*Journal of Interdisciplinary Mathematics*
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 28 (2025), No. 1, pp. 261–266
DOI : 10.47974/JIM-1866

# Inside of nano topological spaces : A novel generalized open and closed nano operator

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## Abstract

In this study, a novel class of operators in Nano topological spaces is defined a novel Open and Closed Nano operators (briefly, (Ɲˆ (Ā) and Ɲˆ (Ā)), proved properties and theorems in Nano topological spaces.

***Subject Classification:*** *54A05, 54A10.*

***Keywords:*** *: Nano operator open Ɲˆ (Ā), Nano operator close Ɲˆ (Ā).*

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## 1. Introduction

Crossley and Hildebrand [3] published their study in 1971 semi Closure. The closure operator C* idea was defined by Dunham [9]. with some properties. In 2011 S. Bhattacharya [2] defined operator of regular closed sets. Savita R. [4], study the Soft topological spaces with both soft $\omega$-Cl. and soft $\omega$-int. When examining Soft g* closed in "Soft topological spaces", Kalavathi, A. and Krishnan, G. [1]. In topological spaces, $\Lambda$ regular generalized* operators are a novel class of operators introduced by Siham I. Aziz and Nabila I. Aziz [5, 6, 7]. closure regular generalized*$(\alpha, \beta)$ Closure Regular Generalized* and introduce a new class operators on regular generalized closed sets respectively. Nano closure and Nano Interior are developed in (2014) in "Nano topological spaces" by Thivagar M. Lellis and Carmel Rechard [8-13]. In this work, a new class of operators will be defined. These nano topological spaces are referred to as a Nano operator open and Nano operator close. (briefly, ($\hat{\mathrm{Ñ}}$ ($\bar{\mathrm{A}}$) and $\breve{\mathrm{Ñ}}$ ($\bar{\mathrm{A}}$)) Nano topological theories and properties have been proven.

## 2. Preliminaries

**Definition 2.1 [1]:** Let $\mathfrak{R}$ be a relation of equivalence on $\mho$, and let $\mho$ be the universe., and $\mathcal{X}\subseteq \mho$. According to $\mathcal{X}$, the Nano topology on $\mho$ is defined by $\mathfrak{T}\mathfrak{R}(\mathcal{X})$={$\mho$, $\emptyset$, $\mathcal{L}\,\mathfrak{R}\,(\mathcal{X})$, $\mathcal{U}\,\mathfrak{R}\,(\mathcal{X})$, $\mathfrak{B}\mathfrak{R}\,(\mathcal{X})$} and ($\mho$, $\mathfrak{T}\mathfrak{R}(\mathcal{X})$)

**Definition 2.2 [1]**: With respect to $\mathcal{X}$, $\mathcal{X} \subseteq \mho$, ($\mho$, $\mathfrak{T}\mathfrak{R}(\mathcal{X})$) constitute a Nano topological space.. If $\mho \supseteq \Phi$, then the Nano interior of is the Union of all Nano open subsets included in $\Phi$., which is indicated by ɲint($\Phi$). the meeting point of all nano-closure-containing subsets of $\Phi$ recognized as the nano closure of and it is represented by the symbol ɲ$cl$($\Phi$).

**Definition 2.3 [2]:** Given that ($\mathcal{X}$, $\mathfrak{T}$) is a topological space and the operator $\Lambda_{\mathrm{G\ddot{R}}}$ ($\mathfrak{W}$) $= \sqcap\{\mathscr{b}: \mathfrak{W} \subseteq \mathscr{b}, \mathscr{b}$ is generalized regular open set $\subseteq \mathcal{X}\}$

## 3. Inside of Nano topological spaces, a novel, generalized Open and Closed Nano operators.

**Definition 3.1:** Assume that Ѝ $\subseteq \mathfrak{U}$, ($\mathfrak{U}$, $\mathfrak{T}\mathfrak{R}(\mathcal{X})$) is a nano topological space. We defined 1-$\hat{\mathrm{Ñ}}$(Ѝ)=Intersection {$\mathfrak{w}$: Ѝ$\subseteq$ $\mathfrak{w}$, $\mathfrak{w}$ element of $\mathcal{N}$ o($\mathfrak{U}$, $\mathcal{X}$)},

2-$\breve{\mathrm{Ñ}}$ (Ѝ)=Union {$\mathfrak{w}$: $\mathfrak{w} \subseteq$ Ѝ, $\mathfrak{w}$ element of $\mathcal{N}$c ($\mathfrak{U}$, $\mathcal{X}$)}.

**Example 3.2:** $\mathfrak{U}$={$\varpi$, ƥ, ę, ɋ} with $\mathfrak{U}$ /$\mathfrak{R}$={{$\varpi$, ƥ}, {ę}, {ɋ}}, $\chi$={ę},

$\mathfrak{T}\mathfrak{R}(\mathcal{X})$)={$\mathfrak{U}$, {ę}, $\phi$}, Ѝ={ę}, $\hat{\mathrm{Ñ}}$(Ѝ)={ę}, $\breve{\mathrm{Ñ}}$ (Ѝ)=$\phi$.

**Lemma 3.3:** *Consider the space of nano topology ($\mathfrak{U}$, $\mathfrak{T}\mathfrak{R}(\mathcal{X})$).*

The operator must satisfy the following conditions, and that operator is denoted by

$$\widehat{\mathrm{N}}: \wp(\mathcal{X}) \longrightarrow \wp(\mathcal{X}).$$

i. $\widehat{\mathrm{N}}$ (Empty set)=Empty set, $\widehat{\mathrm{N}}(\mathcal{X})=\mathcal{X}$.

ii. $\widehat{\mathrm{N}}(\grave{И}) \supseteq \grave{И}$.

iii. $\widehat{\mathrm{N}}\,\widehat{\mathrm{N}}(\grave{И})= \widehat{\mathrm{N}}(\grave{И})$.

iv. If $\grave{И} \subseteq Ķ$, $\widehat{\mathrm{N}}(\grave{И}) \subseteq \widehat{\mathrm{N}}(Ķ)$.

v. $\widehat{\mathrm{N}}(\grave{И} \cap Ķ) \subseteq \widehat{\mathrm{N}}(\grave{И}) \cap \widehat{\mathrm{N}}(Ķ)$.

vi. $\widehat{\mathrm{N}}(\grave{И} \cup Ķ) \supseteq \widehat{\mathrm{N}}(\grave{И}) \cup \widehat{\mathrm{N}}(Ķ)$.

vii. $[\mathfrak{U} - \widehat{\mathrm{N}}(\grave{И})]=\widehat{\mathrm{N}}(\mathfrak{U} - \grave{И})]$.

***Proof:***

i. $\grave{И} \subseteq Ҵ$, and $Ҵ$ is an open nano set$\Rightarrow \widehat{\mathrm{N}}(\grave{И}) \supseteq \grave{И}$.

iii. $\widehat{\mathrm{N}}\widehat{\mathrm{N}}(\grave{И})=\cap\, [\cap\{Ҵ: \grave{И} \subseteq Ҵ,\ Ҵ$ element of $\mathrm{N}o\,(\mathfrak{U}, \mathcal{X})\}]=\cap\{Ҵ: \grave{И} \subseteq Ҵ,\ Ҵ$ element of $\mathrm{N}\, o\,(\mathfrak{U}, \mathcal{X})\}=\widehat{\mathrm{N}}(\grave{И})$.

iv. If $\grave{И} \subseteq Ķ$, $\widehat{\mathrm{N}}(\grave{И}) \subseteq \widehat{\mathrm{N}}(Ķ)$, $\because \grave{И} \subseteq Ķ$, $\cap\{Ҵ: \grave{И} \subseteq Ҵ,\ Ҵ$ element of $\mathrm{N}o\,(\mathfrak{U}, \mathcal{X})\} \subseteq \cap\{Ҵ: Ķ \subseteq Ҵ,\ Ҵ$ element of $\mathrm{N}o\,(\mathfrak{U}, \mathcal{X})\} \Rightarrow \widehat{\mathrm{N}}(\grave{И}) \subseteq \widehat{\mathrm{N}}(Ķ)$.

v. $\because \grave{И} \cap Ķ \subseteq \grave{И}, \grave{И} \cap Ķ \subseteq Ķ$. Then $\widehat{\mathrm{N}}(\grave{И} \cap Ķ) \subseteq \widehat{\mathrm{N}}(\grave{И}), \widehat{\mathrm{N}}(\grave{И} \cap Ķ) \subseteq \widehat{\mathrm{N}}(Ķ)$

Hence $\widehat{\mathrm{N}}(\grave{И} \cap Ķ) \subseteq \widehat{\mathrm{N}}(\grave{И}) \cap \widehat{\mathrm{N}}(Ķ)$.

vi. Since $\grave{И} \cup Ķ \supseteq \grave{И}, Ķ \cup \grave{И} \supseteq Ķ$ then $\widehat{\mathrm{N}}(\grave{И} \cup Ķ) \supseteq \widehat{\mathrm{N}}(\grave{И}), \widehat{\mathrm{N}}(Ķ \cup \grave{И}) \supseteq \widehat{\mathrm{N}}(Ķ)$. Thus $\widehat{\mathrm{N}}(\grave{И} \cup Ķ) \supseteq \widehat{\mathrm{N}}(\grave{И}) \cup \widehat{\mathrm{N}}(Ķ)$.

vii. $\mathfrak{U} - \widehat{\mathrm{N}}(\grave{И})=\widehat{\mathrm{N}}(\mathfrak{U}\text{-}\grave{И}) \Rightarrow \mathfrak{U} - \widehat{\mathrm{N}}(\grave{И})=\mathfrak{U}\text{-}[\cap\{Ҵ: \grave{И} \subseteq Ҵ, Ҵ \in \mathrm{N}o\,(\mathfrak{U}, \mathcal{X})\}=\cup\,\{\mathfrak{U}\text{-}Ҵ: \mathfrak{U}\text{-}Ҵ \subseteq \mathfrak{U}\text{-}\grave{И}, (\mathfrak{U}\text{-}Ҵ) \in \mathrm{N}\,C(\mathfrak{U}, \mathcal{X})\})=\widehat{\mathrm{N}}(\mathfrak{U}\text{-}\grave{И})$.

**Lemma 3.4:** *Consider the nano topological space $\breve{\mathrm{N}}: \mathcal{P}(\mathcal{X}) \longrightarrow \mathcal{P}(\mathcal{X})$ in $(\mathfrak{U}, \mathfrak{TR}(\mathcal{X}))$. Be an operator with the following characteristics.*

i. $\breve{\mathrm{N}}(\emptyset)=\emptyset$, $\breve{\mathrm{N}}(\mathfrak{U})=\mathfrak{U}$.

ii. $\mathfrak{X} \supseteq \breve{\mathrm{N}}(\mathfrak{X})$.

iii. $\breve{\mathrm{N}}\ \breve{\mathrm{N}}(\mathfrak{X})= \breve{\mathrm{N}}(\mathfrak{X})$.

iv. If $\mathfrak{X} \subseteq Ḍ$, $\breve{\mathrm{N}}(\mathfrak{X}) \subseteq \breve{\mathrm{N}}(Ḍ)$.

v. $\breve{\mathrm{N}}(\mathfrak{X} \cap Ḍ) \subseteq \breve{\mathrm{N}}(\mathfrak{X}) \cap \breve{\mathrm{N}}(Ḍ)$.

vi. $\breve{\mathrm{N}}(\mathfrak{X} \cup Ḍ) \supseteq \breve{\mathrm{N}}(\mathfrak{X}) \cup \breve{\mathrm{N}}(Ḍ)$.

vii. $[\mathfrak{U} - \breve{\mathrm{N}}(\mathfrak{X})]=\widehat{\mathrm{N}}(\mathfrak{U} - \mathfrak{X})]$.

***Proof:*** This case can be demonstrated by describing oneself and using the same evidence as the prior case.

**Remark 3.5:** The following example demonstrates that lemma [3. 3, 3. 4]'s converse part (5-6) is untrue.

**Example 3.6:** $\mathcal{U}$={ș, p̧, ņ, m̧}, $\mathcal{U}$/R̂={{ș, p̧}, {ņ}, {m̧}}, Ẋ={p̧, ņ},. Ṫ R̂($\mathcal{X}$)={$\mathcal{U}$, ∅, {p̧}, {ș, p̧, ņ}, {ș, ņ}}. Ṫ$^C$ R̂($\mathcal{X}$)={$\mathcal{U}$, ∅, {m̧}, {p̧, m̧}, {ș, ņ, m̧}}.

1- $\mathcal{W}$={p̧, ņ}, $\mathcal{Z}$={ș, p̧}, N̂($\mathcal{W}$)={ș, p̧, ņ}, N̂($\mathcal{Z}$)={ș, p̧, ņ}, N̂($\mathcal{W}$) ∩ N̂($\mathcal{Z}$)={ș, p̧, ņ}.
   N̂($\mathcal{W}$ ∩ B)=N̂{p̧}={p̧}, then N̂($\mathcal{W}$ ∩ $\mathcal{Z}$) is not equal N̂($\mathcal{W}$) ∩ N̂($\mathcal{Z}$).
2- N̂($\mathcal{W}$ ∪ $\mathcal{Z}$)=N̂($\mathcal{W}$) ∪ N̂($\mathcal{Z}$). Ḩ={ș}, Ķ={∅}, N̂(Ḩ)={ș, ņ}, N̂(Ķ)={p̧}, N̂(Ḩ) ∪ N̂(Ķ)={ș, ņ, p̧}, N̂(Ḩ ∪ Ķ)={ș, ņ}, then N̂(Ḩ ∪ Ķ) is not equal N̂(Ḩ) ∪ N̂(Ķ)
3- Ḩ={ș, p̧, m̧}, Ķ={ș, ņ, m̧}, Ň(Ḩ)={p̧, m̧}, Ň(Ķ)={ș, ņ, m̧}, Ň(Ḩ) ∩ Ň(Ķ)={m̧}, Ň(Ḩ ∩ Ķ)= Ň{ș, m̧}={m̧}, then Ň(Ḩ ∩ Ķ) is not equal Ň(Ḩ) ∩ Ň(Ķ)
4- Ḝ={ș, m̧}, Ḑ={ņ, m̧}, Ň(Ḝ)={m̧}, Ň(Ḑ)={m̧}, Ň(Ḝ) ∪ Ň(Ḑ)={m̧}, Ň(Ḝ ∪ Ḑ)={ș, ņ, m̧}, then Ň(Ḝ ∪ Ḑ) is not equal Ň(Ḝ) ∪ Ň(Ḑ).

**Theorem 3.7:** *$\mathcal{Q}$ Is the open set of Nano, and N̂ ($\mathcal{Q}$)=$\mathcal{Q}$. $\mathcal{N}$ o($\mathcal{U}$, $\mathcal{X}$) in a nutshell*

***Proof***: Make $\mathcal{Q}$ a Nano open set.

As a result of $\mathcal{Q}$ ⊆ N̂($\mathcal{Q}$), ɨnt($\mathcal{Q}$) ⊆ 𝔓, 𝔓 being a Nano open set, N̂($\mathcal{Q}$) ⊆ $\mathcal{Q}$ and so N̂($\mathcal{Q}$)=$\mathcal{Q}$.

Alternatively: if N̂($\mathcal{Q}$)=$\mathcal{Q}$. showing that $\mathcal{Q}$ is a Nano open set

Due to the fact that The Nano open set includes every intersection. $\mathcal{Q}$ is then an open set for Nano.

**Theorem 3.8:** Ň*(𝔎)=𝔎 is a member of the closed nano set. In a nutshell, $\mathcal{N}$c ($\mathcal{U}$, $\mathcal{X}$)*

***Proof:*** A closed nano set is an example 𝔎. Ň(𝔎)=𝔎 because

$\breve{\mathcal{N}}$ (𝔎) ⊆ 𝔎, Ġ ⊆ Cl (𝔎), Ġ are Nano closed sets, respectively.

In contrast, if Ň(𝔎)=𝔎. To demonstrate that 𝔎 is a nano closed set Due to the fact that each union of a nano closed set is a nano closed set. As a result, 𝔎 is a Nano closed set.

## 4. Conclusion

Obtaining a new operators in Nano topological spaces, called Nano operator open and Nano operator close (briefly, (N̂ (Ā) and $\breve{\mathcal{N}}$ (Ā)) and expecting that there are relationships between them in Nano topological spaces.

## References

[1] A. Kalavathi and G. Krishnan, "Soft g* closed and soft g* open sets in soft topological spaces," *J. Interdiscip. Math.,* vol. 19, no. 1, pp. 65-82 (2016). [Online]. Available: https://doi.org/10.1080/09720502.2015.1103110.

[2] S. Bhattacharya, "On generalized regular closed sets," *Contemp. Math. Sciences,* vol. 6, no. 3, pp. 145-152 (2011).

[3] S. G. Crossley and S. K. Hildebrand, "Semi-closure," *Texas Journal Sci.*, vol. 22, pp. 99-112 (1971).

[4] S. Rathee, R. Girdhar, and K. Dhingra, "On soft ω-interior and soft ω-closure in soft topological spaces," *J. Interdiscip. Math.,* vol. 23, no. 6 (2020). [Online]. Available: https://doi.org/10.1080/09720502.2020.1815399.

[5] S. I. Aziz and N. I. Aziz, "On some generalized recent operators in topological spaces," *Tikrit Journal of Pure Science,* vol. 27, no. 4, pp. 92-94 (2022). [Online]. Available: https://doi.org/10.25130/tjps.v27i4.40.

[6] S. I. Aziz and N. I. Aziz, "On some generalized α and β closure in topological spaces," *J. Interdiscip. Math.*, vol. 27, no. 7, pp. 1851-1854 (2021). [Online]. Available: https://doi.org/10.1080/09720502.2021.1963521.

[7] S. I. Aziz and N. I. Aziz, "New generalized operator in topological spaces," V. International Scientific Congress of Pure, Applied and Technological Sciences (2022).

[8] S. M. Kadham, M. A. Mustafa, N. K. Abbass, and S. Karupusamy, "Comparison between of fuzzy partial H-transform and fuzzy partial Laplace transform in x-ray images processing of acute interstitial pneumonia," *International Journal of System Assurance Engineering and Management,* Vol 2023, pp 1-9 (2023), doi: https://doi.org/10.1007/s13198-023-02001-3.

[9] M. A. Mustafa, M. A. M. M. Rahman, S. M. S. Islam, S. M. A. R. Iqbal, and A. M. A. S. Hossain, "A novel fuzzy M-transform technique for

sustainable ground water level prediction," *Appl. Geomatics,* vol. 16, no. 1, pp. 9–15 (Jan. 2023), doi: 10.1007/s12518-022-00486-4.

[10] A. H. Alwan, "Small intersection graph of subsemimodules of a semimodule," *Commun. Combinatorics Cryptogr. Comput. Sci.,* vol. 2022, no. 1, pp. 15-22 (2022).

[11] S. Ediz, M. Cancan, and M. R. Farahani, "Eccentric connectivity and connective eccentricity indices of generalized Petersen graphs," *Commun. Combinatorics Cryptogr. Comput. Sci.,* vol. 2021, no. 1, pp. 1-6 (2021).

[12] R. Ponraj, S. Prabhu, and M. Sivakumar, "Pair mean cordial labeling of total graph of some graphs and prism-related graphs," *Commun. Combinatorics Cryptogr. Comput. Sci.,* vol. 2024, no. 2, pp. 181-192 (2024).

[13] F. O. Oduol and I. O. Okoth, "Counting vertices among all noncrossing trees by levels and degrees," *Commun. Combinatorics Cryptogr. Comput. Sci.,* vol. 2024, no. 2, pp. 193-212 (2024).

*Received June, 2024*