Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 28 (2025), No. 1, pp. 123–130
DOI : 10.47974/JIM-1802

Derivation of Cauchy dispersion relation dependent on temperature for silicon

Iqbal Adil Jbain *
Sundus Y. Hasan †
Department of Physics
College of Education for Girls
Kufa University
Najaf
Iraq

Abstract

In order to study the optical characteristics of silicon at different temperatures, a general Cauchy dispersion equation for Silicon material that depends on temperature and wavelength was derived in this paper. The wavelengths vary between 1.2 and 14 μm, and the temperatures vary from 100 to750K. There was a great deal of agreement between the computed refractive index of three digits and previously published data. In addition, the phase velocity for the waves in the material was measured and, when compared to actual values, The results demonstrated good concordance.

Subject Classification (2020): *30E20, 32A26.*

Keywords: *Ca dispersion Eq, Silicon, Refractive-Index,* v_p.

1. Introduction

The study of the optical characteristics of Silicon is of great importance in multiple fields. Understanding these properties helps develop photonic and electronic technology, such as in photovoltaic [1], detectors [2], filters [3]. Silicon can also be used in the manufacture of lenses [4] and other optical devices, Understanding these properties helps improve performance and develop future applications The refractive index of

† *ORCID-ID:* 0000-0001-9736-3964

* *E-mail:* iqbala.alghanami@student.uokufa.edu.iq (Corresponding Author)
† *E-mail:* Sundus.Y.Hasan@student.uokufa.edu.iq

© Taru Publications

Silicon was calculated in this study using Cauchy's dispersion relation, a theoretical relationship which the mathematician Augustin-Louis Cauchy named and developed in 1837 that describes how a material's refractive index varies with light wavelength. In 1980, Li studied Refractive index compiled and analyzed. Recommended values of refractive index for the transparent spectral region were generated in the range (1.2 to 14μm) and 100-750 K. [5] In 2008, Martin A. G studied of the optical properties of intrinsic Silicon, Improved values of refractive index at 300 K are tabulated over the 0.25-1.45 mm wavelength range at 0.01 mm intervals [6]. In 2006, Bradley J. F et.al studied the refractive index of Silicon making use of the high-accuracy cryogenic. The refractive index measurement system (CHARMS), as a function of both wavelength and temperature, we reported absolute refractive- index at temperatures ranging from 20 to 300 K at wavelengths from 1.1 to 5.6μm [7]. In 2022, Douglas. B. L et. study of cryogenic refractive index of Silicon has been carried out for sample materials produced in the 2019-2020.Represents the first significant study of interspecies index variability of Si using CHARMS. These new measurements spanned the wavelength region from 1.11-5.43 μm for Si [8]. In 2023, Allyson. T and Nuggehalli. M. R study the Forouhi-Bloomer dispersion equations to calculate refractive index of Silicon for the wavelength spectrum 1-10 μm range [9, 12-17].

2. Results

The study used the Cauchy dispersion equation to calculate the refraction of the Silicon index across a wide range of temperature and wave length regions. The values of the refractive index were gained by applying the Cauchy formula of equation (1):

$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \frac{D}{\lambda^6} \quad (1)[10]$$

To be able to solve the four-parameter equation (A, B, C, and D), Four of values refractive- index with wavelengths for each temperature must be taken into consideration when solving simultaneous equations. The values of the four parameters and a Matlab code are needed to solve these equations, as shown at the table 1.

Table 1

The values of the parameters in Equation. (1) at 14 various temperatures

T(K)	A(μm^2)	B(μm^2)	C(μm^2)	D(μm^2)
100	3.389451	0.139125	-0.00461	0.001841
150	3.394324	0.142277	-0.0035	0.000213
200	3.400305	0.144616	-0.00142	-0.00173
250	3.406976	0.148181	-0.00334	0.000252
293	3.413456	0.150573	-0.00152	-0.00209
350	3.422827	0.154085	-0.00319	0.00029
400	3.431699	0.15747	-0.00372	0.000267
450	3.44097	0.161035	-0.00564	0.00225
500	3.450742	0.16442	-0.00617	0.002228
550	3.460922	0.166759	-0.00409	0.000283
600	3.471196	0.169911	-0.00298	-0.00134
650	3.481668	0.173296	-0.00351	-0.00137
700	3.492247	0.175816	-0.00282	-0.00131
750	3.502718	0.17938	-0.00475	0.000677

The values measured from Equation. 1 were compared to the useful values of reference so as to check the accuracy of these parameters. [5] As shown in Figs. (1) and (2), it is obvious that the values reach 3 digits.

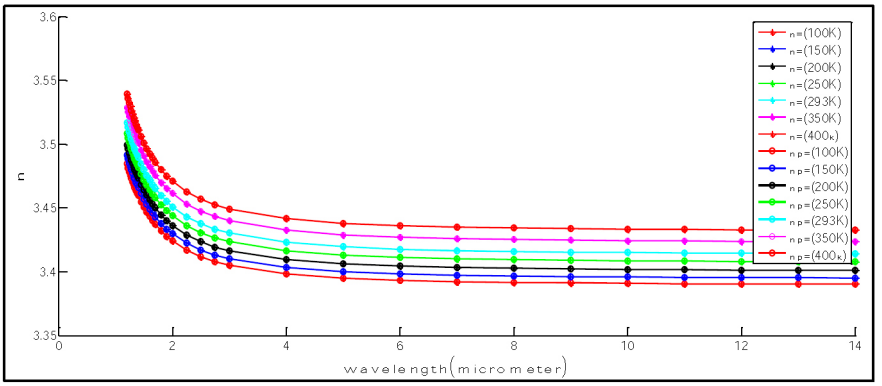


Figure 1

Refractive index values were calculated by Equation. 1, n, and the values were calculated by Li [5], np. at (100, 150, 200, 250, 293, 350, 400, 450) K.

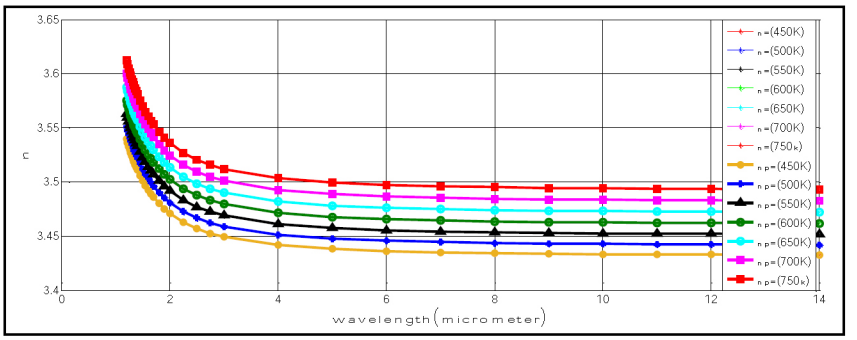


Figure 2

Refractive index values were calculated by Eq. (1), n, and the values were calculated by Li. [5], np. At (450, 500, 550, 600, 650, 700, 750) K.

3. Equation for Cauchy dispersion as a function of(T)

The coefficients were expressed of temperature in order to develop the Cauchy equation with four coefficients in terms of wavelength and temperature. He form of an equation with four constants, as follows:

$$\begin{aligned} A &= a_1T^3 + a_2T^2 + a_3T + a_4 \\ B &= b_1T^3 + b_2T^2 + b_3T + b_4 \\ C &= c_1T^3 + c_2T^2 + c_3T + c_4 \\ D &= d_1T^3 + d_2T^2 + d_3T + d_4 \end{aligned} \tag{2}$$

Hence, the following is the form of the dependent on temperature Cauchy dispersion relation:

$$n(\lambda,T) = a_1T^3 + a_2T^2 + a_3T + a_4 + \frac{b_1T^3+b_2T^2+b_3T+b_4}{\lambda^2} + \frac{c_1T^3+c_2T^2+c_3T+c_4}{\lambda^4} + \frac{d_1T^3+d_2T^2+d_3T+d_4}{\lambda^6} \tag{3}$$

In order to obtain these sixteen parameters from Eq. (3), four equations need to be solved for each of the four parameters at the four temperatures (200, 400, 600, and 700 K), and the calculated values are shown in Table 2.

To assure the correctness of equation (3), the refractive index values were calculated at temperatures as shown in Figures (3, 4) where we compare the practical values and the fig Fig2: Refractive index values were calculated by Eq. (1), n, and the actual values were taken from ref. [5], np. at temperatures (450, 500, 550, 600, 650, 700, 750) K.

Table 2

Parameters values of Equation (3)

A		B		C		D	
a_1	-1.15738*10^{-10}	b_1	-1.07728*10^{-11}	c_1	*9.00465310^{-11}	d_1	*1.46586110^{-11}
a_2	2.40175*10^{-7}	b_2	7.7650126*10^{-9}	c_2	1.459764*10^{-7}	d_2	10* 2.210402^{7}
a_3	4.52707*10^{-5}	b_3	6.2628154*10^{-5}	c_3	7.38523*10^{-5}	d_3	0.00010157442
a_4	3.38257	b_4	0.131866	c_4	0.00822973781	d_4	0.0143774566-

Theoretical values, and they agree well. Thus, the equation is suitable for all temperatures, and the values reach 3 digits.

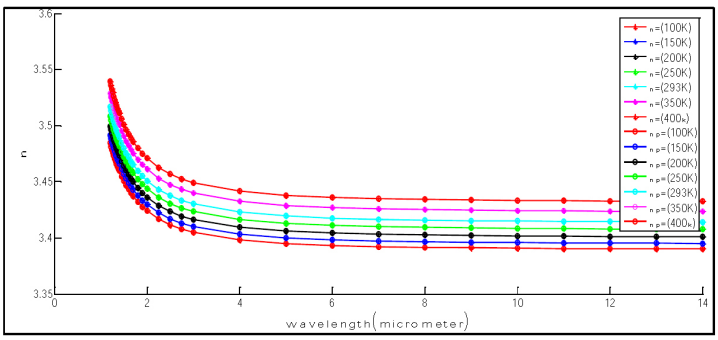


Figure 3

Refractive index values were calculated by Eq. (3), n, and the values were calculated by Li. [5], np. at temperatures (100, 150, 200, 250, 293, 350, 400)K

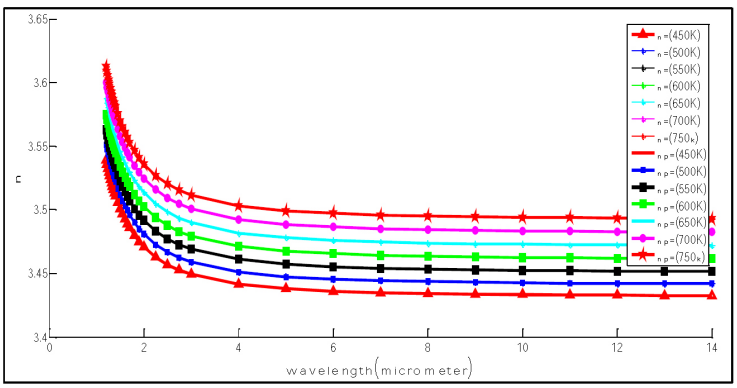


Figure 4

Refractive index values were calculated by Eq. (3), n, and the values were calculated by Li. [5], np. at temperatures (450, 500, 550, 600, 650, 700, 750)K.

4. Phase velocity

Phase velocity is the velocity at which a wave's phase moves through space. v_p can be expressed in terms of the wave number K and the angular frequency w:

$v_P = \frac{w}{k}$ [11] Considering that 3×10^8m/sec is the speed of light, the phase velocity can be expressed:

$$vp = \frac{c}{n} = \frac{c}{A+\frac{B}{\lambda^2}+\frac{C}{\lambda^4}+\frac{D}{\lambda^6}} \tag{4}$$

The phase velocity was calculated from the practical values of the refractive index as in Equation 5

$$vp_p = \frac{c}{n_p} \tag{5}$$

When comparing the practical values of phase velocity that were calculated from Equation 5 and the theoretical values that were calculated from Equation (4), they agree well and that the values reached 3 reach to 3digits, As demonstrated in Fig (5), demonstrates that (v_P)is inversely proportional to temperature and directly with wavelength.

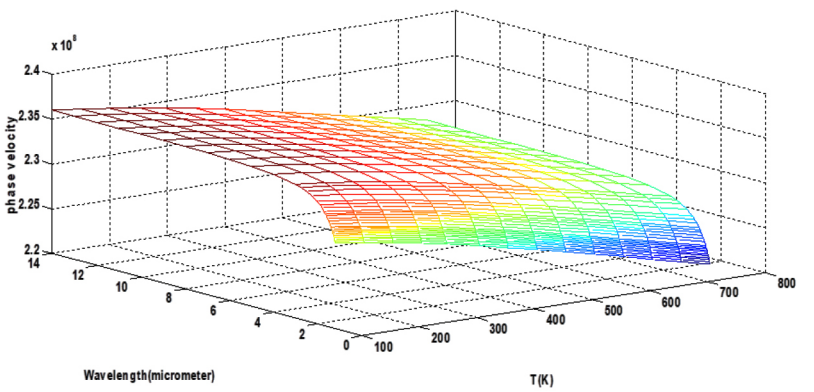


Figure 5

3-D representing the phase velocity is dependent on temperature and wavelength.eq5.

5. Conclusions

A Cauchy equation was used in this study to examine the optical attributes of silicon, and the results may be summarized as follow, the

Cauchy formula is suitable for studying the optical attributes of Silicon in the spectral region, which were generated in the ranges of 1.2 to 14 μm and temperatures of 100-750 K, the refractive index of silicon is inversely correlated with wavelength and directly related to temperature and the phase velocity of Silicon is directly proportional to $\lambda(\mu m)$and inversely with (T). The time derivative-dependent Cauchy equation is derived in this work.

References

[1] P. Singh and N. M. Ravindra, "Temperature dependence of solar cell performance—an analysis," *Solar Energy Materials and Solar Cells,* vol. 101, pp. 36-45 (2012).

[2] M. P. Lepselter, A. T. Fiory, and N. M. Ravindra, "Platinum and rhodium silicide-germanide optoelectronics," *Journal of Electronic Materials,* vol. 37, pp. 403-416 (2008).

[3] F. Ming Tong, N. M. Ravindra, and W. F. Kosonocky, "Design and simulation of permissible tolerance during fabrication of infrared-induced transmission filters," *Optical Engineering,* vol. 36, no. 2, pp. 549-557 (1997).

[4] V. V. Aristov, M. Grigoriev, S. Kuznetsov, L. Shabelnikov, V. Yunkin, C. Rau, A. A. Snigirev, I. Snigireva, T. Weitkamp, and M. Hoffmann, presented at the Advances in X-Ray Optics (2001) (unpublished).

[5] H. H. Li, "Refractive index of silicon and germanium and its wavelength and temperature derivatives," *Journal of Physical and Chemical Reference Data*, vol. 9, no. 3, pp. 561-658 (1980).

[6] M. A. Green, "Self-consistent optical parameters of intrinsic silicon at 300 K including temperature coefficients," *Solar Energy Materials and Solar Cells*, vol. 92, no. 11, pp. 1305-1310 (2008).

[7] B. J. Frey, D. B. Leviton, and T. J. Madison, presented at the Optomechanical Technologies for Astronomy (2006) (unpublished).

[8] D. B. Leviton, M. A. Quijada, and K. H. Miller, presented at the Advances in Optical and Mechanical Technologies for Telescopes and Instrumentation V (2022) (unpublished).

[9] A. Tarifa and N. M. Ravindra, presented at the TMS Annual Meeting & Exhibition (2023) (unpublished).

[10] V. Bonal, J. A. Quintana, J. M. Villalvilla, R. Muñoz-Mármol, J. C. Mira-Martínez, P. G. Boj, M. E. Cruz, Y. Castro, M. A. Díaz-García,

"Simultaneous determination of refractive index and thickness of submicron optical polymer films from transmission spectra," Polymers, vol. 13, no. 15, p. 2545 (2021).

[11] L. Brillouin, Wave Propagation and Group Velocity, Academic Press (2013).

[12] M. R. Farahani, S. Jafari, S. A. Mohiuddine, and M. Cancan, "Intuitionistic fuzzy stability of generalized additive set-valued functional equation via fixed point method," *Mathematical Statistician and Engineering Applications,* vol. 71, no. 3s3, pp. 142-154 (2022). [Online]. Available: https://www.philstat.org.ph/index.php/MSEA/article/view/355.

[13] M. R. Farahani, J. E. N. Valdes, M. Cancan, S. Jafari, and C. Park, "Fuzzy stability in the sense of Hyers-Ulam-Rassias of a functional equation on quadratic forms," *J. Interdiscip. Math.*, vol. 26, no. 6, pp. 1355-1365 (2023). [Online]. Available: https://doi.org/10.47974/JIM-1633.

[14] M. A. Saeed, H.K. Alkhayyat, and S.M. Kadham, "Enhance MRI images of lung cancer using hybrid transform," *J. Discrete Math. Sci. Cryptogr.,* vol. 26, no. 7, pp. 1897–1902 (Jan. 2023), [Online]. Available https://doi.org/10.47974/jdmsc-1682.

[15] S.N.J. Alamiry, S.M. Kadham, M. A. Mustafa, and N. K. Abbass, "Encryption and enhance medical image using hybrid transform (Ã-module and partial fuzzy Ȟ-transform)," *J. Discrete Math. Sci. Cryptogr.*, vol. 26, no. 7, pp. 1903–1910 (2023), [Online]. Available https://doi.org/10.47974/jdmsc-1683.

[16] A.M. Darweesh, R.H. Hasan, and S.M. Kadham, "Results of Fourth-Order Differential Superordination and Subordination for Univalent Functions Defined by Integral Operator," *Mathematical Modelling and Engineering Problems,* vol. 11, no. 2, pp. 385–392 (Feb. 2024), doi: https://doi.org/10.18280/mmep.110210.

[17] S. Jafari, M. R. Farahani, and M. Rajesh, "Double fuzzy rarely continuous functions," *Annals of Fuzzy Mathematics and Informatics,* vol. 26, no. 1, pp. 35-45 (2023).

Received June, 2024