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# A new optimal Dai-Liao algorithm for solving iteration problems

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**Abstract**

The conjugate gradient approach is fascinating when applied to mathematical formulations. An innovative formula based on curvature needs has been developed. In contrast to the Dai-Liao method, which uses gradient and difference point data in its computations, the improved approach will show more efficiency by using function information. Experimental numerical studies are approved to demonstrate the efficiency of the suggested method.

***Subject Classification:*** *90C30, 65K05, 49M37.*

***Keywords:*** *Optimal Dai-Liao, Experimental numerical, Iteration problems.*

## 1. Introduction

In order to address the problem of unconstrained optimization, we developed the conjugate gradient method:

$$Minf(x), \mathrm{x} \in R^n. \tag{1}$$

where $f$ is a smooth function, [1]. Initially, we examine the conjugate gradient method (CG), which produces a series of iterations repeatedly by:

$$x_{k+1} = x_k + \alpha_k d_k \tag{2}$$

If the step size is denoted by $\alpha_k$ and the search direction, $d_k$, is defined as:

$$d_{k+1} = -g_{k+1} + \beta_k s_k \tag{3}$$

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where $\beta_k$ is a parameter that specifies the conjugate gradient method [2]. While some of these methods are useful conceptually, others are effective quantitatively. When Hestenes-Stiefel [3] first proposed the nonlinear conjugate gradient approach, $\beta_k$ looked like this:

$$\beta_k^{HS} = g_{k+1}^T y_k / d_k^T y_k \tag{4}$$

Length of step For iterative techniques, $\alpha_k$ is usually utilized to satisfy specific needs [4]. Among the introduced requirements in the field of convergence analysis and conjugate gradient technique implementations, the Wolfe conditions are popular and need the following:

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k \tag{5}$$

$$d_k^T g(x_k + \alpha_k d_k) \geq \sigma \, d_k^T g_k \tag{6}$$

where $0 < \delta < \sigma < 1$, see [1]. Sadly, all of the methods may not be able to use them convergently when using the Wolfe line search with a nonconvex objective function.

An additional effort was made to attain the theoretical advantages and computational returns of the conjugate gradient methods projected Dai-Liao in [5], we get:

$$\beta_k^{DL} = \frac{g_{k+1}^T y_k - t g_{k+1}^T s_k}{d_k^T y_k} \tag{7}$$

Several conjugate gradient methods were subsequently explored, which were generated from different modified secant equations [6].

We present the derivation of the updated conjugate gradient formula. A more thorough explanation of the computational experiment's technique is provided in the section that follows. New conjugate gradient parameters are being built utilising the modified secant equation in order to increase the CG technique's accuracy and accomplish more effective minimisation, particularly in complex and large-scale scenarios requiring unconstrained optimisation.

## 2. An Optimal Dai-Liao Method:

As is widely known, the development of conjugate gradient approaches has attracted special attention due to the Dai-Liao methodology [5]. Here, based on the Taylor series, the curvature condition, [7] known as:

$$s_k^T B_{k+1} s_k \cong (f_{k+1} - f_k) + 3/2 s_k^T y_k \tag{8}$$

By algebraic operations on (8) it is:

$$d_{k+1}^T B_{k+1} s_k \cong [3/2 + (f_{k+1} - f_k)/s_k^T y_k] d_{k+1}^T y_k \tag{9}$$

Based on the Perry's conjugacy condition ($d_{k+1}^T y_k = -g_{k+1}^T s_k$) and (3), we obtained:

$$d_{k+1}^T B_{k+1} s_k \cong -[2/3 + 2/3(f_k - f_{k+1})/s_k^T y_k] g_{k+1}^T s_k \tag{10}$$

Using the search direction in (7), we have:

$$(-g_{k+1} + \beta_k s_k)^T y_k \cong -\left[\frac{2}{3} + \frac{2}{3}\frac{(f_k - f_{k+1})}{s_k^T y_k}\right] g_{k+1}^T s_k \tag{11}$$

This implies:

$$\beta_k = \frac{g_{k+1}^T y_k - \left[\frac{2}{3} + \frac{2(f_k - f_{k+1})}{3\, s_k^T y_k}\right] g_{k+1}^T s_k}{s_k^T y_k} \tag{12}$$

For convenience of usage, these tactics are referred to as Dai-Liao (BB). Dai-Liao (BB) Algorithm Update:

**Algorithm Dai-Liao (BB).**

**Stage 1.** Set the initial t $x_0 \in R^n$, $k = 0$, $0 < \delta < \sigma < 1, d_0 = -g_0$.

**Stage 2.** Stop, if $\|g_k\| \leq \varepsilon$.

**Stage 3.** Use (5), (6) to calculate$\alpha_k$ :

**Stage 4.** Update the $x_{k+1} = x_k + \alpha_k d_k$,.

**Stage 5.** Update the $d_{k+1} = -g_{k+1} + \beta_k d_k$.

**Stage 6**. Put $k = k + 1$ and energy to stage 4.

## 4. Experimental numerical

This section uses a series of well-known unconstrained optimisation test functions to demonstrate the performance of the FORTRAN implementation of the algorithm HS. We have taken into consideration numerical experiments with n=100 and 1000 variables for each function. The test questions come from [8, 9]. We contrasted the algorithm HS's performance with the well-known formula HS, which is established in (4). The typical Wolfe line search criteria, $\delta_1 = 0.001$ and $\delta_2 = 0.9$, the halting condition, $\|g_{k+1}\| \leq 10^{-6}$, are used to build all of these algorithms. The following are included in the comparison:

NI : number of iterations, NF : function evaluations.

The Dolan and More [10-18] presentation profile was used to compare the BB approach with the HS. Figures 1 and 2 illustrate the plotting of a fraction P(t) of the problems for which the approach is within a factor t of the optimal time for each solution. The vertical axis on the left side of the curves indicates the fastest percentage of test problems for an approach. We conclude that for the majority of tested issues under the merely Wolfe line search, the Dai-Liao (BB) algorithm outperforms the HS approach.

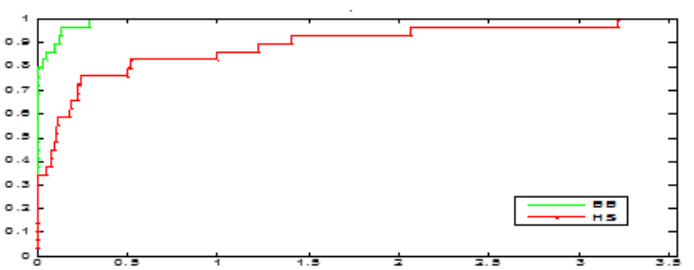



**Figure 1**
**Shows the NI-based performance profile**

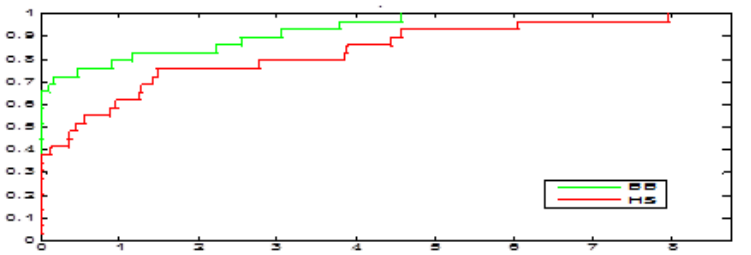



**Figure 2**
**Shows the NF-based performance profile**

## 4. Conclusions

We presented an effective Dai-Liao approach in this research. Additionally, our numerical findings demonstrated that the Dai-Liao (BB) technique is more efficient than the HS method in terms of restarts, function evaluations, iterations.

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