\alpha) \in \mathfrak{V}^* \ni \mathfrak{V}_\alpha \in \mathfrak{V}, \alpha \in \Delta$, there is $\mathfrak{g}^{-1}(\mathfrak{H}_\lambda) \in \mathfrak{H}^*, \lambda \in \Lambda$ such that $\mathfrak{V}_\alpha - \mathfrak{g}^{-1}(\mathfrak{H}_\lambda) \in \mathcal{J}$, thus $\mathfrak{g}(\mathfrak{V}_\alpha) - \mathfrak{H}_\lambda \in \mathfrak{g}(\mathcal{J})$. Since $\mathfrak{X} - \bigcup_{\alpha \in \Delta} \mathfrak{V}_\alpha \in \mathcal{J}$, then $\mathfrak{g}(\mathfrak{X} - \bigcup_{\alpha \in \Delta} \mathfrak{V}_\alpha) \in \mathfrak{g}(\mathcal{J})$, thus $\mathcal{Y} - \bigcup_{\alpha \in \Delta} \mathfrak{g}(\mathfrak{V}_\alpha) \in \mathfrak{g}(\mathcal{J})$. Hence $(\mathcal{Y}, \Upsilon)$ is $\mathfrak{g}(\mathcal{J})D$-paracompact space.

Proposition 3. 14: Let $\mathfrak{h}: (\mathfrak{X}, \mathfrak{T}) \to (\mathcal{Y}, \Upsilon)$ be homeomorphism function. If $(\mathfrak{X}, \mathfrak{T}, \mathcal{J}_1)$ is $\mathcal{J}_1 D$-paracompact space such that $\mathcal{J}_2 = \{\mathfrak{C} \subseteq \mathcal{Y} : \mathfrak{h}^{-1}(\mathfrak{C}) \in \mathcal{J}_1\}$, then $(\mathcal{Y}, \Upsilon, \mathcal{J}_2)$ is $\mathcal{J}_2 D$-paracompact space.

Proof: Similar proof to the Proposition (3. 13).

Proposition 3.15: If $\mathfrak{C}$ and $\mathfrak{D}$ are $\mathcal{J}D$-paracompact sets in ideal space $(\mathfrak{X}, \mathfrak{T}, \mathcal{J})$, then $\mathfrak{C} \cup \mathfrak{D}$ is $\mathcal{J}D$-paracompact set in $\mathfrak{X}$. since $\mathfrak{H}$ and $\mathcal{U}$ are $\mathcal{J}$-refinement of $\mathfrak{V}$

Proof: Let $\mathfrak{V} = \{\mathfrak{V}_\alpha : \alpha \in \Delta\}$ be open cover of $\mathfrak{C} \cup \mathfrak{D}$ in $\mathfrak{X}$, since $\mathfrak{C}$ and $\mathfrak{D}$ are $\mathcal{J}D$-paracompact sets in $\mathfrak{X}$, there is $\mathcal{J}$-Locally finite open $\mathcal{J}$-refinement sets $\mathfrak{H} = \{\mathfrak{H}_\delta : \delta \in \Omega\}$ and $\mathcal{U} = \{\mathcal{U}_\lambda : \lambda \in \Delta\}$ of $\mathfrak{V}$ such that $\mathfrak{C} - \bigcup_{\delta\in\Omega} \mathfrak{H}_\delta \in \mathcal{J}$ and $\mathfrak{D} - \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda \in \mathcal{J}$. Suppose that $\mathfrak{L} = \{\mathfrak{H}_\delta : \delta \in \Omega\} \cup \{\mathcal{U}_\lambda : \lambda \in \Delta\}$, to prove that $\mathfrak{L}$ is $\mathcal{J}$-Locally finite $\mathcal{J}$-refinement of $\mathfrak{W}$. Let $\mathfrak{H}_\delta \cup \mathcal{U}_\lambda \in \mathfrak{L}$ such that $\mathfrak{H}_\delta \in \{\mathfrak{H}_\delta : \delta \in \Omega\}$ and $\mathcal{U}_\lambda \in \{\mathcal{U}_\lambda : \lambda \in \Delta\}$, there is $\mathfrak{V}_\alpha \in \mathfrak{V}$ and $(\mathfrak{H}_\delta \cup \mathcal{U}_\lambda) - \mathfrak{V}_\alpha = (\mathfrak{H}_\delta - \mathfrak{V}_\alpha) \cup (\mathcal{U}_\lambda - \mathfrak{V}_\alpha)$, but $\mathfrak{H}_\delta - \mathfrak{V}_\alpha \in \mathcal{J}$ and $\mathcal{U}_\lambda - \mathfrak{V}_\alpha \in \mathcal{J}$, thus $(\mathfrak{H}_\delta \cup \mathcal{U}_\lambda) - \mathfrak{V}_\alpha \in \mathcal{J}$, but, thus $\mathfrak{L}$ is $\mathcal{J}$-refinement of $\mathfrak{V}$.

Let $x \in \mathfrak{X}$, since $\mathfrak{H}$ and $\mathcal{U}$ are $\mathcal{J}$-Locally finite of $\mathfrak{X}$, there exists $\mathfrak{B}$ and $\mathcal{K}$ are neighborhood of x with $\mathfrak{B} \cap \mathfrak{H}_\delta \notin \mathcal{J}$ and $\mathcal{K} \cap \mathcal{U}_\lambda \notin \mathcal{J}$, for at most finitely many indices δ and λ, respectively. But $(\mathfrak{B} \cup \mathcal{K}) \cap (\mathfrak{H}_\delta \cup \mathcal{U}_\lambda) = ((\mathfrak{B} \cap \mathfrak{H}_\delta) \cup (\mathcal{K} \cap \mathfrak{H}_\delta)) \cup ((\mathfrak{B} \cap \mathcal{U}_\lambda) \cup (\mathcal{K} \cap \mathcal{U}_\lambda))$, since $(\mathfrak{B} \cap \mathfrak{H}_\delta) \subseteq ((\mathfrak{B} \cap \mathfrak{H}_\delta) \cup (\mathcal{K} \cap \mathfrak{H}_\delta)) \cup ((\mathfrak{B} \cap \mathcal{U}_\lambda) \cup (\mathcal{K} \cap \mathcal{U}_\lambda))$ and $\mathfrak{B} \cap \mathfrak{H}_\delta \notin \mathcal{J}$, then $(\mathfrak{B} \cup \mathcal{K}) \cap (\mathfrak{H}_\delta \cup \mathcal{U}_\lambda) \notin \mathcal{J}$, for at most finitely many indices δ and λ, respectively, thus $\mathfrak{L}$ is $\mathcal{J}$-Locally finite of $\mathfrak{X}$. But $(\mathfrak{C} \cup \mathfrak{D}) - (\bigcup_{\delta\in\Omega} \mathfrak{H}_\delta \cup \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda) = ((\mathfrak{C} - \bigcup_{\delta\in\Omega} \mathfrak{H}_\delta) \cap (\mathfrak{C} - \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda)) \cup ((\mathfrak{D} - \bigcup_{\delta\in\Omega} \mathfrak{H}_\delta) \cap (\mathfrak{D} - \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda))$, since $(\mathfrak{C} - \bigcup_{\delta\in\Omega} \mathfrak{H}_\delta) \cap (\mathfrak{C} - \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda) \subseteq (\mathfrak{C} - \bigcup_{\delta\in\Omega} \mathfrak{H}_\delta)$ and $(\mathfrak{D} - \bigcup_{\delta\in\Omega} \mathfrak{H}_\delta) \cap (\mathfrak{D} - \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda) \subseteq (\mathfrak{D} - \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda)$, then $(\mathfrak{C} - \bigcup_{\delta\in\Omega} \mathfrak{H}_\delta) \cap (\mathfrak{C} - \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda) \in \mathcal{J}$ and $(\mathfrak{D} - \bigcup_{\delta\in\Omega} \mathfrak{H}_\delta) \cap (\mathfrak{D} - \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda) \in \mathcal{J}$, thus $(\mathfrak{C} \cup \mathfrak{D}) - (\bigcup_{\delta\in\Omega} \mathfrak{H}_\delta \cup \bigcup_{\lambda\in\Delta} \mathcal{U}_\lambda) \in \mathcal{J}$. Hence $\mathfrak{C} \cup \mathfrak{D}$ is $\mathcal{J}D$-paracompact set in $\mathfrak{X}$.

Corollary 3.16: *If $\mathfrak{g} : (\mathfrak{X}, \mathfrak{T}) \to (\mathcal{Y}, \Upsilon, \mathcal{J})$ is homeomorphism function and $(\mathcal{Y}, \Upsilon)$ is $\mathcal{J}D$-paracompact space, then $(\mathfrak{X}, \mathfrak{T})$ is $\mathfrak{g}^{-1}(\mathcal{J})D$-paracompact space.*
Proof: *By definition of homeomorphism function and Proposition (3. 13).*

4. Conclusion and future work:

1. A new definition of ID-paracompact space was presented based on $\mathcal{J}$-refinement and $\mathcal{J}$-local finiteness. Moreover, their most important topological and algebraic properties were examined.
3. If $\mathcal{J}=\{\emptyset\}$, then paracompact (resp. $\mathcal{J}$-paracompact) space and $\{\emptyset\}$ D-paracompact space are equivalent.
4. $\mathcal{J}$D-paracompact space can be applied in fuzzy set theory and proximity theory.

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