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Derivation of a numerical method for calculating single integrals

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Abstract

The major goal of this research is to find a numerical method that determines the values of single integrals when they are continuous integrals but their derivatives are not defined at one end of the integral. The error value (correction limits) is additionally calculated using the Lagrange formula, and the method is then contrasted with well-known numerical methods devoid of acceleration techniques. After using accelerated techniques, the procedure delivered excellent results with great precision in condensed time intervals. We labeled it with the letter NK2.

Subject Classification (2010): *34Hxx, 34-XX, 34Lxx.*

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1. Introduction

Numerical integration is numerical analysis. It is an area of mathematics that links computers with analytical mathematics. In applications related to engineering and physics, numerical integration is

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crucial. Dentists have recently used numerical approaches to find the surface area of the cross section of the root canal by using numerical integration in dentistry. The researchers used many different numerical methods to calculate the values of single integrations identified in their periods of integration and improved the results in multiple evasive ways. Usually we get a rough approximation or a rough solution using these methods. We now have an error ratio as a result of this. Numerous academics have worked in this area to identify the approximate solution to a problem and assess the error in numerical analysis and numerical integration in particular. AlSharifi [1], Mohammed, Fox [2] and others [3, 5] since the constraints of computer memory prevent it from fully storing or representing numbers. Therefore, we must develop numerical techniques to locate accurate results using the fewest bits and mattresses possible. In this study, single integrals were calculated using a novel numerical method based on the Lagrange formula and compared to previous methods without the use of acceleration techniques. The findings were excellent, and in a few partial intervals we obtained a high degree of accuracy and an accelerated method for calculating the real value after applying Romberg acceleration, [4] determining the limits of correction, and comparing it to other methods.

1.1 *Method of derivation*

Let's g say a function is defined on the field R and assume that χ_0, χ_1, …, χ_n it is $n+1$ from the different points in R, so there is a multi-boundary $\mathrm{P}(\chi)$ of the class $n \geq$ so that

$$g(\chi) = \mathrm{P}(\chi) + \mathrm{E}(\chi) = \sum_{k=0}^{n} g(\chi_k) L_k(\chi) + \mathrm{E}(\chi)$$

where $k = 0, 1, 2, \ldots n$ and L_k is a polynomial of degree n ≥ in which

$$L_k(\chi_j) = \begin{cases} 1 & \text{if } j=k \\ 0 & \text{if } j \neq k \end{cases}$$

$\mathrm{E}(\chi)$ the amount of error.

A polynomial is $L_k(\chi)$ what are the Lagrange's knowledge of this

$$\frac{(\chi-\chi_0)\ldots(\chi-\chi_{k-1})(\chi-\chi_{k+1})\ldots(\chi-\chi_n)}{(\chi_k-\chi_0)\ldots(\chi_k-\chi_{k-1})(\chi_k-\chi_{k+1})\ldots(\chi_k-\chi_n)}$$

it's mean

$$L_k(\chi)=\prod_{\substack{j=0\\ j\neq k}}^{n}\frac{(\chi-\chi_j)}{(\chi_k-\chi_j)}.$$

If we integrate the following:

$Nk2=\Psi=\int_{\chi_0}^{\chi_n} g(\chi)d\chi$ the integration interval $[\chi_0,\chi_n]$ will be divided into n subintervals of equal length. $h=\frac{\chi_n-\chi_0}{n}$

$$\Psi=\int_{\chi_0}^{\chi_n} g(\chi)d\chi=\sum_{k=0}^{n} g(\chi_k)\int_{\chi_0}^{\chi_n} L_k(\chi)d\chi+\mathrm{E}(\chi)$$

$$\Psi=\int_{\chi_0}^{\chi_n} g(\chi)d\chi=\sum_{k=0}^{n} g(\chi_k)\int_{\chi_0}^{\chi_n} L_k(\chi)d\chi+\frac{\overset{(n+1)}{g(\chi)}}{(n+1)!}\prod_{i=0}^{n}(\chi-\chi_i)$$

A number with n is odd.

$$\Psi=\int_{\chi_0}^{\chi_n} g(\chi)d\chi=\sum_{k=0}^{n} g(\chi_k)\int_{\chi_0}^{\chi_n} L_k(\chi)d\chi+\frac{\overset{(n+1)}{g(\zeta)}}{(n+1)!}\int_{\chi_0}^{\chi_n}\prod_{i=0}^{n}(\chi-\chi_i)d\chi$$

n is an even number in this case

$$\Psi=\int_{\chi_0}^{\chi_n} g(\chi)d\chi=\sum_{k=0}^{n} g(\chi_k)\int_{\chi_0}^{\chi_n} L_k(\chi)d\chi+\frac{\overset{(2n)}{g(\zeta)}}{(2n)!}\int_{\chi_0}^{\chi_n}\prod_{i=0}^{n}(\chi-\chi_i)d\chi$$

where $\zeta\in[\chi_0,\chi_n]$, if we divide the period by taking the points so $\overline{\chi_j}=\frac{\chi_{j+1}-\chi_j}{2}$

We get multiple borders (approximation of Lagrange) with other coefficients, in general, the approximation of the Lagrange can be written in the form

$$Q(h)=h(\lambda g_0+\overline{\lambda}g_1+\overline{\overline{\lambda}}g_2+\cdots+\overline{\lambda}g_{n-1}+\lambda g_n).$$

that is $g_r=g(\chi_r)$, $\chi_r=\chi_0+rh$ and $r=0,1,\cdots n$.

The weight coefficients take order $(\lambda,\overline{\lambda},\overline{\overline{\lambda}},\overline{\lambda},\overline{\overline{\lambda}},\cdots,\overline{\overline{\lambda}},\overline{\lambda},\lambda)$

where $Q(h)$ the value of integration is without using the correction limits, when the weights of the first case of fragmentation form (0,1,0.5,1,0.5 …, 0.5,1,0). Additionally, in the second situation of the divisor $(1,1,1,\ldots,1)$, where the weight coefficients are close to the Lagrange.

$$\Psi=\int_{\chi_0}^{\chi_n} g(\chi)d\chi=\frac{h}{3}\left[\frac{g(\chi_n)+g(\chi_0)}{2}+\sum_{i=1}^{n-1} g(\chi_i)+2\sum_{i=0}^{n-1} g(\chi_i+h/2)\right]+$$
$$a_1h^2(g^{(1)}(\chi_n)-g^{(1)}(\chi_0))+a_2h^4(g^{(3)}(\chi_n)-g^{(3)}(\chi_0))+\cdots$$

where $a_1, a_2, a_3, \cdots$ is constants.

$$\Psi=\int_{\chi_0}^{\chi_n} g(\chi)d\chi=\frac{h}{3}\left[\frac{g(\chi_n)+g(\chi_0)}{2}+\sum_{i=1}^{n-1} g(\chi_i)+2\sum_{i=0}^{n-1} g(\chi_i+h/2)\right]$$
$$+\Phi_1h^2+\Phi_2h^4+\Phi_3h^6+\cdots$$

Where $\Phi_1, \Phi_2, \Phi_3, \cdots$ the constants of their values do not depend on h but depend on the value of derivatives first and top of the function at the border of integration.

This rule applies to integrations whose interactions are continuous during the integration period now, if the integrand is singular at one end or both of limits of the region or if the integrand is continuous at the ends of integration but the derivative is singular at one or both ends of integration, then there are three cases of limits of correction.

1. The first case

Since the integrator is defined at every point during the integration period aside from the point where it is unknown when it is derived, we can decompose the integration as follows:

$$\Psi=\int_{\chi_0}^{\chi_n} g(\chi)\ d\chi=\int_{\chi_0}^{\chi_{n-1}} g(\chi)\ d\chi+\int_{\chi_{n-1}}^{\chi_n} g(\chi)\ d\chi \tag{1}$$

Calculate the second integration of the equation (1), and then publish the function g with a Tyler series around the point $\chi=\chi_{n-1}$ you get the

$$g(\chi)=\sum_{i=0}^{\infty}\frac{g^{(i)}(\chi_{n-1})}{i!}(\chi-\chi_{n-1})^i \tag{2}$$

Additionally, by integrating equation (2) for the variable χ indicated period $[\chi_{n-1}, \chi_n]$
we get:

$$\int_{\chi_{n-1}}^{\chi_n} g(\chi)\,d\chi = \sum_{i=0}^{\infty} \frac{g^{(i)}(\chi_{n-1})}{(i+1)!}(\chi_n - \chi_{n-1})^{(i+1)} \tag{3}$$

and using equation (3), we obtain a correction for $(\chi_n - \chi_{n-1})$ in h.

$$\int_{\chi_{n-1}}^{\chi_n} g(\chi)\,d\chi = \sum_{i=0}^{\infty} \frac{g^{(i)}(\chi_{n-1})}{(i+1)!} h^{(i+1)} \tag{4}$$

Then we make up for χ in χ_n Once and in $\chi_{n-1} + \frac{h}{2}$ again in equation (2) we then compensate the output for $(\chi_n - \chi_{n-1})$ in h and $(\chi_n - \chi_{n-1} + \frac{h}{2})$ in $h/2$ we get

$$g(\chi_n) = \sum_{i=0}^{\infty} \frac{g^{(i)}(\chi_{n-1})}{i!} h^i \tag{5}$$

$$g(\chi_{n-1} + h/2) = \sum_{i=0}^{\infty} \frac{g^{(i)}(\chi_{n-1})}{i!} (h/2)^i \tag{6}$$

Multiply equations (5) and (6) in $\frac{-h}{3}$ we combine it with equation (4)

$$\int_{\chi_{n-1}}^{\chi_n} g(\chi)\,d\chi = \frac{h}{3}\left[\frac{g(\chi_n) + g(\chi_{n-1})}{2} + 2g(\chi_{n-1} + \tfrac{h}{2})\right] + \sum_{i=1}^{\infty} h^{(i+1)} g^{(i)}(\chi_{n-1}) a_i \tag{7}$$

$$\int_{\chi_0}^{\chi_{n-1}} g(\chi)d\chi = \frac{h}{3}\left[\frac{g(\chi_0) + g(\chi_{n-1})}{2} + \sum_{i=1}^{n-2} g(\chi_i) + 2\sum_{i=0}^{n-2} g(\chi_i + h/2)\right] + \Phi_1 h^2 + \Phi_2 h^4 + \Phi_3 h^6 + \cdots \tag{8}$$

Where a_i Constants do not affect the value of integration as for the first integration $\int_{\chi_0}^{\chi_{n-1}} g(\chi)\ d\chi$ In equation (1) it is continuous and its value is determined by the rule of continuous Integration Its $\Phi_1, \Phi_2, \Phi_3, \cdots$ constants depend on the derivative values of the function g of the variable χ By combining equations (7) and (8) we get

$$\int_{\chi_0}^{\chi_n} g(\chi)d\chi = \frac{h}{3}\left[\frac{g(\chi_0) + g(\chi_n)}{2} + \sum_{i=1}^{n-1} g(\chi_i) + 2\sum_{i=0}^{n-1} g(\chi_i + h/2)\right] + \Phi_1 h^2 + \Phi_2 h^4 + \Phi_3 h^6 + \cdots + \sum_{i=1}^{\infty} h^{(i+1)} g^{(i)}(\chi_{n-1}) a_i \tag{9}$$

Note: the symbols used in the table below represent numerical methods and methods derived by researchers where the values obtained were compared using the NK2 method with them and have given good results and a few partial periods compared to others:

Sue represents the proposed method of researcher Mohammed [6], F is the derived method by AlSharifi [1].

The symbols (M, T, S) represent the Newton Cuts (Simpson, Trapezoidal, and the middle point

2. Examples

Note from table 1 that the given functions are continuous and can be derived during the integration period.

When using the method on examples (1,2,3,4,6), We found partial separations between 44 and 222 and precision between 10 and 11 decimal places. We obtained the same accuracy but partial intervals while using the S (Simpson) rule. As a result, the NK2 rule outperforms the S rule. For rules (M, T, S, Su, F), we found accuracies ranging from 4 to 7 and partial separations ranging from 135 to 688. As a result, rules (M, T, S, Su, F) About the superiority of the NK2 technique in terms of accuracy and number of sub periods.

We will only compare the NK2 rule and the S rule using the Romberg acceleration to improve the results, as the Simpson technique is the closest rule to the NK2 rule in terms of accuracy and number of sub-periods.

In Table 2, which details the outcomes of applying the rules NK2 and S along with Romberg's acceleration, we observe that the accompanying correction limits for the first integrated integral integration during the integration period are as follows:

1. Accompanied correction limits of the base $R-NK2$ is

$$E(X)=h^2\,,\,h^4\,,\,h^6\,,\,h^8\,,\cdots$$

2. Accompanied correction limits of the base $R-S$ is

$$E(X)=h^4\,,\,h^6\,,\,h^8\,,\cdots$$

With the NK2 method, we were able to reach nine decimal places, and with the Romberg acceleration, we achieved fourteen decimal places at n = 16.

NK2 favours the rule over the S rule since we obtained twelve orders using R-S while having the same number of subperiods.

The limits of correction for the second integration, whose integrand is continuous throughout the integration period but whose derivative is singular at one or both integration endpoints, are as follows:

1. Accompanied correction limits of the base $R-NK2$ is:

$$E(X)=h^{1.5},\ h^{2.5},\ h^{3.5},\ h^{4.5},\ h^{5.5},\ h^{6.5},h^{7.5},\ h^{8.5}$$

2. Accompanied correction limits of the base $R-S$ is:

$$E(X)=h^{2.5},\ h^{3.5},\ h^{4.5},\ h^{5.5},\ h^{6.5},\ h^{7.5},\ h^{8.5},\ h^{9.5},\cdots$$

At N = 256 we obtained five correct decimal places when using the NK2 rule while the correct number of decimal places became fifteen decimal places after using the Romberg's acceleration [7-15], while using the S base we obtained four correct decimal places and after improving the results using R-S acceleration we obtained three decimal mattresses true decimal.

As for integration (3) whose integration is singularity derivatives at the bottom of the integration, the accompanying correction limits are:

1. Accompanied correction limits of the base $R-NK2$ is:

$$E(X)=h^{1},h^{1},h^{2},h^{2},h^{2},h^{3},h^{3},h^{4},h^{4},h^{4},\cdots$$

2. Accompanied correction limits of the base $R-S$ is:

$$E(X)=h^{1},h^{1},h^{2},h^{2},h^{3},h^{3},h^{4},h^{4},h^{4},\cdots$$

At n = 512 we obtained correct decimal places when using base NK2 while the correct number of decimal places became eight decimal places after the use of Romberg's acceleration. When using the S base we obtained the correct decimal places. After improving the results using R-S acceleration we obtained seven correct decimal places.

As for integration (4) is not known as the analytical value and is continuous integrator during the integration period for which the correction limits are for integration:

1. Accompanied correction limits of the base $R-NK2$ is:

$$E(X)=h^{2},h^{4},h^{6},h^{8},h^{10},h^{12},h^{14},h^{16},h^{18},\cdots$$

2. Accompanied correction limits of the base $R-S$ is:

$$E(X)=h^{4},h^{6},h^{8},h^{10},h^{12},h^{14},h^{16},h^{18},\cdots$$

When n = 256, we can see from Table 3 and the R-NK2 method that the horizontal results (for four columns) are equal and tend to be close to a specific value of 0.02532866677508314, rounded to sixteen decimal places.

Table 1

Gives the value of the precise integration and the ratio of the total mattresses to the total number of partial integrations. periods using rules *NK*2, *S, T, M* Su,F

	integration	Analytical value	$NK2/n$	S/n	T/n	M/n	F/n	Su/n
1	$\int_1^2 (\cos(x)/x)dx$	0.085576905873897	11/204	11/408	6/382	6/326	6/270	6/191
2	$\int_2^3 e^{(1-x^2)}dx$	0.011215515289360	11/106	11/210	7/688	7/356	7/488	7/344
3	$\int_1^2 \ln(x+x^2)dx$	1.295836866004329	11/94	11/186	6/390	6/210	6/276	6/195
4	$\int_1^2 \frac{dx}{x+x^2}dx$	0.287682072451781	11/156	11/310	6/346	6/212	6/246	6/173
5	$\int_2^3 x^2e^x dx$	85.649572418077042	10/222	10/442	4/512	4/672	4/362	4/256
6	$\int_1^2 \frac{dx}{\sqrt[4]{1+x}}$	0.796952301929798	11/44	11/88	7/270	7/184	7/192	7/135

Table 2

Displays the value of the precise integration as well as the number of mattresses obtained, the number of partial periods using the rules S and NK2, and the application of Romberg's acceleration using R-S and R-NK2.

	Integration	Analytical value for integration	n	NK2	R–NK2	S	R–S	Integral state
1	$\int_3^4 e^{\sqrt[3]{\chi}} d\chi$	4.56316466131465	16	9	14	8	12	Continuous integration during the term of integration

Contd...

2	$\int_0^1 \sqrt{1-\chi}\sqrt{\chi}d\chi$	0.392699081698724	256	5	15	4	13	While integration is ongoing,the singularity derivatives at the bottom of the upper and lower integration
3	$\int_0^1 -\ln(\chi)/(1+\chi)d\chi$	0.822467033424113	512	2	8	2	7	The bottom integration limit is a singularity during the integration phase.
4	$\int_2^3 \sin(\pi\chi^2)/\chi^2 d\chi$	unknown-real values	256	9	16	8	15	Continuous integration during the term of integration

From Table 4 of the method, the value of the results appears horizontally (for three columns) rounded to fifteen 0.025328667750831 decimal places at the same number of sub-periods n = 256. Therefore, the value of the integral is very close to the series of correction limits (0.02532 86677508314).

The tables clearly show the NK2 method's numerical accuracy in calculating single integrations whether the integrand is continuous or ontinuous but the derivative is singular or the analytical value is unknown

3. Conclusion

It has been shown that the method is highly efficient and accurate in calculating the values of integrals, whether the integral is continuous or continuous, but the single derivative is in one of the limits of the integral. From the table, the method gave good results in terms of getting close to the true value after using the Ramburk acceleration. In the first and fourth examples, where the integrals were continuous, as well as in the second and third examples, where the integral is not defined at one of the integration limits. As a result, one can trust this method of calculating integrals as in Table 2

Table 3

R – NK 2

n	NK2	h^2	h^4	h^6	h^8	h^{10}	h^{12}	h^{14}	h^{16}
1	0.0754247233265651								
2	-0.0372194776666687	-0.0747675446644133							
4	0.0286518477722220	0.0506089562518523	0.0589673896462700						
8	0.0254481156760903	0.0243802049773798	0.0226316215590816	0.0220548633354754					
16	0.0253353902345329	0.0252978150873471	0.0253589890946782	0.0254022806428623	0.0254154077695580				
32	0.0253290776939140	0.0253269735137077	0.0253289174087984	0.0253284400804511	0.0253281505096181	0.0253280652141539			
64	0.0253286932174772	0.0253285650586650	0.0253286711616621	0.0253286672529774	0.0253286681438501	0.0253286686498464	0.0253286687972055		
128	0.0253286693400961	0.0253286613809691	0.0253286678024560	0.0253286677491353	0.0253286677510810	0.0253286677506971	0.0253286677504775	0.0253286677504136	
256	0.0253286678501230	0.0253286673534653	0.0253286677516317	0.0253286677508250	0.0253286677508316	0.0253286677508314	0.0253286677508314	0.0253286677508314	0.0253286677508314

Table 4

R – S

n	S	h^2	h^4	h^6	h^8	h^{10}	h^{12}	h^{14}
2	0.075424723326565							
4	-0.037219477666669	-0.044729091066218						
8	0.028651847772222	0.033043269468148	0.034277751381392					
16	0.025448115676090	0.025234533536348	0.025110585346955	0.025074635676231				
32	0.025335390234533	0.025327875205096	0.025329356818885	0.025330214746226	0.025330464579139			
64	0.025329077693914	0.025328656857873	0.025328669265060	0.025328666568770	0.025328665055400	0.025328664615956		
128	0.025328693217477	0.025328667585715	0.025328667755998	0.025328667750080	0.025328667751235	0.025328667751893	0.025328667752085	
256	0.025328669340096	0.025328667748271	0.025328667750851	0.025328667750831	0.025328667750832	0.025328667750831	0.025328667750831	0.025328667750831

Also, when using the rule NK2, it gave good results compared to other rules. Also, when using the rule, it gave good results compared to other rules, so it can be relied upon in calculating single integrals as in table 1.

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