

On Zweier $\mathcal{J}$ -convergent sequence spaces defined by Orlicz function in neutrosophic normed linear spaces

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Abstract

In this article we define and develop Zweier $\mathcal{J}$ -convergent sequence spaces: $\mathcal{Z}_{(m, n, s)}(\mathcal{J} : O)$ and $\mathcal{Z}_{(m, n, s)}(\mathcal{J} : O)$ using Orlicz function in neutrosophic normed linear spaces. We also study some topological property of these spaces.

Subject Classification: 40A35, 40A05, 46B20.

Keywords: $\mathcal{J}$ -convergent, Orlicz function, Neutrosophic normed linear spaces.

1. Introduction

In 1965, Zadeh [33] introduced the idea of fuzzy sets which are successfully applied in many engineering fields, including social dynamics [5], computer science [11], irregular dynamic system [13], chaos control [9]. Some generalizations of fuzzy sets have been appeared in literature (see [4], [8], [27], [1-3], [6], [12], [15], [20], [32]).

The idea of sequence spaces has been introduced in generalized normed spaces such as fuzzy, intuitionistic fuzzy and neutrosophic normed linear spaces. For a detail study on these spaces, we refer [10], [12], [17], [20]-[23], [25], [26], [29]-[31]. In this paper we define some Zweier $\mathcal{J}$ -convergent space in neutrosophic normed spaces and study some of their topological properties.

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2. Preliminaries

We begin with the following definition of statistical convergence due to Fast [7]. A sequence (x_k) of numbers is said to be statistically convergent to x_0 if for every $\epsilon > 0$ the set $K = \{n \in \mathbb{N} : |x_k - x_0| \geq \epsilon\}$ has a natural density zero. Kostyrko et al [19], generalized statistical convergence by using ideal as follows: Let X be a non-empty set and power set and the power set of X is $P(X)$. A family of sets $\mathcal{I} \subseteq P(X)$ is said to be an ideal on X , if $\mathcal{I}$ is additive, i.e., $D, E \in \mathcal{I} \Rightarrow D \cup E \in \mathcal{I}$ and hereditary, i.e., $D \in \mathcal{I}, E \subseteq D \Rightarrow E \in \mathcal{I}$. A family of sets $\mathcal{F} \subset P(X)$ is said to be filter on X , if $\emptyset \notin \mathcal{F}$, for $D, E \in \mathcal{F}$ we have $D \cap E \in \mathcal{F}$ and for each $D \in \mathcal{F}$ and $E \supset D \Rightarrow E \in \mathcal{F}$. A sequence (x_k) of numbers is said to be ideal convergent to x_0 if for every $\epsilon > 0$ the set $K = \{k \in \mathbb{N} : |x_k - x_0| \geq \epsilon\} \in \mathcal{I}$.

By an Orlicz function we mean a function $O: [0, \infty) \rightarrow [0, \infty)$, which is continuous, non-decreasing and convex with $O(0) = 0, O(p) > 0$ for $p > 0$ and $O(p) \rightarrow \infty$ as $p \rightarrow \infty$ [19].

Şengönül [28] used Z^p transformation of the sequence $x = (x_k)$ to define a new sequence $y = (y_k)$ by

$$y_k = px_k + (1-p)x_{k-1}$$

Where $x_{k-1} = 0, 1 < p < \infty$ and $Z^p = (z_{kj})$ defined by

$$z_{kj} = \begin{cases} p, & (k = j) \\ 1 - p, & (k - 1 = j: (k, j \in \mathbb{N})) \\ 0, & \text{otherwise} \end{cases}$$

Using Z^p transform Şengönül [28] define the following spaces, $Z = \{x = (x_k) \in \xi : Z^p(x) \in c, Z_0 = \{x = (x_k) \in \xi : Z^p(x) \in c_0\}$. Where, ξ denotes set of all the sequence in neutrosophic normed linear space. After their pioneer work several papers have been appeared to define new sequence spaces by use of Z^p transform, Orlicz function and certain generalized convergences such as statistical and ideal convergence. A few among these can be seen in [14]-[16].

Kirişçi and Şimşek used t -norm and t -conorm to define neutrosophic norm linear spaces. for detail study of neutrosophic normed linear spaces, we refer [18][24].

Let $(X, \mathfrak{M}, \mathfrak{N}, \mathfrak{P}, *, *)$ be a NNLS, $\eta \in (0, 1), \zeta > 0$ and $u \in X$. Then the set

$$\mathcal{S}_u(\epsilon, \zeta) = \{v \in X : \{k \in \mathbb{N} : \mathfrak{M}(u - v, \zeta) \leq 1 - \epsilon \text{ or } \mathfrak{N}(u - v, \zeta) \geq \epsilon, \mathfrak{P}(u - v, \zeta) \geq \epsilon\} \in \mathcal{I}\}$$

is define as open ball with center u and radius ϵ with respect to ζ .

3. Main Results

In this section we give our main results. We start by defining the following sequence spaces in neutrosophic normed linear spaces.

$$Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{I}; O) =$$

$$\left\{ (u_k) \in \xi : \left\{ k \in \mathbb{N} : O\left(\frac{\mathfrak{M}(Z^p(u_k) - u_0, \zeta)}{\varsigma}\right) \leq 1 - \eta \text{ and } O\left(\frac{n(Z^p(u_k) - u_0, \zeta)}{\varsigma}\right) \geq \eta, \right. \right. \\ \left. \left. O\left(\frac{\mathfrak{P}(Z^p(u_k) - u_0, \zeta)}{\varsigma}\right) \geq \eta \right\} \in \mathcal{I} \right\},$$

$$Z_{0(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{I}; O) =$$

$$\left\{ (v_k) \in \xi : \left\{ k \in \mathbb{N} : O\left(\frac{\mathfrak{M}(Z^p(v_k) - v_0, \zeta)}{\varsigma}\right) \leq 1 - \eta \text{ and } O\left(\frac{n(Z^p(v_k) - v_0, \zeta)}{\varsigma}\right) \geq \eta, \right. \right. \\ \left. \left. O\left(\frac{\mathfrak{P}(Z^p(v_k) - v_0, \zeta)}{\varsigma}\right) \geq \eta \right\} \in \mathcal{I} \right\}.$$

Now for the author convenient and throughout the paper we will use $Z^p(u_k) = u'_k$ and $Z^p(v_k) = v'_k$ for $u, v \in \xi$.

$$Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{I}; O) =$$

$$\left\{ (u_k) \in \xi : \left\{ k \in \mathbb{N} : O\left(\frac{\mathfrak{M}(u'_k - u_0, \zeta)}{\varsigma}\right) \leq 1 - \eta \text{ and } O\left(\frac{n(u'_k - u_0, \zeta)}{\varsigma}\right) \geq \eta, \right. \right. \\ \left. \left. O\left(\frac{\mathfrak{P}(u'_k - u_0, \zeta)}{\varsigma}\right) \geq \eta \right\} \in \mathcal{I} \right\},$$

$$Z_{0(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{I}; O) =$$

$$\left\{ (v_k) \in \xi : \left\{ k \in \mathbb{N} : O\left(\frac{\mathfrak{M}(v'_k - v_0, \zeta)}{\varsigma}\right) \leq 1 - \eta \text{ and } O\left(\frac{n(v'_k - v_0, \zeta)}{\varsigma}\right) \geq \eta, \right. \right. \\ \left. \left. O\left(\frac{\mathfrak{P}(v'_k - v_0, \zeta)}{\varsigma}\right) \geq \eta \right\} \in \mathcal{I} \right\}.$$

Theorem 3.1: $Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{I}; O)$ and $Z_{0(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{I}; O)$ are normed linear spaces.

Proof: Omitted.

Theorem 3.2: Every open ball $\mathcal{S}_{u'}(\varepsilon, \varsigma)$ is an open set in $Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{I}; O)$.

Proof: Let $\mathcal{S}_{u'}(\varepsilon, \varsigma)$ be an open ball with center u' and ε with respect to ς that is

$$\mathcal{S}_{u'}(\varepsilon, \varsigma) =$$

$$\left\{ v \in X : \left\{ k \in \mathbb{N} : O\left(\frac{\mathfrak{M}(u'_k - v'_k, \zeta)}{\varsigma}\right) \leq 1 - \varepsilon \text{ or } O\left(\frac{n(u'_k - v'_k, \zeta)}{\varsigma}\right) \geq \varepsilon \text{ and } O\left(\frac{\mathfrak{P}(u'_k - v'_k, \zeta)}{\varsigma}\right) \geq \varepsilon \right\} \in \mathcal{I} \right\}.$$

$$\text{Let } v' \in \mathcal{S}_{u'}^c(\varepsilon, \varsigma). \text{ Then } O\left(\frac{\mathfrak{M}(u' - v', \zeta)}{\varsigma}\right) > 1 - \varepsilon \text{ and } O\left(\frac{n(u' - v', \zeta)}{\varsigma}\right) < \varepsilon, \\ O\left(\frac{\mathfrak{P}(u' - v', \zeta)}{\varsigma}\right) < \varepsilon.$$

Since $O\left(\frac{\mathfrak{M}(u' - v', \zeta)}{\varsigma}\right) > 1 - \varepsilon$, there exist $\zeta_0 \in (0, 1)$ such that $O\left(\frac{\mathfrak{M}(u' - v', \zeta_0)}{\varsigma}\right) > 1 - \varepsilon$ and $O\left(\frac{n(u' - v', \zeta_0)}{\varsigma}\right) < \varepsilon$, $O\left(\frac{\mathfrak{P}(u' - v', \zeta_0)}{\varsigma}\right) < \varepsilon$. Putting $\varepsilon = \left(\frac{\mathfrak{M}(u' - v', \zeta)}{\varsigma}\right)$ then $\varepsilon_0 > 1 - \varepsilon$. Thus $\exists 0 < s < 1$ such that $\varepsilon_0 > 1 - s > 1 - \varepsilon$. For $\varepsilon_0 > 1 - s, \exists \varepsilon_1, \varepsilon_2 \in (0, 1)$ such that $\varepsilon_0 * \varepsilon_1 > 1 - s$ and $(1 - \varepsilon_0) *$

$(1 - \varepsilon_2) \leq s$. Putting $\varepsilon_3 = \max\{\varepsilon_1, \varepsilon_2\}$ and consider the ball $\mathcal{S}_{u'}^c(1 - \varepsilon_3, \zeta - \zeta_0)$. We prove that $\mathcal{S}_{v'}^c(1 - \varepsilon_3, \zeta - \zeta_0) \subset \mathcal{S}_{u'}^c(\varepsilon, \varsigma)$.

Let $z' \in \mathcal{S}_{v'}^c(1 - \varepsilon_3, \zeta - \zeta_0)$. Then $O\left(\frac{\Re(v' - z', \zeta - \zeta_0)}{\varsigma}\right) > \varepsilon_3$ and $O\left(\frac{\Re(v' - z', \zeta - \zeta_0)}{\varsigma}\right) < \varepsilon_3$, $O\left(\frac{\Im(v' - z', \zeta - \zeta_0)}{\varsigma}\right) < \varepsilon_3$.
Thus, $O\left(\frac{\Re(u' - z', \zeta)}{\varsigma}\right) \geq O\left(\frac{\Re(u' - v', \zeta_0)}{\varsigma}\right) \star O\left(\frac{\Re(v' - z', \zeta - \zeta_0)}{\varsigma}\right) (1 - \varepsilon_0) \star (1 - \varepsilon_3) \geq (\varepsilon_0 \star \varepsilon_0) \geq 1 - s > 1 - \varepsilon$.
And $O\left(\frac{\Re(u' - z', \zeta)}{\varsigma}\right) \leq O\left(\frac{\Re(u' - v', \zeta_0)}{\varsigma}\right) \star O\left(\frac{\Re(v' - z', \zeta - \zeta_0)}{\varsigma}\right) \leq (1 - \varepsilon_0) \star (1 - \varepsilon_2) \leq (1 - \varepsilon_0) \star (1 - \varepsilon_2) \leq s < \varepsilon$,
 $O\left(\frac{\Im(u' - z', \zeta)}{\varsigma}\right) \leq O\left(\frac{\Im(u' - v', \zeta_0)}{\varsigma}\right) \star O\left(\frac{\Im(v' - z', \zeta - \zeta_0)}{\varsigma}\right) \leq (1 - \varepsilon_0) \star (1 - \varepsilon_2) \leq (1 - \varepsilon_0) \star (1 - \varepsilon_2) \leq s < \varepsilon$,

So $z' \in \mathcal{S}_{v'}^c(1 - \varepsilon_3, \zeta - \zeta_0)$. Hence $\mathcal{S}_{v'}^c(1 - \varepsilon_3, \zeta - \zeta_0) \subset \mathcal{S}_{u'}^c(\varepsilon, \varsigma)$.

Theorem 3.3: A sequence $u = (u_n) \in Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)$ $\mathcal{J}$ -converges iff for every $\eta > 0$ and $\zeta > 0 \exists$ a number $N = N(u, \eta, \varsigma)$ such that

$$\left\{ k \in \mathbb{N} : O\left(\frac{\Re(u'_k - u_0, \frac{\zeta}{2})}{\varsigma}\right) \leq 1 - \eta \text{ or } O\left(\frac{\Re(u'_k - u_0, \frac{\zeta}{2})}{\varsigma}\right) \geq \eta, O\left(\frac{\Im(u'_k - u_0, \frac{\zeta}{2})}{\varsigma}\right) \geq \eta \right\} \in \mathcal{F}(\mathcal{J}).$$

Proof: Omitted.

Remark 3.4: Let $Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)$ be a NNLS. Define $\tau_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O) = \{A \subset [Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)] : \text{for each } u \in A, \text{ we can find } \zeta \text{ and } 0 < \varepsilon < 1 \text{ such that } [\mathcal{S}_{u'}^c(\mathcal{J}; O)] \subset A\}$. Then $\tau_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)$ is a topology on $Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)$.

Remark 3.5: Since $\{\mathcal{S}_{u'}^c\left(\frac{1}{n}, \frac{1}{n}\right)(\mathcal{J}; O) : n \in \mathbb{N}\}$ is a local base at u' , the topology $\tau_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)$ on $Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)$ is first countable.

Theorem 3.6: $Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)$ and $Z_{0(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)$ are Housdorff spaces.

Proof: Let $u', v' \in [Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}; O)]$ such that $u' \neq v'$. Then $0 < O\left(\frac{\Re(u' - v', \zeta)}{\varsigma}\right) < 1$ and $0 < O\left(\frac{\Re(u' - v', \zeta)}{\varsigma}\right) < 1$, $0 < O\left(\frac{\Im(u' - v', \zeta)}{\varsigma}\right) < 1$. Putting $\varepsilon_1 = O\left(\frac{\Re(u' - v', \zeta)}{\varsigma}\right)$ and $\varepsilon_2 = O\left(\frac{\Re(u' - v', \zeta)}{\varsigma}\right)$, $\varepsilon_3 = O\left(\frac{\Im(u' - v', \zeta)}{\varsigma}\right)$ and $\varepsilon = \max\{\varepsilon_1, 1 - \varepsilon_2, 1 - \varepsilon_3\}$. Then for each $\varepsilon_0 \in (\varepsilon, 1)$, $\exists \varepsilon_4$ and ε_5 such that $\varepsilon_4 \star \varepsilon_5 \geq \varepsilon_0$ and $(1 - \varepsilon_4) \star (1 - \varepsilon_5) \leq (1 - \varepsilon_0)$

Putting $\varepsilon_6 = \max\{\varepsilon_4, 1 - \varepsilon_5\}$. And consider the open ball $\mathcal{S}_{u'}^c(1 - \varepsilon_6, \frac{\zeta}{2})$ and $\mathcal{S}_{v'}^c(1 - \varepsilon_6, \frac{\zeta}{2})$ then the clearly $\mathcal{S}_{u'}^c(1 - \varepsilon_6, \frac{\zeta}{2}) \cap \mathcal{S}_{v'}^c(1 - \varepsilon_6, \frac{\zeta}{2}) = \phi$.

For if $\exists w' \in \mathcal{S}_u^c \left(1 - \varepsilon_6, \frac{\zeta}{2}\right) \cap \mathcal{S}_{v'}^c \left(1 - \varepsilon_6, \frac{\zeta}{2}\right)$, then

$$\begin{aligned} \varepsilon_1 = O\left(\frac{\mathfrak{M}(u'-v', \zeta)}{\varsigma}\right) &\geq O\left(\frac{\mathfrak{M}(u'-w', \frac{\zeta}{2})}{\varsigma}\right) \star O\left(\frac{\mathfrak{M}(w'-v', \frac{\zeta}{2})}{\varsigma}\right) \\ &\geq \varepsilon_6 \star \varepsilon_6 \geq \varepsilon_4 \star \varepsilon_4 \geq \varepsilon_0 > \varepsilon_1 \end{aligned}$$

And

$$\begin{aligned} \varepsilon_2 = O\left(\frac{\mathfrak{M}(u'-v', \zeta)}{\varsigma}\right) &\geq O\left(\frac{\mathfrak{M}(u'-w', \frac{\zeta}{2})}{\varsigma}\right) \star O\left(\frac{\mathfrak{M}(w'-v', \frac{\zeta}{2})}{\varsigma}\right) \\ &\geq (1 - \varepsilon_6) \star (1 - \varepsilon_6) \geq (1 - \varepsilon_5) \star (1 - \varepsilon_5) \geq (1 -) > \varepsilon_1, \\ \varepsilon_3 = O\left(\frac{\mathfrak{P}(u'-v', \zeta)}{\varsigma}\right) &\geq O\left(\frac{\mathfrak{P}(u'-w', \frac{\zeta}{2})}{\varsigma}\right) \star O\left(\frac{\mathfrak{P}(w'-v', \frac{\zeta}{2})}{\varsigma}\right), \\ &\geq (1 - \varepsilon_6) \star (1 - \varepsilon_6) \geq (1 - \varepsilon_5) \star (1 - \varepsilon_5) \geq (1 -) > \varepsilon_1. \end{aligned}$$

Contradict our assumption. So $[Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{I}: O)]$ is a Housdroff space. Similarly, $[Z_{(\mathfrak{M}, \mathfrak{N}, \mathfrak{P})}(\mathcal{J}: O)]$ is a Housdroff space.

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Received July, 2024