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Double INEM-transform integral for solving second order partial differential equation

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Abstract

In this study, a brand-new double transform known as the double INEM transform is introduced. Combined with the definition and essential features of the proposed double transform, new findings on partial derivatives, Heaviside function, are also presented. Additionally, we solve several symmetric applications to show how effective the provided transform is at resolving partial differential equation.

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1. Introduction

The most effective technique for resolving partial differential equations is regarded as integral transformations. To solve partial, fractional, and integral DE numerous integral transforms have been developed and used. We provide some instances here [1-3]. The authors of [4-7] introduced an INEM integral transform]. This article explores a new double transform known as the Double INEM integral transform. We will present and demonstrate the fundamental DINEM properties and theorems

Definition 1: [5]. The INEM-transform of the function $f(t)$ is

$$E\big(f(t)\big) = I(v) = p(v)\int_0^{\infty} f(t)e^{-(q(v))^{\frac{1}{n}}t}\,dt \qquad \ldots(1)$$

Definition 2: The DINEM-transform of the function $f(x,t)$,$x>0,t>0$ is

$$E_xE_t\{f(x,t)\} = I(s,v) =$$

$$p_1(s)p_2(v)\int_0^{\infty}\int_0^{\infty} f(x,t)\,e^{-\left[[q_1(s)]^{\frac{1}{n}}x+[q_2(v)]^{\frac{1}{m}}t\right]}dxdt, \qquad \ldots(2)$$

where $p_1(s), q_1(s)$, are functions of parameters, $s, p_2(v), q_2(v)$ are functions of parameters v, $n,m \in \mathbb{Z}$. Clearly the DINEM-transform is an integral transformation that is linear, showing below

$$E_xE_t\{af(x,t)+bh(x,t)\} = p_1(s)p_2(v)\int_0^{\infty}\int_0^{\infty}\{af(x,t)+bh(x,t)\}$$

$$e^{-\left[[q_1(s)]^{\frac{1}{n}}x+[q_2(v)]^{\frac{1}{m}}t\right]}dxdt = a\,E_xE_t\{f(x,t)\}+bE_xE_t\{h(x,t)\}$$

Definition 3: the invers DINEM-transform is defined by the following

$$E_x^{-1}E_t^{-1}\{I(s,v)\} =$$

$$\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} p_1(s)\,e^{-[q_1(s)]^{\frac{1}{n}}x}\,ds\ \frac{1}{2\pi i}\int_{\omega-i\infty}^{\omega+i\infty} p_2(v)\,e^{-[q_2(v)]^{\frac{1}{m}}t}\,I(s,v)dv \qquad \ldots(3)$$

Property 1: If $f(x,t)=\beta(x)\delta(t)$,where $x,t>0$ then,$E_xE_t\{f(x,t)\} = E_x\{\beta(x)\}E_t\{\delta(t)\}$

2. Double INEM-transform integral of some basic function

(1) Let $f(x,t)=1$, then

$$I(s,v) = \frac{p_1(s)p_2(v)}{[q_1(s)]^{\frac{1}{n}}\,[q_2(v)]^{\frac{1}{m}}} \qquad (4)$$

(2) Let $f(x,t)=x^a.t^b$ where a and b are positive constants by property (1) we can $E_xE_t\{x^a.t^b\} = E_x\{x^a\}.E_t\{t^b\}$. By properties of INEM-transform we have

$$= \frac{a!b!\,p_1(s)p_2(v)}{\left[[q_1(s)]^{\frac{1}{n}}\right]^{a+1}\left[[q_2(v)]^{\frac{1}{m}}\right]^{b+1}}$$

$$I(s,v)=\frac{p_1(s)p(v)}{\left([q_1(s)]^{\frac{1}{n}}-a\right)\left([q_2(v)]^{\frac{1}{m}}-b\right)} \tag{5}$$

(4) Let $f(x,t)=cos(k)$, a,b is constants, $k=ax+bt$

$$E_xE_t\{cos(k)\}=\frac{p_1(s)p_2(v)\left[[q_1(s)]^{\frac{1}{n}}\,[q_2(v)]^{\frac{1}{m}}-ab\right]}{\left([q_1(s)]^{\frac{2}{n}}+a^2\right)\left([q_2(v)]^{\frac{2}{m}}+b^2\right)}$$

(5) Let $f(x,t)=sinh(k)\,, a,b>0, k=ax+bt$

$$E_xE_t\{sinh(k)\}=\frac{p_1(s)p_2(v)\left(b[q_1(s)]^{\frac{1}{n}}+a[q_2(v)]^{\frac{1}{m}}\right)}{\left([q_1(s)]^{\frac{2}{n}}-a^2\right)\left([q_2(v)]^{\frac{2}{m}}-b^2\right)}$$

Theorem 1: *If a function $f(x,t)$ is a continuous function in every finite interval $(0,X)$ and $(0,T)$ of exponential order e^{ax+bt}, then the DINEM transform of $f(x,t)$ exists for all $[q_1(s)]^{\frac{1}{n}}$ and $[q_2(v)]^{\frac{1}{m}}$ provided Re $\left|[q_1(s)]^{\frac{1}{n}}\right|>a$ and Re $\left|[q_2(v)]^{\frac{1}{m}}\right|>b$.*

3. Some Basic theorem of DINEM

Theorem 2 (derivative properties): *If $E_xE_t\{f(x,t)\}=I(s,v)$, then*

(1) $E_xE_t\left\{\frac{\partial f(x,t)}{\partial t}\right\}=[q_2(v)]^{\frac{1}{m}}\,I(s,v)-p_2(v)\,I(s,0)$

(2) $E_xE_t\left\{\frac{\partial f(x,t)}{\partial x}\right\}=[q_1(s)]^{\frac{1}{n}}\,I(s,v)-p_1(s)\,I(0,v)$

(3) $E_xE_t\left\{\frac{\partial^2 f}{\partial x^2}\right\}=[q_1(s)]^{\frac{2}{n}}\,I(s,v)-[q_1(s)]^{\frac{1}{n}}p_1(s)\,I(0,v)-p_1(s)\,I_x(0,v)$

(4) $E_xE_t\left\{\frac{\partial^2 f}{\partial t^2}\right\}=[q_2(v)]^{\frac{2}{m}}\,I(s,v)-[q_2(v)]^{\frac{1}{m}}\,p_2(v)\,I(s,0)-p_2(v)\,I_t(s,0)$

(5) $E_xE_t\left\{\frac{\partial^2 f}{\partial x\partial t}\right\}=[q_1(s)]^{\frac{1}{n}}\,[q_2(v)]^{\frac{1}{m}}\,I(s,v)-p_2(v)I_x(s,0)-[q_2(v)]^{\frac{1}{m}}$ $p_1(s)\,I(0,v)$

Proof:

$$E_xE_t\left\{\frac{\partial f(x,t)}{\partial t}\right\}=p_1(s)p_2(v)\int_0^\infty\int_0^\infty\left(\frac{\partial f(x,t)}{\partial t}\right)e^{-\left([q_1(s)]^{\frac{1}{n}}x+[q_2(v)]^{\frac{1}{m}}t\right)}dxdt$$

$$=p_1(s)\int_0^\infty e^{-[q_1(s)]^{\frac{1}{n}}x}dx\;p_2(v)\left\{\int_0^\infty\left(\frac{\partial f(x,t)}{\partial t}\right)e^{-[q_2(v)]^{\frac{1}{m}}t}dt\right\}$$

Part $p_2(v)\left\{\int_0^\infty\left(\frac{\partial f(x,t)}{\partial t}\right)e^{-[q_2(v)]^{\frac{1}{m}}t}\,dt\right\}$

$$=p_2(v)\left(-f(x,0)+[q_2(v)]^{\frac{1}{m}}\int_0^\infty e^{-[q_2(v)]^{\frac{1}{m}}t}f(x,t)\,dt\right)$$

$$E_xE_t\left\{\frac{\partial f(x,t)}{\partial t}\right\}=[q_2(v)]^{\frac{1}{m}}\,I(s,v)-P_2(v)\,I(s,0)$$

Proof:

$$E_xE_t\left\{\frac{\partial^2 f(x,t)}{\partial x^2}\right\}=p_1(s)p_2(v)\int_0^\infty\int_0^\infty\frac{\partial^2 f(x,t)}{\partial x^2}e^{-\left([q_1(s)]^{\frac{1}{n}}x+[q_2(v)]^{\frac{1}{m}}t\right)}dxdt$$

$$=p_1(s)\int_0^\infty\frac{\partial^2 f}{\partial x^2}e^{-[q_1(s)]^{\frac{1}{n}}x}dx\,p_2(v)\int_0^\infty e^{-[q_2(v)]^{\frac{1}{m}}t}dt$$

$$p_1(s)\int_0^\infty\frac{\partial^2 f(x,t)}{\partial x^2}e^{-[q_1(s)]^{\frac{1}{n}}x}\,dx$$

$$=p_1(s)\left(\frac{-\partial f(0,t)}{\partial x}+[q_1(s)]^{\frac{1}{n}}\int_0^\infty e^{-[q_1(s)]^{\frac{1}{n}}x}\,\frac{\partial f(x,t)}{\partial x}dx\right)$$

Using the result in part (2), we have

$$E_xE_t\left\{\frac{\partial^2 f(x,t)}{\partial x^2}\right\}=[q_1(s)]^{\frac{2}{n}}\,I(s,v)-[q_1(s)]^{\frac{1}{n}}\,p_1(s)\,I(0,v)-p_1(s)\,I_x(0,v)$$

Similar arguments can be used to prove the remaining relationships.

Theorem 3 (Heaviside function): *If* $E_xE_t\{f(x,t)\}=I(s,v)$, *then*

$$E_xE_t\{f(x-\gamma,t-\alpha)I(x-\gamma,t-\alpha)\}=e^{-[q_1(s)]^{\frac{1}{n}}\gamma-[q_2(v)]^{\frac{1}{m}}\alpha}I(s,v)$$

where $I(x,t)$ is Heaviside unite step defined by

$$I(x-\gamma,t-\alpha)=\begin{Bmatrix}t\ x>\gamma,t>\alpha\\0\ other\ wise\end{Bmatrix}$$

Proof: $E_xE_t\{f(x-\gamma,t-\alpha)\}I(x-\gamma,t-\alpha)\}=$

$$=e^{-[q_1(s)]^{\frac{1}{n}}\gamma-[q_2(v)]^{\frac{1}{m}}\alpha}\,p_1(s)p_2(v)\int_0^\infty\int_0^\infty e^{-[q_1(s)]^{\frac{1}{n}}w_1-[q_2(v)]^{\frac{1}{m}}w_2}\,f(w_1,w_2)dw_1dw_2$$

$=e^{-[q_1(s)]^{\frac{1}{n}}\gamma-[q_2(v)]^{\frac{1}{m}}\alpha}I(s,v)$, where $w_1=x-\gamma, w_2=t-\alpha$

Theorem 4 (periodic function): *If* $E_xE_t\{f(x,t)\}$ *exist, where* $f(x,t)$ *is periodic function of periods* a *and* b *such that* $f(x+a,t+b)=f(x,t)\ \forall x,t$, *then*

$$E_xE_t\{f(x,t)\}=\frac{p_1(s)p_2(v)}{\left(1-e^{-[q_1(s)]^{\frac{1}{n}}a-[q_2(v)]^{\frac{1}{m}}b}\right)}$$

Definition 4: The double convolution of $f(x,t)$ and $h(x,t)$ is given

$$(f**h)(x,t)=\int_0^x\int_0^t f(x-\gamma,t-\alpha)\,h(\gamma,\alpha)d\gamma d\alpha$$

Theorem 5: (convolution theorem). *If* $E_xE_t\{f(x,t)\}=I_1(s,v)$, $E_xE_t\{h(x,t)\}=I_2(s,v)$ *are exist, then* $E_xE_t\{(f**h)(x,t)\}=I_1(s,v)I_2(s,v)/P_1(s)P_2(v)$

Proof: $E_x E_t\{(f ** h)(x,t)\}$

$$= p_1(s)p_2(v)\int_0^\infty\int_0^\infty (f ** h)(x,t)\, e^{-\left([q_1(s)]^{\frac{1}{n}}x+[q_2(v)]^{\frac{1}{m}}t\right)}dxdt$$

$= \int_0^\infty\int_0^\infty h(\gamma,\alpha)d\gamma d\alpha \left\{p_1(s)p_2(v)\int_0^\infty\int_0^\infty e^{-[q_1(s)]^{\frac{1}{n}}x-[q_2(v)]^{\frac{1}{m}}t} f(x-\gamma, t-\right.$ $\alpha)\, I(x-\gamma, t-\alpha)dxdt.$

By the (Heaviside function) gives

$$\int_0^\infty\int_0^\infty h(\gamma,\alpha)d\gamma d\alpha\left\{e^{-[q_1(s)]^{\frac{1}{n}}\gamma-[q_2(v)]^{\frac{1}{m}}\alpha}\, I_1(s,v)\right\} = \frac{1}{p_1(s)p_2(v)} I_1(s,v)I_2(s,v)$$

4. Comparative analysis of the DINEM and other transforms

This section compares a few different DINEM and some other double transform

(1) The double Laplace transformation is

$$L_x L_t[q(x,t)] = \int_0^\infty\int_0^\infty e^{-(v\,x+st)} f(x,t)dxdt$$

(2) The double ARA- transform is

$$G_x G_t\{f(x,t)\} = vs\int_0^\infty\int_0^\infty e^{-(v\,x+st)} f(x,t)dxdt$$

(3) The double formable transform is

$$\mathcal{R}^2\{f(x,t)\} = sv\int_0^\infty\int_0^\infty e^{-(v\,x+st)} f(rx,ut)dxdt$$

(4) The double ElZaki transform is

$$E_x E_t\{f(x,t),u,v\} = uv\int_0^\infty\int_0^\infty f(x,t)e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}.\,dxdt$$

Basic Principle of DINEM Transform:

The technique for solving PDE in two variables $x$and t works in general by

$$AZ_{xx}(x,t) + BZ_{xt}(x,t) + CZ_{tt}(x,t) + DZ_x(x,t) + EZ_t(x,t) + FZ(x,t) = g(x,t)$$

With the Initial conditions $z(x,0) = f_1(x), z_t(x,0) = f_2(x)$, and boundary conditions $z(0,t) = f_3(x), z_x(0,t) = f_4(x)$, And assume that $Z(0,0) = \tau$, where $Z(x,t)$ is dependent variable $g(x,t)$ is the original term and A,B,C,D,E,F and τ are constants, Utilizing the INEM transform under the specified conditions results in

$E[z(x,0)] = E[f_1(x)] = I_1(s), E[z_t(x,0)] = E[f_2(x)] = I_2(s),$
$E[z(0,t)] = E[f_3(t)] = I_3(v), E[z_x(0,t)] = E[f_4(t)] = I_4(v)$

$$E_x E_t\{AZ_{xx}(x,t) + BZ_{xt}(x,t) + CZ_{tt}(x,t) + DZ_x(x,t) + EZ_t(x,t) + FZ(x,t)\} = E_x E_t\{g(x,t)\}$$

The derivative properties of DINEM transform and the above conditional yield.

$$A\left([q_1(s)]^{\frac{2}{n}} I(s,v) - [q_1(s)]^{\frac{1}{n}} p_1(s)I_3(v) - p_1(s)I_4(v)\right) +$$

$$B\left([q_2(v)]^{\frac{1}{m}}[q_1(s)]^{\frac{1}{n}}I(s,v) - p_2(v)[q_1(s)]^{\frac{1}{n}}I_1(s) + p_2(v)Z(0,0) - [q_2(v)]^{\frac{1}{m}}p_1(s)I_3(v)\right) + C\left([q_2(v)]^{\frac{2}{m}}I(s,v) - [q_2(v)]^{\frac{1}{m}}p_2(v)I_1(s) - p_2(v)I_2(s)\right) + D\left([q_1(s)]^{\frac{1}{n}}I(s,v) - p_1(s)I_3(v)\right) + E\left([q_2(v)]^{\frac{1}{m}}I(s,v) - p_2(v)I_1(s)\right) + FI(s,v) = G(s,v)$$

$$I(s,v) = \frac{1}{A[q_1(s)]^{\frac{2}{n}} + B[q_1(s)]^{\frac{1}{n}}[q_2(v)]^{\frac{1}{m}} + C[q_2(v)]^{\frac{2}{m}} + D[q_1(s)]^{\frac{1}{n}} + E[q_2(v)]^{\frac{1}{m}} + F}.$$

$$\left\{\left(Bp_2(v)[q_1(s)]^{\frac{1}{n}} + C[q_2(v)]^{\frac{1}{m}}p_2(v) + Ep_2(v)\right)I_1(s) + \left(Cp_2(v)\right)I_2(s) + \left([q_1(s)]^{\frac{1}{n}}p_1(s)A + [q_2(v)]^{\frac{1}{m}}p_1(s)B + p_1(s)D\right)I_3(v) + p_1(s)AI_4(v) - p_2(v)\{C + G(s,v)\}\right\}$$

$$Z(x,t) = E_x^{-1}E_t^{-1}\left[\frac{1}{A[q_1(s)]^{\frac{2}{n}} + B[q_2(v)]^{\frac{1}{m}}[q_1(s)]^{\frac{1}{n}} + C[q_2(v)]^{\frac{2}{m}} + D[q_1(s)]^{\frac{1}{n}} + E[q_2(v)]^{\frac{1}{m}} + F}.\right.$$

$$\left.\left\{\left(Bp_2(v)[q_1(s)]^{\frac{1}{n}} + C[q_2(v)]^{\frac{1}{m}}p_2(v) + Ep_2(v)\right)I_1(s) + \left(Cp_2(v)\right)I_2(s) + \left([q_1(s)]^{\frac{1}{n}}p_1(s)A + [q_2(v)]^{\frac{1}{m}}p_1(s)B + p_1(s)D\right)I_3(v) + p_1(s)AI_4(v) - p_2(v)\{C + G(s,v)\}\right\}\right]$$

Where $Z(x,t)$ represents the term arising from the function g(x,t) and all conditions

5. Elucidative examples

Example 1: Consider the homogenous wave equation: $Z_{tt} = Z_{xx}$

With the conditions: $Z(x,0) = sin\,x\,, Z_t(x,0) = 1\; Z(0,t) = t, Z_x(0,t) = cos\,t$

Substituting

$$I_1(s) = \frac{P_1(s)}{[q_1(s)]^{\frac{2}{n}}+1} =, I_2(s) = \frac{P_1(s)}{[q_1(s)]^{\frac{1}{n}}}, I_3(v) = \frac{P_2(v)}{[q_2(v)]^{\frac{2}{m}}}, I_4(v) = \frac{P_2(v)[q_2(v)]^{\frac{1}{m}}}{[q_2(v)]^{\frac{2}{m}}+1}$$

$$Z(x,t) = E_x^{-1}E_t^{-1}\left[\frac{P_2(v)}{[q_2(v)]^{\frac{2}{m}}} + \frac{P_1(s)P_2(v)[q_2(v)]^{\frac{1}{m}}}{\left([q_1(s)]^{\frac{2}{n}}+1\right)\left([q_2(v)]^{\frac{2}{m}}+1\right)}\right] = t + sin\,x\,cos\,t$$

Example 2: Consider the heat equation : $Z_t = 2Z_{xx}$, With the conditions : $Z(x,0) = x + \sinh x\,, Z_t(x,0) = 2\sinh x\,, Z(0,t) = 0, Z_x(0,t) = 1 + e^{2t}$
Substituting

$I_1(s) = \frac{P_1(s)}{[q_1(s)]^{\frac{2}{n}}} + \frac{P_1(s)}{[q_1(s)]^{\frac{2}{n}-1}} =, I_2(s) = \frac{2P_1(s)}{[q_1(s)]^{\frac{2}{n}-1}}, I_4(v) = \frac{P_2(v)}{[q_2(v)]^{\frac{1}{m}}} + \frac{P_2(v)}{[q_2(v)]^{\frac{1}{m}-2}}$

$$Z(x,t) = E_x^{-1}E_t^{-1}\left[\frac{P_1(s)}{[q_1(s)]^{\frac{2}{n}}} + \frac{P_1(s)P_2(v)}{\left([q_2(v)]^{\frac{1}{m}-2}\right)\left([q_1(s)]^{\frac{2}{n}-1}\right)}\right] = x + e^{2t}\sinh x$$

Example 3: Consider the following Laplace equation: $Z_{xx} + Z_{tt} = 0$
The conditions: $Z(x,0) = e^{2x} - 4, Z_t(x,0) = e^{2x} - 4, Z(0,t) = 1 - 4e^t, Z_x(0,t) = 2 - 4e^t$

$I_1(s) = \frac{P_1(s)}{[q_1(s)]^{\frac{1}{n}-2}} - \frac{4P_1(s)}{[q_1(s)]^{\frac{1}{n}}} = I_2(s), I_3(v) = \frac{P_2(v)}{[q_2(v)]^{\frac{1}{m}}} - \frac{4P_2(v)}{[q_2(v)]^{\frac{1}{m}-1}}, I_4(v) = \frac{2P_2(v)}{[q_2(v)]^{\frac{1}{m}}} - \frac{4P_2(v)}{[q_2(v)]^{\frac{1}{m}-1}}$

$$Z(x,t) = E_x^{-1}E_t^{-1}\left[\frac{P_1(s)}{[q_1(s)]^{\frac{1}{n}-2}} - \frac{4P_2(v)}{[q_2(v)]^{\frac{1}{m}-1}}\right] = e^{2x} - 4e^t$$

Conclusions

In this paper, we introduced a new double transform called the DINEM transform, several properties and theorems of the double transform. Finally, we applied DINEM to solve some applications.

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