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An innovative secant technique for minima one variable problems

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Abstract

The classical Newtonian method is the most often used in single-variable scenarios. The second-order Taylor expansion, which the new secant technique is built on, does away with the need to compute the second derivative. Comparing the New Secant strategy to the current Newton method, numerical testing has demonstrated that it is both numerical and effective.

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1. Introduction

One kind of minimizing technique using only one variable is single-variable function minimization. In the following description of single-variable function minimizations, the function is considered to be of the type $\xi = \xi(\varsigma)$, which is comparable to finding the root of the nonlinear equation:

$$\xi'(\varsigma^*) = 0 \tag{1}$$

according to [1], as this is a necessary requirement for the extremum of $\xi(\varsigma)$. Actually, finding the minimum of the objective function using Newton's method, which is the most enticing in terms of convergence, is one of the simplest ways to reduce the functions of a single variable, as demonstrated in [2]. An improved version of Newton's function minimization technique is as follows:

$$\varsigma_{\tau+1} = \varsigma_\tau - \frac{\xi'(\varsigma_\tau)}{\xi''(\varsigma_\tau)}. \tag{2}$$

Due to its quadratic rate of convergence towards the minimum and its utilization of second derivative details regarding the function ξ, this methodology is categorized as a second-order technique. In certain practical situations, obtaining this derivative analytically may not be feasible. Moreover, the approach has the drawback of slow convergence when the second derivative tends to zero, see [3].

It is a good idea to employ Secant techniques since most of the approaches used to determine the lowest value of single functions, as seen in [4-5], are based on Newton's method to find the stationary point of a function, when the derivative is zero. They do not require the compute matrix. Using a forward-difference approximation for the second derivative in Newton's method, we may obtain the following results:

$$\varsigma_{\tau+1} = \varsigma_\tau - \frac{\xi'(\varsigma_\tau)(\varsigma_\tau - \varsigma_{\tau-1})}{\xi'(\varsigma_\tau) - \xi'(\varsigma_{\tau-1})}. \tag{3}$$

We call this method the secant technique. Compared to Newton's method, it is usually much faster. As demonstrated in [6,7], a successful convergence to the minimum requires that the function's second derivative be greater than zero. This idea was explored and presented as a secant method in several articles; for more details, see [9-20]. The explored secant techniques do not use matrix computation.

An efficient approximation-optimal secant strategy is being developed to investigate their main elements for single-variable issues and to provide a more robust solution. The numerical results proved the superiority of the Newton technique.

2. Derivation of the new secant method:

Effective improvement of the secant technique may be achieved by using a second-order Taylor series. The second-order Taylor's series [14], is located approximately at point x_k:

$$\xi(\varsigma) \cong \xi(\varsigma_\tau) + \xi'(\varsigma_\tau)(\varsigma - \varsigma_\tau) + \frac{1}{2}\xi''(\varsigma_\tau)(\varsigma - \varsigma_\tau)^2 \tag{4}$$

Use the following result to equalize the derivative by zero to obtain the minimum:

$$\xi'(\varsigma) \cong \xi'(\varsigma_\tau) + \xi''(\varsigma_\tau)(\varsigma - \varsigma_\tau) = 0 \tag{5}$$

Based on (5) and (4), we arrived to the following:

$$\xi''(\varsigma_\tau)(\varsigma - \varsigma_\tau)^2 \cong \xi(\varsigma_\tau) - \xi(\varsigma) - \frac{1}{2}\xi'(\varsigma_\tau)(\varsigma - \varsigma_\tau) \tag{6}$$

We calculate the estimated value as follows:

$$\xi''(\varsigma_\tau)(\varsigma - \varsigma_\tau)^2 = 2/3(\xi(\varsigma_\tau) - \xi(\varsigma)) - 2/3\xi'(\varsigma_\tau)(\varsigma - \varsigma_\tau) \tag{7}$$

The above equation may be rewritten as follows by changing $\varsigma_{\tau-1}$ to ς:

$$\xi''(\varsigma_\tau) = \frac{2/3(\xi(\varsigma_\tau) - \xi(\varsigma_{\tau-1})) - 2/3\xi'(\varsigma_\tau)(\varsigma_{\tau-1} - \varsigma_\tau)}{(\varsigma_{\tau-1} - \varsigma_\tau)^2} \tag{8}$$

By performing mathematical operations on (8), we get:

$$\xi''(\varsigma_\tau) = \frac{\frac{2/3(\xi(\varsigma_\tau) - \xi(\varsigma_{\tau-1}))}{(\varsigma_{\tau-1} - \varsigma_\tau)} - 2/3\xi'(\varsigma_\tau)}{(\varsigma_{\tau-1} - \varsigma_\tau)} \tag{9}$$

When (9) is added to (1), we obtain:

$$\varsigma_{\tau+1} = \varsigma_\tau - \frac{\xi'(\varsigma_\tau)(\varsigma_{\tau-1} - \varsigma_\tau)}{\frac{2/3(\xi(\varsigma_\tau) - \xi(\varsigma_{\tau-1}))}{(\varsigma_{\tau-1} - \varsigma_\tau)} - 2/3\xi'(\varsigma_\tau)} \tag{10}$$

2.1 *Innovative Secant Algorithm*

Stage 1. To put it $\boldsymbol{\varepsilon}, \boldsymbol{\varsigma_0}$ and $\boldsymbol{\xi(\varsigma_0)}$, set $\tau = 0$.
Stage 2. Let $\tau = \tau + 1$.
Stage 3. Suppose $\varsigma_\tau = \varsigma_{\tau-1} - \frac{\xi'(\varsigma_{\tau-1})}{\xi''(\varsigma_{\tau-1})}$.
Stage 4. Calculate $\varsigma_{\tau+1} = \varsigma_\tau - \frac{\xi'(\varsigma_\tau)(\varsigma_{\tau-1} - \varsigma_\tau)}{\frac{2/3(\xi(\varsigma_\tau) - \xi(\varsigma_{\tau-1}))}{(\varsigma_{\tau-1} - \varsigma_\tau)} - 2/3\xi'(\varsigma_\tau)}$
Stage 5. Stop, if the absolute value of $|\varsigma_{\tau+1} - \varsigma_\tau| \le \varepsilon$.

3. Analysis of convergence

The goal of optimization algorithm is to provide a sequence of approximations that will hopefully lead to the right answer over time. With the new secant technique, it is possible to prove the following truth.

Theorem 3.1: For an open interval I, let $\varsigma^* \in I$ be the zero of a sufficiently smooth function $\xi: I \to R$. The iterative approach offers second-order convergence if ς_0 is near enough to ς^*.

Proof: Assuming a fresh iterative schema, as:

$$\varsigma_{\tau+1} = \varsigma_\tau - \frac{\xi'(\varsigma_\tau)(\varsigma_{\tau-1} - \varsigma_\tau)}{\frac{2/3(\xi(\varsigma_\tau) - \xi(\varsigma_{\tau-1}))}{(\varsigma_{\tau-1} - \varsigma_\tau)} - 2/3\xi'(\varsigma_\tau)} \tag{11}$$

Assuming that $e_{\tau+1}$and e_τ are errors τ^{th} and $(\tau + 1)^{th}$ respectively, after which, we get the updated iterative schema:

$$e_{\tau+1} = e_\tau - \frac{\xi'(\varsigma_\tau)(e_{\tau-1}-e_\tau)}{\frac{2/3(\xi(\varsigma_\tau)-\xi(\varsigma_{\tau-1}))}{(e_{\tau-1}-e_\tau)} - 2/3\xi'(\varsigma_\tau)} \quad (12)$$

Now, using Taylor's theorem's mean value form for some υ between ς^* and ς_τ, we obtain:

$$\xi(\varsigma^*) = \xi(\varsigma_\tau) + \xi'(\varsigma_\tau)(\varsigma^* - \varsigma_\tau) + \frac{1}{2!}\xi''(\varsigma_\tau)(\varsigma^* - \varsigma_\tau)^2 + \frac{1}{3!}\xi'''(\upsilon)(\varsigma^* - \varsigma_\tau)^2 \quad (13)$$

The following results from the previous equation's derivative equaling zero:

$$-\xi'(\varsigma_\tau) = -e_\tau \xi''(\varsigma_\tau) + \frac{e_\tau^2}{2}\xi'''(\upsilon) \quad (14)$$

Applying (14) to (11) yields:

$$e_{\tau+1} = \frac{e_\tau^2}{2}\frac{\xi'''(\upsilon)}{\xi''(\varsigma_\tau)} \quad (15)$$

Second order convergence results from the new iterative method.

4. Application Examples

Our computational investigations are primarily focused on examining the effectiveness of algorithms for resolving six test issues from[8]. The parameters $\varepsilon = 10^{-10}$ are used in all of the numerical experiment procedures. The apps are all written in Matlab. For simplicity, we denote the Newton method by NWM, the number of iterations by BNIT and execution time by BET.

Example 1: Function $\xi(\varsigma) = e^{\varsigma} - 3\varsigma^2$, Initial. Approximation $\varsigma_0 = 0.25$.

Methods	BNIT	ς_k	BET
NWM	.4	.0.2045	.0.2660
Innovative S.	.3	.0.2033	.0.2030

Example 2: Function $\xi(\varsigma) = e^{-\varsigma} + \varsigma^2$,Initial. Approximation $\varsigma_0 = 1$.

Methods	BNIT	ς_τ	BET
NWM	.5	.0.3517	.0.3130
Innovative S.	.5	.0.4293	.0.2340

Example 3: Function $\xi(\varsigma) = -\varsigma e^{-\varsigma}$, Initial. Approximation $\varsigma_0 = 0$.

Methods	BNIT	ς_τ	BET
NWM	.7	.1	.0.3900
Innovative S.	.5	.1.1002	.0.3440

Example 4: Function $\xi(\varsigma) = 0.65 - 0.75/(1+\varsigma^2) - 0.65\varsigma \, tan^{-1}(1/\varsigma)$, Initial Approximation $\varsigma_0 = 0.1$.

Methods	BNIT	ς_τ	BET
NWM	.6	.0.4809	.0.7970
Innovative S.	.5	.0.5390	.0.4690

Example 5: Function $\xi(\varsigma) = 0.5\varsigma^2 - sin(\varsigma)$, Initial. Approximation $\varsigma_0 = 2$.

Methods	BNIT	ς_τ	BET
NWM	.5	.0.7391	.0.4210
Innovative S.	.3	.0.7467	.0.2040

Example 6: Function $\xi(\varsigma) = \varsigma^4 + 2\varsigma^2 - \varsigma - 3$, Initial. Approximation $\varsigma_0 = 1$.

Methods	BNIT	ς_τ	BET
NWM	.7	.0.2367	.0.6870
Innovative S.	.5	.0.1966	.0.3440

Conclusion

Our new iterative method for solving nonlinear equations is based on the Taylor series formula and only requires the use of up to two terms. Based on the numerical computation, this method is more effective than the well-known Newton's method. Comprehensive numerical simulations demonstrate the technology's theoretical characteristics as well as its real-world performance. Not every strategy has to converge to the global minimum; in fact, some might not even succeed in doing so.

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