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Innovative conjugate gradient methods for image impulse noise removal

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Abstract

Conjugate gradient approaches focus on the conjugate of the coefficient. In this study, we provide an innovative coefficient conjugate gradient approach for eliminating impulse noise pictures using the Taylor series. We have suggested an intriguing conjugate gradient approach and a search strategy based on the new coefficient conjugate. We demonstrate that the suggested approach is globally convergent under the right circumstances. Our numerical findings demonstrate the effectiveness of this technique for picture restoration.

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Keywords: *Innovative conjugate gradient, Convergence analysis, Image problems.*

1. Introduction

In actuality, many intricate optimization problems in engineering and management, such as the noisy pixel cleaning model, may be boiled down to optimization problem and resolved:

$$f_\alpha(u) = \sum_{(i,j)\in N}\left[\left|u_{i,j} - y_{i,j}\right| + \frac{\beta}{2}(2\times S_{i,j}^1 + S_{i,j}^2)\right]. \tag{1}$$

where $S_{i,j}^1 = 2\sum_{(m,n)\in P_{i,j}\cap N^c}\phi_\alpha(u_{i,j} - y_{m,n}), S_{i,j}^2 = \sum_{(m,n)\in P_{i,j}\cap N}\phi_\alpha(u_{i,j} - y_{m,n})$, see [1, 2]. In image restoration, $u_{i,j} = \left[u_{i,j}\right]_{(i,j)\in N}$ is the picture's pixel value at

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location (i,j), $\phi_\alpha = \sqrt{\alpha + x^2}$ is the edge-preserving potential function with parameter α > 0, β is the parameter under the right circumstances, and $y_{i,j}$ is a column direction of length |N| sorted lexicographically [2,3].

Optimization issues are a good fit for conjugate gradient techniques because they can be readily iterated and need little memory:

$$f(u^*) = \min_{x \in R^N} f(u). \tag{2}$$

where f is a smooth model, see [3]. This work uses the nonlinear conjugate gradient technique to solve four different kinds of problems:

$$u_{k+1} = u_k + \alpha_k d_k. \tag{3}$$

with α_k as a step length, this produces the direction d_{k+1} as follows:

$$d_{k+1} = -g_{k+1} + \beta_k d_k \tag{4}$$

where β_k is the conjugate coefficient (see [4, 5])Fa. Notable are the convolutional gradient algorithms' global convergence characteristics. The technique proposed by Flatcher-Reeves (FR) yielded the superlative property results [6]. Nevertheless, the Hastenes-Stiefal (HS) approach, one of the best-performing CG algorithms, was unable to meet the necessity for global convergence in classical line search [7]. For instance, procedures:

$$\beta_k^{HS} = \frac{y_k^T g_{k+1}}{d_k^T y_k}, \beta_k^{FR} = \frac{\|g_{k+1}\|^2}{\|g_k\|^2} \tag{5}$$

where $y_k = g_{k+1} - g_k$. Profound study materials that describe contemporary computer graphics methods with remarkable outcomes may be located in [8]. Being able to meet the conjugacy requirements is one of the factors supporting the validity of the Hestenes-Stiefel formula. One factor that gives the formula's validity the greatest degree of trust is this one. This directly leads to it developing into a very useful mathematical tool. You may expedite the Newton method by utilizing these many ways. According to the quasi-Newton direction principle, d_{k+1} may be used to approximate the quasi-Newton method in (4) and get the result that is most comparable to the other possibilities. A parameter β_k that as a result:

$$-Q_{k+1}^{-1} g_{k+1} = -g_{k+1} + \beta_k s_k. \tag{6}$$

with Q_{k+1} is second drivative, see Nazareth [9]. The goal of this latest study is to offer a search strategy that adheres to globally convergent descent. Idea-rich techniques, such as [10], function well both theoretically and practically. The innovative technique [1] is more efficient than the optimization method [11], as demonstrated by the numerical results. As can be seen below, a range of line search methods, including exact line search, are employed in [3] to calculate the step size:

$$\alpha_k = -\frac{g_k^T d_k}{d_k^T G d_k} \tag{7}$$

Taking this into consideration, we decided to try a few additional enhancements, thinking that if we were successful, they may help with the numerical performance. A Taylor series was used to determine if the changes were necessary once the conjugate gradient parameter of a new conjugate gradient was established.

2. Using the Taylor series in deriving new parameter:

The Taylor series is a key idea in order to determine the new conjugate gradient parameter. As we move forward:

$$f(x) = f(x_{k+1}) - g_{k+1}^T s_k + \frac{1}{2} s_k^T Q(u_{k+1}) s_k \tag{8}$$

One way to express the gradient is as follows:

$$g_{k+1} = g_k + Q(u_k) s_k \tag{9}$$

The twist may be determined using equations (8) and (9):

$$s_k^T Q(u_k) s_k = 2(f_k - f_{k+1}) + 2y_k^T s_k + 2g_k^T s_k \tag{10}$$

We can find out by applying some algebra:

$$s_k^T Q(u_k) s_k = 1/2 y_k^T s_k + (f_{k+1} - f_k) - g_k^T s_k. \tag{11}$$

The $Q(u_k)$ matrix solution that results is as follows:

$$Q(u_k) = \frac{1/2 y_k^T s_k + (f_{k+1} - f_k) - g_k^T s_k}{s_k^T s_k} I_n. \tag{12}$$

It has the identity matrix as I_n. Using (6) and (12), we get:

$$\beta_k^{BBI} = \left(1 - \frac{s_k^T s_k}{1/2 y_k^T s_k + (f_{k+1} - f_k) - g_k^T s_k}\right) \frac{g_{k+1}^T y_k}{s_k^T y_k}, \tag{13}$$

For convenience of usage, these tactics are referred to as BBI. BBI Algorithm Update:

Algorithm BBI.

Stage 1. Put $x_0 \in R^n$, set$k = 0, 0 < \delta < \sigma < 1, d_0 = -g_0$.

Stage 2. Put an stop, if $\|g_k\| \leq \varepsilon$.

Stage 3. Use (9), (12) and (13) to calculateα_k:

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k \tag{14}$$

$$d_k^T g(x_k + \alpha_k d_k) \geq \sigma\, d_k^T g_k \tag{15}$$

Stage 4. Compute β_k by (13) and $x_{k+1} = x_k + \alpha_k d_k$,.

Stage 5. Calculate $d_{k+1} = -g_{k+1} + \beta_k s_k$.

Stage 6. Put $k = k + 1$ and energy to stage 4.

3. Convergence analysis

The analysis of BBI algorithms global convergence is our next topic of study. The following assumption is made:

i. $f(x)$ has bounds set $\Psi = \{x \in R^n : f(x) \leq f(x_0)\}$.
ii. Because g is Lipschitz, a non-negative stable L exits such that:

$$\|g(q) - g(j)\| \leq L\|q - j\|, \forall q, j \in R^n. \tag{16}$$

In this instance, if specific function assumptions are met, there is a stable $\Gamma \geq 0$ such that $\|\nabla f(x)\| \leq \Gamma$, see [12].

Take a look at the theorem to see how useful the descent condition may be.

Theorem 1: *If* $s_k^T y_k \neq 0$, *then the search directions produced by (4) and (13) are descent directions.*

Proof: The product of (4) by g_{k+1}, is:

$$d_{k+1}^T g_{k+1} = -g_{k+1}^T g_{k+1} + \left(1 - \frac{s_k^T s_k}{1/2 y_k^T s_k + (f_{k+1} - f_k) - g_k^T s_k}\right) \frac{y_k^T g_{k+1}}{s_k^T y_k} s_k^T g_{k+1} \tag{17}$$

From (9) and (11), we get:

$$s_k^T y_k = 1/2 y_k^T s_k + (f_{k+1} - f_k) - g_k^T s_k. \tag{18}$$

Lipschitz condition results in: $y_k^T g_{k+1} \leq L s_k^T g_{k+1}$ and $s_k^T y_k \leq L s_k^T s_k$. So, we could write:

$$d_{k+1}^T g_{k+1} \leq -\|g_{k+1}\|^2 + \left(\frac{L s_k^T s_k - s_k^T s_k}{s_k^T y_k}\right) \frac{L (s_k^T g_{k+1})^2}{s_k^T y_k}. \tag{19}$$

Since L and α_k^2 are very small, it is important to observe that:

$$d_{k+1}^T g_{k+1} \leq 0. \tag{20}$$

The final proof has been completed.

Similarly, based on the idea of the methods in [11], we prove global convergence to the new method.

4. Numerical Results

We exhibit the successful decrease in salt-and-pepper impulse noise through the utilization of the BBI algorithm (3) in this research. These algorithms apply distinct parameters and techniques as described in Table 1. The research flow diagram and the initial test images are depicted in Table 1. All computations were conducted on a computer running MATLAB, [13-14]. Comparisons are given between the FR technique and the performance evaluations of the BBI methodology. The key goal of this study is to ascertain the timeframe for reducing carbon emissions. The Signal-to-Noise Ratio (SNR) is used to evaluate the quality of the recovered pictures:

$$PSNR = 10 \log_{10} \frac{255^2}{\frac{1}{MN} \sum_{i,j} (u_{i,j}^r - u_{i,j}^*)^2} \tag{21}$$

where the pixel values of the original and restored images are meant by $u_{i,j}^r$ and $u_{i,j}^*$, individually. The conditions listed below are those that must be met in order to come to a halt in either approach:

$$\frac{|f(u_k) - f(u_{k-1})|}{|f(u_k)|} \leq 10^{-4} \text{ and } \|f(u_k)\| \leq 10^{-4}(1 + |f(u_k)|). \tag{22}$$

In Table 1 and Figure 1, displays the results acquired from the tests. It encompasses the PSNR, which represents the peak signal-to-noise ratio, along with the count of iterations (NI) and function calculations (NF).

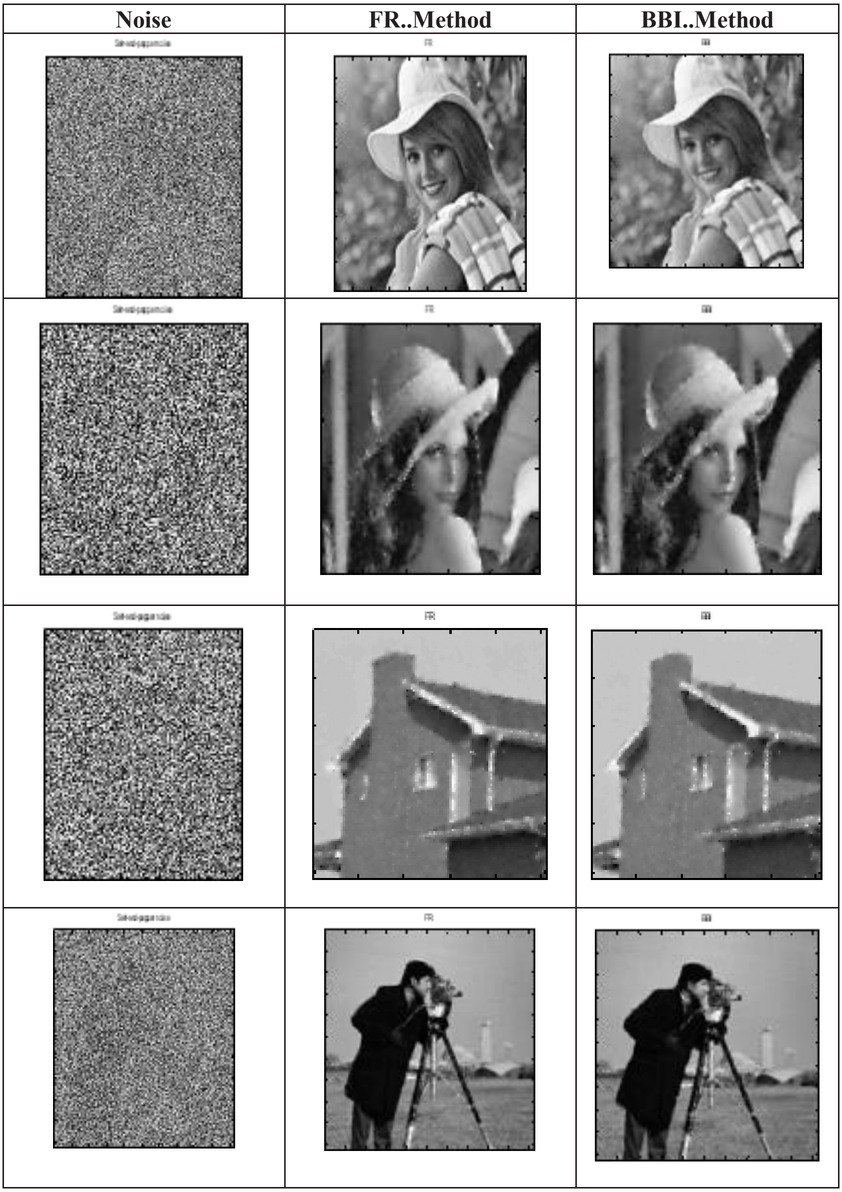

Figure 1
Displays the results of the FR and BBI 256 * 256 algorithms of Elaine, Lena, House, and Cameraman

Table 1
Compare between FR and BBI algorithms.

Noise (%)	Image	FR..Method			BBI..Method		
		NI	NF	PSNR (dB)	NI	NF	PSNR (dB)
50	El	35	36	33.9129	34	35	33.8619
	Le	82	153	30.5529	57	105	30.5000
	Ho	52	53	30.6845	38	61	34.7648
	c512	59	87	35.5359	43	47	35.3617
70	El	38	39	31.864	38	39	31.7898
	Le	81	155	27.4824	54	88	27.4719
	Ho	63	116	31.2564	49	75	31.0513
	c512	78	142	30.6259	44	72	30.5033
90	El	65	114	28.2019	52	53	28.1708
	Le	108	211	22.8583	60	95	22.8120
	Ho	111	214	25.287	59	93	25.2550
	c512	121	236	24.3962	58	116	25.0599

5. Conclusions

To sum up, we have introduced a novel altered conjugate gradient equation alongside the BBI method. Through the utilization of the line search parameters, we successfully determined its global convergence. The simulations have demonstrated that BBI have the potential to significantly decrease the comparative tool while preserving the image quality.

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