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On Δ^m (J) summability in neutrosophic-n-normed linear spaces

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Abstract

For the difference operator Δ^m and the admissible ideal $J \subseteq P(N)$, we aim in present work to introduce Δ^m (J_N^*)-summability and Δ^m (J_N^*)-summability in neutrosophic- n-normed linear spaces (briefly called *N-n-NLS*). We explore some basic properties of Δ^m (J_N^*)-summability and Δ^m (J_N^*)-summability and find conditions on J for which the two summability methods coincide. Finally, we define Δ^m (J_N)-Cauchy sequences in *N-n-NLS* and obtain the Cauchy convergence criteria.

Subject Classification: *40A35, 40A05.*

Keywords: *Neutrosophic-n-normed linear spaces, Statistical convergence, J-convergence, J-Cauchy sequences.*

1. Introduction

Fast [6] and Schoenberg [28] independently introduced statistical convergence which was further developed by many mathematicians including Fridy [7], Šalát [30], Connor [2], Maddox [20], Vijay and Mursaleen [14]. "The natural density for any set A $\subseteq \mathbb{N}$ is defined by $\delta(A) = \lim_{n\to\infty} \frac{1}{n} |\{k \leq n: k \in A\}|$ provided the limit exists. Furthur a number sequence $x = (x_k)$ is called statistical convergent to x_0 if for each $\gamma > 0, \lim_{n\to\infty} \frac{1}{n} |\{k \leq n: |x_k - x_0| \geq \gamma\}| = 0$ i. e. $\delta(A_\gamma) = 0$ where $A_\gamma = \{k \leq n: |x_k - x_0| \geq \gamma\}$. In this case, we

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write $S-\lim_{k\to\infty} x_k = x_0$." Kostyrko [18] defined "I-convergence" using the concept of an admissible ideal J. For a deep understanding of the topic, we refer to the reader [3], [4], [13], [15], and [17], [21] On the other hand, fuzzy set were invented by Zadeh [35] to reduce the vagueness which the crisp set theory could not address. Atanassov [1] is credited with one of the most significant generalizations of fuzzy sets which deals with uncertain situations. Neutrosophic sets were initially defined by Smarandache [29] and subsequently developed in [10], [16], [22], [25], [26], and [27]. In [12] Kirişci and Şimşek studied statistical convergence in neutrosophic normed space. Later, many authors studied these spaces, see [8], [11], [23], [31], [32], [33], [34] etc. Kumar et al. [19] defined and studied N-n-NLS.

We now recall the following terminology. A non-empty family of sets $\boldsymbol{F} \subseteq \boldsymbol{P(A)}$ is called a filter on A if and only if (i) $\boldsymbol{\phi} \notin \boldsymbol{F}$, (ii) $\boldsymbol{A_1}, \boldsymbol{A_2} \in \boldsymbol{F}$ implies that $\boldsymbol{A_1} \cap \boldsymbol{A_2} \in \boldsymbol{F}$ and (iii) For $\boldsymbol{A_1} \in \boldsymbol{F}$ **and** $\boldsymbol{A_2} \supseteq \boldsymbol{A_1}$, we have $\boldsymbol{A_2} \in \boldsymbol{F}$. An ideal $\boldsymbol{J}$ is called non-trivial if $\boldsymbol{J} \neq \boldsymbol{\phi}$ and $\boldsymbol{A} \notin \boldsymbol{J}$. Obviously, $\boldsymbol{J} \subseteq \boldsymbol{P(A)}$ is a non-trivial ideal if and only if the class $\boldsymbol{F} = \boldsymbol{F(J)} = \{\boldsymbol{A} - \boldsymbol{A_1} : \boldsymbol{A_1} \in \boldsymbol{J}\}$ is a filter on A. The filter $\boldsymbol{F} = \boldsymbol{F(J)}$ is called the filter associated with the ideal $\boldsymbol{J}$. A non-trivial ideal $\boldsymbol{J} \subseteq \boldsymbol{P(A)}$is called admissible ideal in $\boldsymbol{A}$ if and only if it contains all singletons i.e., if it contains $\{\{\boldsymbol{a}\} : \boldsymbol{a} \in \boldsymbol{A}\}$. We now give the lemma 4 of [24]

Lemma 1.1: *Let $\{E_j\}_{j=1}^{\infty}$ be a countable collection of subsets of $\mathbb{N}$ such that $E_j \in F(J)$ for each j where $F(J)$ is a filter associated with the admissible ideal J satisfying property (AP). Then there exists a set $E \subseteq \mathbb{N}$ such that $E \in F(J)$ and the set $E - E_j$ is finite for each j. Et and Colak [5] introduced the generalized concept of difference sequence spaces as-*

Let $\boldsymbol{w}$ be the set of all sequences in the neutrosophic-$\boldsymbol{n}$-normed linear space $\boldsymbol{X} = (\boldsymbol{V}, \boldsymbol{B}, \boldsymbol{D}, \boldsymbol{H}, \circledast, \star)$. Define $\boldsymbol{\Delta^m} : \boldsymbol{w} \to \boldsymbol{w}$ by $\boldsymbol{\Delta^0 x_k} = \boldsymbol{x_k}; \boldsymbol{\Delta^1 x_k} = \boldsymbol{x_k} - \boldsymbol{x_{k+1}}; \ldots; \boldsymbol{\Delta^m x_k} = \boldsymbol{\Delta^{m-1}(\Delta x_k)} = \boldsymbol{\Delta^{m-1}(x_k - x_{k+1})}; \boldsymbol{m} \geq \boldsymbol{2}$ and for all $\boldsymbol{k} \in \mathbb{N}$. In present study, we define and study $\boldsymbol{\Delta^m(J_N)}$-convergence, $\boldsymbol{\Delta^m(J_N^*)}$-convergence and $\boldsymbol{\Delta^m(J_N)}$-Cauchy sequences in $\boldsymbol{N}$-$\boldsymbol{n}$-$\boldsymbol{NLS}$.

2. $\Delta^m(\boldsymbol{J_N})$-Convergence in $\boldsymbol{N}$-$\boldsymbol{n}$-$\boldsymbol{NLS}$.

Definition 2.1: A sequence $x = (x_k)$ in X is called $\Delta^m(J_N)$ convergent to $x_0 \in X$ if $\forall\ \gamma \in (0,1)$ and $\alpha > 0$, $A(\gamma) = \{k \in \mathbb{N} : B\ (\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \leq 1-\gamma$ or $D(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \geq \gamma, H(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \geq \gamma\} \in J$. In this case, we write: $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_0$.

Theorem 2.1: *If $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_1$ and $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_2$ then $x_1 = x_2$.*

Proof: Suppose that $x_1 \neq x_2$ and let $\gamma > 0$ Choose $\beta > 0$ such that,

$(1-\gamma) \circledast (1-\gamma) > 1-\beta$ and $\gamma \star \gamma < \beta$. For $\alpha > 0$, let

$K_{B_1}(\gamma,\alpha) = \left\{ k \in \mathbb{N} : B\left(\Delta^m x_k - x_1, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) \leq 1 - \gamma \right\}$, $K_{B_2}(\gamma,\alpha) = \left\{ k \in \mathbb{N} : B\left(\Delta^m x_k - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) \leq 1 - \gamma \right\}$, $K_{D_1}(\gamma,\alpha) = \{ k \in \mathbb{N} : D\left(\Delta^m x_k - x_1, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) \geq \gamma \}$, $K_{D_2}(\gamma,\alpha) = \{ k \in \mathbb{N} : D\left(\Delta^m x_k - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) \geq \gamma \}$, $K_{H_1}(\gamma,\alpha) = \{ k \in \mathbb{N} : H\left(\Delta^m x_k - x_1, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) \geq \gamma \}$, $K_{H_2}(\gamma,\alpha) = \{ k \in \mathbb{N} : H\left(\Delta^m x_k - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) \geq \gamma \}$,

Since, $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_1$ and $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_2$, So, $K_{B_1}(\gamma,\alpha), K_{D_1}(\gamma,\alpha), K_{H_1}(\gamma,\alpha), K_{B_2}(\gamma,\alpha), K_{D_2}(\gamma,\alpha)$ and $K_{H_2}(\gamma,\alpha)$ belongs to J. Define a set, $K_N(\beta,\alpha) = \left\{ K_{B_1}(\gamma,\alpha) \cup K_{B_2}(\gamma,\alpha) \right\} \cap \left\{ K_{D_1}(\gamma,\alpha) \cup K_{D_2}(\gamma,\alpha) \right\} \cap K_{H_1}(\gamma,\alpha) \cup K_{H_2}(\gamma,\alpha)\}$, then $K_N(\beta,\alpha) \in J$ which implies$\{ \mathbb{N} \setminus K_N(\beta,\alpha)\} \in F(J)$.Then $\{ \mathbb{N} \setminus K_N(\beta,\alpha)\}$ is a non empty set. Let k $\in \{ \mathbb{N} \setminus K_N(\beta,\alpha)\}$, then the following possibilities arise: 1.$k \in \mathbb{N} \setminus \left\{ K_{B_1}(\beta,\alpha) \cup K_{B_2}(\beta,\alpha) \right\}$, 2.$k \in \mathbb{N} \setminus \left\{ K_{D_1}(\beta,\alpha) \cup K_{D_2}(\beta,\alpha) \right\}$, 3.$k \in \mathbb{N} \setminus \{ K_{H_1}(\beta,\alpha) \cup K_{H_2}(\beta,\alpha)\}$, Case 1: Suppose (1.) holds, then $k \notin K_{B_1}(\beta,\alpha) \cap K_{B_2}(\beta,\alpha)$ which gives $k \notin K_{B_1}(\beta,\alpha)$ and $k \notin K_{B_2}(\beta,\alpha)$. So, we have $B\left(\Delta^m x_k - x_1, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) > 1 - \gamma$ and $B\left(\Delta^m x_k - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) > 1 - \gamma$. Now, $B(x_1 - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) \geq B(\Delta^m x_k - x_1, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}) \circledast B(\Delta^m x_k - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}) > (1-\gamma) \circledast (1-\gamma) > 1 - \beta$. Since β is arbitrary and for every $\alpha > 0$, it gives $B(x_1 - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) = 1$ and therefore, $x_1 = x_2$. Case 2: Suppose (2) holds, then $k \notin K_{D_1}(\beta,\alpha) \cap K_{D_2}(\beta,\alpha)$ which gives $k \notin K_{D_1}(\beta,\alpha)$ and k $\notin K_{D_2}(\beta,\alpha)$. So, we have $D\left(\Delta^m x_k - x_1, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) < \gamma$ *and* $D\left(\Delta^m x_k - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}\right) < \gamma$ Now, $D(x_1 - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) \leq D(\Delta^m x_k - x_1, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}) \star D(\Delta^m x_k - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \frac{\alpha}{2}) < \gamma \star \gamma < \beta$. Since β is arbitrary and for every $\alpha > 0$, it gives $D(x_1 - x_2, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) = 1$ and therefore, $x_1 = x_2$. Case 3. Follows similarly as case 2. Hence in all cases, we get $x_1 = x_2$.

Theorem 2.2: *If* $N - \lim_{k\to\infty}(\Delta^m x_k) = x_0$ *then* $(\Delta^m J_N) - \lim_{k\to\infty} x_k = x_0$.

Proof: Suppose $N - \lim_{k\to\infty}(\Delta^m x_k) = x_0$. Then for each $\gamma > 0$ and $\alpha > 0$, $\exists\ k_0 \in \mathbb{N}$ such that $B(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) > 1 - \gamma, D(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) < \gamma, H(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) < \gamma\ \forall\ k \geq k_0$. It follows that, $\{ k \in \mathbb{N} : B(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) \leq 1 - \gamma$ or $D(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) \geq \gamma, H(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha) \geq \gamma\} \in J$.

Hence, $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_0$. We now give the linearity property of $\Delta^m(J_N)$-convergence.

Theorem 2.3: *If* $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_1$ *and* $\Delta^m(J_N) - \lim_{k\to\infty} z_k = x_2$, *then* $\Delta^m(J_N) - \lim_{k\to\infty}(x_k + z_k) = x_1 + x_2$ *and* $\Delta^m(J_N) - \lim_{k\to\infty} (\lambda x_k) = \lambda x_1, \lambda \neq 0$.

Proof: The proof can be obtained analogous to Theorem 2.1 so omitted.

3. $\Delta^m(J_N^*)$ – Convergence in N-n-NLS.

Definition 3.1: $x = (x_k)$ in X is called $\Delta^m(J_N^*)$-convergent to x_0 iff $\exists$ a set $I = \{i_1, i_2, i_3, \ldots\} \in F(J)$ such that $N - \lim_{n\to\infty} \Delta^m x_{i_n} = x_0$.In this case, we write $\Delta^m(J_N^*) - \lim_{k\to\infty} x_k = x_0$.

Theorem 3.1: *If* $\Delta^m(J_N^*) - \lim_{k\to\infty} x_k = x_0$, *then* $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_0$.

Proof: Proof is Omitted.

Theorem 3.2: $\Delta^m(J_N)$-lim *is equivalent to* $\Delta^m(J_N^*)$-lim if $J \subseteq P(\mathbb{N})$ *satisfying* (AP) *property.*

Proof: Since $\Delta^m(J_N^*)$-limit implies $\Delta^m(J_N)$-limit. So, we prove, $\Delta^m(J_N)$-limit implies $\Delta^m(J_N^*)$-limit. Let $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_0$. By definition, for each $\gamma > 0$ and $\alpha > 0$, we have $A(\gamma, \alpha) = \{ k \in \mathbb{N} : B\,(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \leq 1 - \gamma$ or $D\,(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \geq \gamma, H\,(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \geq \gamma\} \in J$. For $\beta \in \mathbb{N}$, we define sets $K(\beta, \alpha) = \{ k \in \mathbb{N} : B\,(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1};\, \alpha) > 1 - \frac{1}{\beta}$ and $D\,(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) < \frac{1}{\beta}$, $H\,(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1};\, \alpha) < \frac{1}{\beta}\}$ and $Q(\beta, \alpha) = \{ k \in \mathbb{N} : B(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \leq 1 - \frac{1}{\beta}$ or $D(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \geq \frac{1}{\beta}, H(\Delta^m x_k - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \geq \frac{1}{\beta}\}$ .Since, $\Delta^m(J_N) - \lim_{k\to\infty} x_k = x_0$, so, for $\alpha > 0$ and $\beta \in \mathbb{N}, Q(\beta, \alpha) \in J$ which immediately gives $K(\beta, \alpha) \in F(J)$. Thus $K(1, \alpha), K(2, \alpha), K(3, \alpha), \ldots \in F(J)$. Then by properties (AP) and lemma 1.1 $\exists$ $K \subseteq \mathbb{N}$ with $K = \{k_1, k_2, \ldots\} \in F(J)$ and the set $\{K - K(\beta, \alpha)\}$ is finite for all β. To prove the rest part, we need to prove that $N - \lim_{i\to\infty} \Delta^m x_{k_i} = x_0$. Let, if possible $N - \lim_{i\to\infty} \Delta^m x_{k_i} \neq x_0$. Then $\exists$ some $\gamma_1 > 0$ and $n_0 \in \mathbb{N}$ such that, $\forall\, i \geq n_0, B\big(\Delta^m x_{k_i} - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha\big) \leq 1 - \gamma_1, D\,\big(\Delta^m x_{k_i} - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha\big) \geq \gamma_1$ *and* $H(\Delta^m x_{k_i} - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) \geq \gamma_1$ which implies that the set $\{k_i \in \mathbb{N} : B(\Delta^m x_{k_i} - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) > 1 - \gamma, D(\Delta^m x_{k_i} - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) < \gamma_1$ and $H(\Delta^m x_{k_i} - x_0, \varrho_1, \varrho_2, \ldots, \varrho_{n-1}; \alpha) < \gamma_1 \}$ is a finite set and must belong to J, which is a contradiction as this set belongs to $F(J)$. Hence $N - \lim_{i\to\infty} \Delta^m x_{k_i} = x_0$.

4. $\Delta^m(J_N)-$ Cauchy sequence in N-n-NLS.

Definition 4. 1. A sequence $x=(x_k)$ in X is called $\Delta^m(J_N)$ $-$Cauchy iff for every $\gamma>0$ and $\alpha>0$, $\exists\, p\in\mathbb{N}$ s.t. $\{k\in\mathbb{N}: B(\Delta^m x_k - x_p, \varrho_1, \varrho_2, \dots, \varrho_{n-1}; \alpha)\le 1-\gamma$ or $D(\Delta^m x_k - x_p, \varrho_1, \varrho_2, \dots, \varrho_{n-1}, \alpha)\ge\gamma, H(\Delta^m x_k - x_p, \varrho_1, \varrho_2, \dots, \varrho_{n-1}, \alpha)\ge\gamma\}\in J$. Next, we give the generalized Cauchy convergence criteria in $N{-}n{-}NLS$.

Theorem 4.1: *If* $\Delta^m(J_N)-lim_{k\to\infty} x_k = x_0$ *then* $x=(x_k)$ *is* $\Delta^m(J_N)$ $-$*Cauchy.*

Proof: The proof can be obtained similar to the proof of the Theorem 5.1 of [31], So, omitted.

Definition 4.2: $x=(x_k)$ in X is called $\Delta^m(J_N^\star)$ $-$Cauchy iff $\exists$ a set $I=\{i_1,i_2,i_3,\dots\}\in F(J)$ such that the subsequence $(\Delta^m x_{i_n})$ is N-Cauchy.

Theorem 4.2: *If* $\Delta^m(J_N^\star)-\lim_{k\to\infty} x_k = x_0$, *then* $x=(x_k)$ is $\Delta^m(J_N^\star)-$ *Cauchy.*

Proof: The proof is omitted.

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