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On S^{β}-summability in neutrosophic-*n*-normed linear spaces

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Abstract

In the present work, we define the notions of statistical convergence, statistical Cauchy and statistical completeness of order β (β ∈ (0,1]) in neutrosophic-n-normed linear spaces (briefly called *N-n-NLS*). We provide examples in support of our notions and obtain some interesting relations in these notions.

Subject Classification: *40A35, 40A05.*

Keywords: *Statistical convergence, Cauchy sequence, n-normed spaces and neutrosophic normed spaces.*

1. Introduction and Preliminaries

Fast [16] introduced the idea of statistical convergence and later it was developed by Steinhaus [38], Fridy [18], [19], Salat [36], Connor [11], Altinok et al. [1], Hazarika and Esi [22], Tripathy and Sen [39]. In 2010, Colak [10] introduced statistical convergence of order β for $\beta \in (0,1]$ and studied some of its properties. Subsequently the notion is developed by different authors in [8], [9], [14], [15].

"For $K \subseteq \mathbb{N}$ and $K_n = \{k \in K : k \leq n\}$, the natural density of order β $(0 < \beta \leq 1)$ of K is defined by $\delta^{\beta}(K) = \lim_{n\to\infty} \frac{|K_n|}{n^{\beta}}$ provided that the limit exists". Here, $|K_n|$ denotes the cardinality of the set K_n. "A sequence (x_k) is said to be statistical convergent of order β $(0 < \beta \leq 1)$ to l if for each $\epsilon > 0$,

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$\delta^{\beta}(A(\epsilon)) = 0$, where $A(\epsilon) = \{k \in \mathbb{N}: |x_k - l| \geq \epsilon\}$". In this case, l is called statistical limit of order β of the sequence (x_k) and symbolically it is denoted by "$x_k \xrightarrow{st^{\beta}} l$".

Recently, the idea of neutrosophic normed linear spaces (briefly called *NNLS*) has been introduced by Kirişçi and Şimşek [27] as a generalization of fuzzy normed linear spaces and studied statistical convergence in these spaces. For sufficient knowledge on *NNLS* and fuzzy normed linear spaces, we refer [2] - [7], [12], [13], [17], [20], [21], [23] - [26], [28] - [35], [37], [40] - [43].

In present work, we define statistical convergence, statistical Cauchy and statistical completeness of order β in N-n-NLS. We also study some of their properties and obtain some relevant connections in N-n-NLS.

2. Statistical convergence of order β in N-n-NLS

In this section, we introduce $S^{\beta}_{(Q,R,T)^n}$- convergence for $0 < \beta \leq 1$ in N-n-NLS and develop some of its characteristics. Throughout present work P will denote the N-n-NLS $(P, Q, R, T, \star, \circledast)$.

Definition 2.1: A sequence $p = (p_k)$ in P is called statistically convergent of order β (or $S^{\beta}_{(Q,R,T)^n}$ convergent) to $p_0 \in P$ w.r.t $(Q, R, T)^n$, if for each $0 < \epsilon < 1, \eta > 0$ and $\varrho_1, \varrho_2, \dots, \varrho_{n-1} \in P$

$\lim_{n\to\infty} \frac{1}{m^{\beta}} |\{k \leq m: Q\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_0, \eta) \leq 1 - \epsilon$ or $R\ (\varrho_1, \varrho_2, \dots,$ $\varrho_{n-1}, p_k - p_0, \eta) \geq \epsilon,\ T\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_0, \eta) \geq \epsilon\}| = 0$. In present case, we write $p_k \longrightarrow p_0 \left(S^{\beta}_{(Q,R,T)^n}\right)$ or $S^{\beta}_{(Q,R,T)^n}\text{–}\lim_{k\to\infty} p_k = p_0$.

Example 2.2: Consider the *n*-normed linear space $(\mathbb{R}^n, \|.\|_n)$ where $\|.\|_n$ is defined by

$$\|p_1, p_2, \dots, p_k\| = abs\left(\begin{vmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{vmatrix}\right)$$

where $p_i = (p_{i_1}, p_{i_2}, \dots, p_{i_n}) \in \mathbb{R}^n$ for each $i = 1,2,\dots,n$ and abs denote the absolute value of the enclosed determinant. Further, for $d, e \in [0,1], \eta > 0$, $\varrho_1, \varrho_2, \dots, \varrho_{n-1} \in P$ if we define the continuous t-norm $d \star e = de$ and the continuous t-conorm $d \circledast e = \min\{d + e, 1\}$, fuzzy sets Q, R and T defined as $Q(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p, \eta) = \frac{\eta}{\eta + \|\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p\|}$, $R(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p, \eta) = \frac{\|\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p\|}{\eta + \|\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p\|}$ and $T(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p, \eta) = \frac{\|\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p\|}{\eta}$, then evidently $(\mathbb{R}^n, Q, R, T, \star, \circledast)$ is a N-n-NLS. We construct a sequence $p = \{p_k\}$ by $p_k = \begin{cases} (k, 0, \dots, 0), & k = n^3\ (n \in \mathbb{N}) \\ (0, 0, \dots, 0), & \text{otherwise} \end{cases}$. For every $0 < \epsilon < 1, \eta > 0$ and $\varrho_1, \varrho_2, \dots, \varrho_{n-1} \in P$, let $K_{\epsilon} = \{k \leq \text{n} : Q(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - 0, \eta) \leq 1 - \epsilon, \text{R}\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - 0, \eta) \geq \epsilon$ and $\text{T}\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - 0, \eta) \geq \epsilon\}$.

Then, $\delta^{\beta}(K_{\epsilon}) = \lim_{n\to\infty} \frac{|K_{\epsilon}|}{n^{\beta}} \leq \lim_{n\to\infty} \frac{\sqrt[3]{n}}{n^{\beta}} = 0$ for $\beta \in \left(\frac{1}{3}, 1\right]$. Hence, $p_k \longrightarrow 0 \left(S^{\beta}_{(Q,R,T)^n}\right)$.

Theorem 2.3: *Let* $p = (p_k)$ *be any sequence in P. If* $S^{\beta}_{(Q,R,T)^n} - lim_{k\to\infty} p_k$ *exists, then it is unique.*

Proof: Suppose that $S^{\beta}_{(Q,R,T)^n} - \lim_{k\to\infty} p_k = p_1$ and $S^{\beta}_{(Q,R,T)^n} - \lim_{k\to\infty} p_k = p_2$. Given $0 < \epsilon < 1$, choose $\epsilon_1 > 0$ such that

$$(1-\epsilon_1) \star (1-\epsilon_1) > 1-\epsilon \text{ and } \epsilon_1 \circledast \epsilon_1 < \epsilon \tag{2.1}$$

For any $\eta > 0$ and $\varrho_1, \varrho_2, \dots, \varrho_{n-1} \in P$, let

$$A_{Q,1}(\epsilon_1,\eta) = \left\{k \in \mathbb{N} : Q\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_1, \frac{\eta}{2}\right) > 1-\epsilon_1\right\},$$

$$A_{Q,2}(\epsilon_1,\eta) = \left\{k \in \mathbb{N} : Q\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_2, \frac{\eta}{2}\right) > 1-\epsilon_1\right\},$$

$$A_{R,1}(\epsilon_1,\eta) = \left\{k \in \mathbb{N} : R\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_1, \frac{\eta}{2}\right) < \epsilon_1\right\},$$

$$A_{R,2}(\epsilon_1,\eta) = \left\{k \in \mathbb{N} : R\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_2, \frac{\eta}{2}\right) < \epsilon_1\right\},$$

$$A_{T,1}(\epsilon_1,\eta) = \left\{k \in \mathbb{N} : T\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_1, \frac{\eta}{2}\right) < \epsilon_1\right\},$$

$$A_{T,2}(\epsilon_1,\eta) = \left\{k \in \mathbb{N} : T\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_2, \frac{\eta}{2}\right) < \epsilon_1\right\}.$$

Let $A_{Q,R,T}(\epsilon,\eta) = \{A_{Q,1}(\epsilon_1,\eta) \cap A_{Q,2}(\epsilon_1,\eta)\} \cup \{A_{R,1}(\epsilon_1,\eta) \cap A_{R,2}(\epsilon_1,\eta)\} \cup \{A_{T,1}(\epsilon_1,\eta) \cap A_{T,2}(\epsilon_1,\eta)\}$. Since $S^{\beta}_{(Q,R,T)^n}–\lim_{k\to\infty} p_k = p_1$, $S^{\beta}_{(Q,R,T)^n} - \lim_{k\to\infty} p_k = p_2$, then clearly $\delta^{\beta}(A_{Q,R,T}(\epsilon_1,\eta)) = 1$.

Case 1: Let $k \in \{A_{Q,1}(\epsilon_1,\eta) \cap A_{Q,2}(\epsilon_1,\eta)\}$ then $k \in A_{Q,1}(\epsilon_1,\eta)$ and $k \in A_{Q,2}(\epsilon_1,\eta)$ and therefore

$Q\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_1, \frac{\eta}{2}\right) > 1-\epsilon_1$ and $Q\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_2, \frac{\eta}{2}\right) > 1-\epsilon_1$

Now $Q(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_1 - p_2, \eta) = Q\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_1 - p_2 + p_k - p_k, \frac{\eta}{2} + \frac{\eta}{2}\right) \geq Q\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_1, \frac{\eta}{2}\right) \star Q\left(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_2, \frac{\eta}{2}\right) > (1-\epsilon_1) \star (1-\epsilon_1) > 1-\epsilon$. As $\epsilon > 0$ is arbitrary, therefore $Q(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_1 - p_2, \eta) = 1$ for all $\eta > 0$ and $\varrho_1, \varrho_2, \dots, \varrho_{n-1} \in P$, which gives $p_1 - p_2 = 0$ $i.e, p_1 = p_2$. Similarly in Case 2 and Case 3, we have $p_1 = p_2$. Hence, limit is unique.

Theorem 2.4: *Let* $q = (q_k)$ *and* $p = (p_k)$ *be two sequences in P such as*

$S^{\beta}_{(Q,R,T)^n} - \lim_{k\to\infty} q_k = q_0$ and $S^{\beta}_{(Q,R,T)^n} - \lim_{k\to\infty} p_k = p_0$. Then,

$(i)\ S^{\beta}_{(Q,R,T)^n} - \lim_{k\to\infty}(q_k + p_k) = q_0 + p_0$

$(ii)\ S^{\beta}_{(Q,R,T)^n} - \lim_{k\to\infty}(\alpha q_k) = \alpha q_0$ where $0 \neq \alpha \in \mathbb{R}$.

Proof: The proof is omitted.

Remark: In *N-n-NLS* , the statistical convergence of order β is well defined particularly for $0 < \beta \leq 1$. For $\beta > 1$, consider the following counterexample:

Example 2.5: Let $(V, \|.\|_n)$ be an n-normed space. Consider the example 2.2 and construct a sequence $p = \{p_k\}$ defined as

$$p_k = \begin{cases} (1,0,\dots,0), & k = 2n\,; (n \in \mathbb{N}) \\ (0,0,\dots,0), & k \neq 2n \end{cases}$$

Therefore, for $\beta > 1$

$\lim_{n\to\infty} \frac{1}{n^{\beta}} |\{k \in \mathbb{N}: Q\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - 0, \eta) \leq 1 - \epsilon,\ R\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - 0, \eta) \geq \epsilon$ and $T\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - 0, \eta) \geq \epsilon\}| \leq \lim_{n\to\infty} \frac{n}{2n^{\beta}} = 0$ and $\lim_{n\to\infty} \frac{1}{n^{\beta}} |\{k \in \mathbb{N}: Q\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - 1, \eta) \leq 1 - \epsilon,\ \ R\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, | p_k - 1, \eta) \geq \epsilon$ and $T\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - 1, \eta) \geq \epsilon\}\ \leq \lim_{n\to\infty} \frac{n}{2n^{\beta}} = 0$ which concludes that $S^{\beta}_{(Q,R,T)^n} - \lim_{k\to\infty} p_k = 0$ and $S^{\beta}_{(Q,R,T)^n} - \lim_{k\to\infty} p_k = 1$ which is not possible.

Theorem 2.6: *Let (p_k) and (q_k) be two any sequences in N-n-NLS P in order that $q_k \longrightarrow p_0 (Q,R,T)^n$ and $\delta^{\beta}(k \in N: p_k \neq q_k) = 0$. Then $p_k \longrightarrow p_0 \left(S^{\beta}_{(Q,R,T)^n}\right)$.*

Proof: Suppose (p_k) and (q_k) are such that $\delta^{\beta}(k \in N: p_k \neq q_k) = 0$ holds and $q_k \longrightarrow p_0 (Q,R,T)^n$. Thus, by definition $\forall\ 0 < \epsilon < 1, \eta > 0$ and $\varrho_1, \varrho_2, \dots, \varrho_{n-1} \in P$, the set $A_{\epsilon} = \{k \in N :\ Q(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_0, \eta) \leq 1 - \epsilon, \mathrm{R}\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_0, \eta) \geq \epsilon$ and $\mathrm{T}\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, p_k - p_0, \eta) \geq \epsilon\}$ consists of finitely many numbers of elements and as a result, we have $\delta^{\beta}(A_{\epsilon}) = 0$. Now, as the containment $B_{\epsilon} = \{k \in N :\ Q(\varrho_1, \varrho_2, \dots, \varrho_{n-1}, q_k - p_0, \eta) \leq\ 1 - \epsilon, \mathrm{R}\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, q_k - p_0, \eta) \geq \epsilon$ and $\mathrm{T}\ (\varrho_1, \varrho_2, \dots, \varrho_{n-1}, q_k - p_0, \eta) \geq \epsilon\} \subseteq A_{\epsilon} \cap \{k \in N: p_k \neq q_k\}$ holds, so we must have $\delta^{\beta}(B_{\epsilon}) = 0$. This shows that, $p_k \longrightarrow p_0 \left(S^{\beta}_{(Q,R,T)^n}\right)$.

2.7 *Statistical Cauchy of order β in N-n-NLS*

Definition 2.7.1: Let $(P, Q, R, T, \star, \circledast)$ be a *N-n-NLS* . A sequence $p = (p_k)$ is known to be statistical Cauchy sequence of order β with respect to neutrosophic -n-norm $(Q, R, T)^n$ if for each $0 < \epsilon < 1, \eta > 0$ and $\varrho_1, \varrho_2, \dots, \varrho_{n-1} \in P$, there exists $m = m(\epsilon) \in N$ so that $\delta^{\beta}(K_{\epsilon}) = 0$ where $K_{\epsilon} = \{k \in N :$

$Q(\varrho_1, \varrho_2, \ldots, \varrho_{n-1}, p_k - p_m, \eta) \leq 1 - \epsilon$ or $R(\varrho_1, \varrho_2, \ldots, \varrho_{n-1}, p_k - p_m, \eta) \geq \epsilon$, $T(\varrho_1, \varrho_2, \ldots, \varrho_{n-1}, p_k - p_m, \eta) \geq \epsilon\}$.

Theorem 2.7.2: *Let $p = (p_k)$ be arbitrary sequence in N-n-NLS P. Then $p = (p_k)$ is statistical convergent of order β $\Leftrightarrow$ it is statistical Cauchy sequence of order β with respect to $(Q, R, T)^n$.*

Proof: The proof is omitted.

2.8 *Statistical Completeness of order β in N-n-NLS*

Definition 2.8.1: A sequence $p = (p_k)$ in P is said to be statistical complete if every statistical Cauchy sequence of order β with respect to neutrosophic n-norm $(Q, R, T)^n$ is statistical convergent of order β in P.

Theorem 2.8.2: *If every statistical Cauchy sequence of order β in P has statistical convergent subsequence of order β, then P is statistical complete.*

Proof: The proof is omitted.

3. Conclusion

Fuzzy functional analysis is a generalization of the classical functional analysis which appeared as an effective tool to resolve many problems which contain huge indeterminacy and vagueness. In this work we provide modified summability methods in a more general context, i.e., in neutrosophic n-normed linear spaces. The results presented here will provide a framework to study many problems of functionals in the neutrosophic environment.

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