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Composition formulas for Saigo k-fractional operator associated with generalized k-Bessel Maitland function

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Abstract

This research focuses on exploring the usage of integral and derivative operator of Saigo's k-fractional type i.e. (S_k*.I.O.*) and (S_k*.D.O.*) that utilize the k-hypergeometric function (k-H.F.). The findings are presented in relation to the Generalized k-Bessel Maitland Function (k-B.M.F.) and articulated in terms of the k-Wright function (k-W.F.), and then the beta transform (B.T.) is applied to obtain the image formulas of integral transforms.

Subject Classification: *44C20, 44A20.*

Keywords: *Saigo's k-fractional operator, Generalized k-Bessel Maitland function, k-Wright function.*

1. Introduction

The concept of fractional calculus (F.C.) was first established in 1695, but it has been in the past two decades that researchers has utilize it effectively, thanks to the availability of computing tool F.C. has proven to be very useful in engineering and science with scholars discovering significant applications in several fields [1-5]. In recent years, several research publications have focused on exploring the generalized classical F.C. operators. One such study was conducted by Mubeen and Habibullah [6-8] who introduced the operator named as k-Reimann-Liouville and its applications. Later on, Dorrego [9], proposed an alternate definition of the operator. Gupta and Parihar [10] presented a concept of S_k.I.O. and S_k.D.O.

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The generalized k-B.M.F. [11] is represented as:

$$J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}(z)=\sum_{n=0}^{\infty}\frac{(\beta)_{qn,k}(-z)^n}{\Gamma_k(\mathrm{u}n+\alpha+1)(\delta)_{pn,k}} \tag{1}$$

where $k\in R_e$, (Real Number) $\mathrm{u},\alpha,\beta,\delta\in C,\mathrm{R}_e(\mathrm{u})>0,\mathrm{R}_e(\alpha)\geq -1,\mathrm{R}_e(\beta)>0,\mathrm{R}_e(\delta)>0; p,q>0,$ and $q<\mathrm{R}_e(\mathrm{u})+p.$

To achieve the desired outcomes, it is essential to use the following Saigo-k fractional integrals and differential operators [10]

Lemma 1.1: *Let* $\sigma,\xi,\varrho,\theta\in C,R_e(\theta)>max[\{0,R_e(\xi-\varrho)]$ *then the following relations are established:*

$$\left(I_{0^+,k}^{\sigma,\xi,\varrho}t^{(\theta/k)-1}\right)(m)=\sum_{n=0}^{\infty}k^n\frac{\Gamma_k(\theta)\Gamma_k(\theta-\xi+\varrho)}{\Gamma_k(\theta-\xi)\Gamma_k(\theta+\sigma+\varrho)}m^{\theta-\xi/k-1} \tag{2}$$

Lemma 1.2: Let $\sigma,\xi,\varrho,\theta\in \mathrm{C},\mathrm{R}_e(\sigma)>0,k\in {\mathrm{R}_e}^+(0,\infty)$and $\mathrm{R}_e(\theta)>\max[\mathrm{R}_e(-\xi),\mathrm{R}_e(-\varrho)]$ then the following relations are established:

$$\left(I_{-,k}^{\sigma,\xi,\varrho}t^{-(\theta/k)}\right)(m)=\sum_{n=0}^{\infty}k^n\frac{\Gamma_k(\theta+\xi)\Gamma_k(\theta+\varrho)}{\Gamma_k(\theta)\Gamma_k(\theta+\sigma+\xi+\varrho)}m^{-\theta-\xi/k} \tag{3}$$

Lemma 1.3: Let $\sigma,\xi,\varrho,\theta\in \mathrm{C},n=[\mathrm{R}_e(\sigma)]+1,k\in {\mathrm{R}_e}^+(0,\infty)$ such that $\mathrm{R}_e(\theta)>\max[0,\mathrm{R}_e(-\sigma-\xi-\varrho)]$ then the following relations are established:

$$\left(D_{0^+,k}^{\sigma,\xi,\varrho}t^{(\theta/k)-1}\right)(m)=\sum_{n=0}^{\infty}\frac{\Gamma_k(\theta)\Gamma_k(\theta+\xi+\varrho+\sigma)}{\Gamma_k(\theta+\varrho)\Gamma_k(\theta+\xi+n-nk)}m^{(\theta+\xi+n/k)-n-1} \tag{4}$$

Lemma 1.4: Let $\sigma,\xi,\varrho,\theta\in \mathrm{C},n=[\mathrm{R}_e(\sigma)]+1,k\in {\mathrm{R}_e}^+,\mathrm{R}_e(\theta)>\max[\mathrm{R}_e(-\sigma-\varrho),\mathrm{R}_e(\xi-nk+n)]$ then the following relations are established:

$$\left(D_{-,k}^{\sigma,\xi,\varrho}t^{-(\theta/k)}\right)(m)=\sum_{n=0}^{\infty}\frac{\Gamma_k(\theta-\xi-n+nk)\Gamma_k(\theta+\sigma+\varrho)}{\Gamma_k(\theta)\Gamma_k(\theta-\xi+\varrho)}m^{(-\theta+\xi+n/k)-n} \tag{5}$$

2. Main results

2.1. *Saigo k-Fractional Operator allied with Generalized k-B.M.F.*

The findings in the subsequent section are exhibited using the S_k.I.O. and S_k.D.O. allied with generalized k-B.M.F.

Theorem 2.1.1: *Let* $\sigma,\xi,\varrho,\theta,\mathrm{u},\alpha,\beta,\delta\in C,\ k\in {R_e}^+$ *such that* $R_e(\theta)>max[\{0,R_e(\xi-\varrho)],R_e(\mathrm{u})>0,R_e(\alpha)\geq-1,R_e(\beta)>0,R_e(\delta)>0.$ *Also let* $p,q>0,$ *and* $q<R_e(\mathrm{u})+p$ *and* $c\in R_e;\eta>-1,$ *then for* $t>0$

$$I_{0^+,k}^{\sigma,\xi,\varrho}\left(t^{(\theta/k)-1}\left(J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}\left(ct^{\eta/k}\right)\right)\right)(m)=\frac{m^{(\theta-\xi/k)-1}\Gamma_k(\delta)}{\Gamma_k(\beta)}\times$$
$${}_3\psi_4^k\left[\begin{matrix}(\beta,qk),(\theta,\eta),(\theta-\xi+\varrho,\eta)\\(\alpha+1,\mathrm{u}),(\delta,pk),(\theta-\xi,\eta),(\theta+\sigma+\varrho,\eta)\end{matrix}:-ckm^{\eta/k}\right] \tag{6}$$

Proof: By applying equation (1) and opting the left-hand integral of k-S_k.I.O. within the summation, the mathematical equation on the LHS of equation (6) can be modified as follows

$$= \sum_{n=0}^{\infty} \frac{(\beta)_{qn,k}(-c)^n}{\Gamma_k(un+\alpha+1)(\delta)_{pn,k}} \left(I_{0^+,k}^{\sigma,\xi,\varrho} \left(t^{\theta+\eta n/k-1} \right)(m) \right) \tag{7}$$

Now, using lemma 1.1 we obtain

$$= \sum_{n=0}^{\infty} \frac{\Gamma_k(\beta+qnk)\Gamma_k(\delta)(-c)^n}{\Gamma_k(\beta)\Gamma_k(un+\alpha+1)\Gamma_k(\delta+pnk)}$$

$$\times \frac{k^n \Gamma_k(\theta+\eta n)\Gamma_k(\theta+\eta n-\xi+\varrho)}{\Gamma_k(\theta+\eta n-\xi)\Gamma_k(\theta+\eta n+\sigma+\varrho)} m^{(\theta+\eta n-\xi/k)-1} \tag{8}$$

$$= \frac{m^{(\theta-\xi/k)-1}\Gamma_k(\delta)}{\Gamma_k(\beta)} \sum_{n=0}^{\infty} \left(-ckn^{\eta/k}\right)^n \frac{\Gamma_k(\beta+qnk)}{\Gamma_k(un+\alpha+1)\Gamma_k(\delta+pnk)}$$

$$\times \frac{\Gamma_k(\theta+\eta n)\Gamma_k(\theta+\eta n-\xi+\varrho)}{\Gamma_k(\theta+\eta n-\xi)\Gamma_k(\theta+\eta n+\sigma+\varrho)} \tag{9}$$

Expressing RHS in respect of k-W.F. ${}_p\psi_q^k$ results in (6). This completes the proof.

Theorem 2.1.2: *Let* $\sigma, \xi, \varrho, \theta, \mathrm{u}, \alpha, \beta, \delta \in C, k \in {R_e}^+(0,\infty)$ *such that* $R_e(\sigma) > 0$ *and* $R_e(\theta) > max[R_e(-\xi), R_e(-\varrho)], R_e(\mathrm{u}) > 0, R_e(\alpha) \geq -1, R_e(\beta) > 0, R_e(\delta) > 0;$ *also let* $p, q > 0,$ *and* $q < R_e(\mathrm{u}) + p$ *and* $c \in R_e; \eta > -1,$ *then for* $t > 0$

$$I_{-,k}^{\sigma,\xi,\varrho} \left(t^{-(\theta/k)} \left(J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}; \left(ct^{-\eta/k}\right) \right) \right)(m) = \frac{m^{-(\theta+\xi/k)}\Gamma_k(\delta)}{\Gamma_k(\beta)}$$

$$\times {}_3\psi_4^k \left[\begin{matrix} (\beta, qk), (\theta+\xi, \eta), (\theta+\varrho, \eta) \\ (\alpha+1, \mathrm{u}), (\delta, pk), (\theta, \eta), (\theta+\xi+\sigma+\varrho, \eta) \end{matrix} : -ckm^{-\eta/k} \right] \tag{10}$$

Proof: The proof closely resembles that of theorem 2.1.1. Hence, we'll omit the specifics.

Theorem 2.1.3: Let $\sigma, \xi, \varrho, \theta, \mathrm{u}, \alpha, \beta, \delta \in \mathrm{C},\ k \in {\mathrm{R}_e}^+(0,\infty)$ *such that*

$R_e(\theta) > max[0, R_e(-\sigma-\xi-\varrho)]$ *and* $R_e(\mathrm{u}) > 0, R_e(\alpha) \geq -1, R_e(\beta) > 0, R_e(\delta) > 0; p, q > 0,$ *and* $q < R_e(\mathrm{u}) + p.$ *Also let* $c \in R_e; \eta > -1,$ *then for* $t > 0$

$$D_{0^+,k}^{\sigma,\xi,\varrho} \left(t^{(\theta/k)-1} \left(J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}; \left(ct^{\eta/k}\right) \right) \right)(m) = \frac{m^{(\theta+\xi/k)-1}\Gamma_k(\delta)}{\Gamma_k(\beta)}$$

$$\times {}_3\psi_4^k \left[\begin{matrix} (\beta, qk), (\theta, \eta), (\theta+\xi+\varrho+\sigma, \eta) \\ (\alpha+1, \mathrm{u}), (\delta, pk), (\theta+\varrho, \eta), (\theta+\xi, \eta+1-k) \end{matrix} : -cm^{(\eta+1/k)-1} \right] \tag{11}$$

Proof: By applying equation (1) and opting the left-hand derivative of k-Saigo fractional derivative operator within the summation, the mathematical equation on the LHS of equation (11) can be modified as follows

$$= \sum_{n=0}^{\infty} \frac{(\beta)_{qn,k}(-c)^n}{\Gamma_k(un+\alpha+1)(\delta)_{pn,k}} \left(D_{0^+,k}^{\sigma,\xi,\varrho} \left(t^{\theta+\eta n/k-1} \right)(m) \right) \tag{12}$$

Now using lemma 1.3 we obtain

$$= \sum_{n=0}^{\infty} \frac{\Gamma_k(\beta+qnk)\Gamma_k(\delta)(-c)^n}{\Gamma_k(\beta)\Gamma_k(un+\alpha+1)\Gamma_k(\delta+pnk)}$$

$$\times \frac{\Gamma_k(\theta+\eta n)\Gamma_k(\theta+\eta n+\xi+\varrho+\sigma)}{\Gamma_k(\theta+\eta n+\varrho)\Gamma_k(\theta+\eta n+\xi+n-nk)} m^{(\theta+\eta n+\xi+n/k)-n-1} \tag{13}$$

$$= \frac{m^{(\theta-\xi/k)-1}\Gamma_k(\delta)}{\Gamma_k(\beta)} \sum_{n=0}^{\infty} \left(-cm^{(\eta+1/k)-1}\right)^n \frac{\Gamma_k(\beta+qnk)}{\Gamma_k(\mathrm{u}n+\alpha+1)\Gamma_k(\delta+pnk)}$$
$$\times \frac{\Gamma_k(\theta+\eta n)\Gamma_k(\theta+\eta n+\xi+\varrho+\sigma)}{\Gamma_k(\theta+\eta n+\varrho)\Gamma_k(\theta+\eta n+\xi+n-nk)} \tag{14}$$

Expressing RHS in respect of k-W.F. ${}_p\psi_q^k$ results in (11). This completes the proof.

Theorem 2.1.4: *Let* $\sigma, \xi, \varrho, \theta, \mathrm{u}, \alpha, \beta, \delta \in C\ k \in R_e{}^+$ *such that* $R_e(\theta) > max[R_e(-\sigma-\varrho), R_e(\xi-nk+n)], R_e(\mathrm{u}) > 0, R_e(\alpha) \geq -1, R_e(\beta) > 0, R_e(\delta) > 0; p, q > 0, and\ q < R_e(\mathrm{u}) + p$. *Also let* $c \in R_e; \eta > -1$, *then for* $t > 0$

$$D_{-,k}^{\sigma,\xi,\varrho}\left(t^{-(\theta/k)}\left(J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}; \left(ct^{-\eta/k}\right)\right)\right)(m) = \frac{m^{(-\theta+\xi/k)}\Gamma_k(\delta)}{\Gamma_k(\beta)}$$
$$\times {}_3\psi_4^k\left[\begin{matrix}(\beta,qk),(\theta-\xi,\eta+k-1),(\theta+\sigma+\varrho,\eta)\\(\alpha+1,\mathrm{u}),(\delta,pk),(\theta,\eta),(\theta-\xi+\sigma,\eta)\end{matrix}: -cm^{(-\eta+1/k)-1}\right] \tag{15}$$

Proof: The proof closely resembles that of theorem 2.1.3. Hence, we'll omit the specifics.

Results provided in (6) and (11) are reduced to the following corollary by allocating some appropriate values to the relevant parameters.

Corollary 2.2.1: *If* $p = 0, \alpha = \alpha - 1, c = -c$ *then (21) leads to the following result.*

$$I_{0^+,k}^{\sigma,\xi,\varrho}\left(t^{(\theta/k)-1}\left(J_{k,\alpha-1,q,}^{\mathrm{u},\beta,\delta}\left(-ct^{\eta/k}\right)\right)\right)(m) = \frac{m^{(\theta-\xi/k)-1}\Gamma_k(\delta)}{\Gamma_k(\beta)}$$
$$\times\ {}_3\psi_4^k\left[\begin{matrix}(\beta,qk),(\theta,\eta),(\theta-\xi+\varrho,\eta)\\(\alpha,\mathrm{u}),(\delta,0),(\theta-\xi,\eta),(\theta+\sigma+\varrho,\eta)\end{matrix}: ckm^{\eta/k}\right] \tag{16}$$

Corollary 2.2.2: *If* $p = 0, \alpha = \alpha - 1, c = -c$ *then (29) leads to the following result.*

$$D_{0^+,k}^{\sigma,\xi,\varrho}\left(t^{(\theta/k)-1}\left(J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}; \left(ct^{\eta/k}\right)\right)\right)(m) = \frac{m^{(\theta+\xi/k)-1}\Gamma_k(\delta)}{\Gamma_k(\beta)}$$
$$\times {}_3\psi_4^k\left[\begin{matrix}(\beta,qk),(\theta,\eta),(\theta+\xi+\varrho+\sigma,\eta)\\(\alpha,\mathrm{u}),(\delta,0),(\theta+\varrho,\eta),(\theta+\xi,\eta+1-k)\end{matrix}: cm^{(\eta+1/k)-1}\right] \tag{17}$$

2.3. *Image formulas in correspondence with integral transformation*

Within this section, we will demonstrate several theorems which involves the preceding section in relation to the integral transform.

The k-B.F. [12] is stated as

$$B_k(w, e) = \frac{1}{k}\int_0^1 \mathfrak{z}^{w-1}(1-\mathfrak{z})^{e-1}\, d\mathfrak{z}, w > 0, e > 0 \tag{18}$$

And another form of k-B.F. is

$$B_k(f(\mathfrak{z}): w, e) = \frac{1}{k}\int_0^1 \mathfrak{z}^{(w/k)-1}(1-\mathfrak{z})^{(e/k)-1} f(\mathfrak{z})\, d\mathfrak{z}, w > 0, e > 0 \quad (19)$$

Theorem 2.3.1: *Let* $\sigma, \xi, \varrho, \theta, \mu, \alpha, \beta, \delta \in C$, $k \in R_e{}^{+}$ *such that* $R_e(\theta) > max[\{0, R_e(\xi - \varrho)\}]$ *and* $R_e(\mathrm{u}) > 0, R_e(\alpha) \geq -1, R_e(\beta) > 0, R_e(\delta) > 0.$

Therefore, the subsequent fractional integral is valid.

$$B_k\left(\left(I_{0^+,k}^{\sigma,\xi,\varrho}\left(t^{(\theta/k)-1} J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}: c(\mathfrak{z}t)^{\eta/k}\right)\right)(m); w, e\right) = \frac{m^{(\theta-\xi/k)-1}\Gamma_k(\delta)}{\Gamma_k(\beta)}$$

$$\times\ {}_4\psi_5^k\left[\begin{matrix}(\beta, qk), (\theta, \eta), (\theta - \xi + \varrho, \eta), (w, \eta)\\ (\alpha+1, \mathrm{u}), (\delta, pk), (\theta - \xi, \eta), (\theta + \sigma + \varrho, \eta), (w + e, \eta)\end{matrix} : -ckm^{\eta/k}\right] \quad (20)$$

Proof: By employing the LHS of equation (20) and applying (19), we have

$$= \frac{1}{k}\int_0^1 \mathfrak{z}^{(w/k)-1}(1-\mathfrak{z})^{(e/k)-1}\left(I_{0^+,k}^{\sigma,\xi,\varrho}\left(t^{(\theta/k)-1} J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}: c\left(\mathfrak{z}t^{\eta/k}\right)\right)\right)(m) d\mathfrak{z} \quad (21)$$

which, under the conditions of Theorem 2.1.1, employing (1) and altering the sequence of integration and summation, yields

$$= \sum_{n=0}^{\infty} \frac{(\beta)_{qn,k}(-c)^n}{\Gamma_k(\mathrm{u}n+\alpha+1)(\delta)_{pn,k}}\left(I_{0^+,k}^{\sigma,\xi,\varrho}\left(t^{\theta+\eta n/k-1}\right)(m)\right)$$

$$\times \frac{1}{k}\int_0^1 \mathfrak{z}^{w+\eta n/k-1}(1-\mathfrak{z})^{e/k-1}\, d\mathfrak{z} \quad (22)$$

By utilizing Lemma 1 and replacing eqn (2) into (22), we can derive the following result.

$$= \frac{m^{(\theta-\xi/k)-1}\Gamma_k(\delta)\Gamma_k(e)}{\Gamma_k(\beta)}\sum_{n=0}^{\infty} \frac{\Gamma_k(\beta+qnk)}{\Gamma_k(\mathrm{u}n+\alpha+1)\Gamma_k(\delta+pnk)}$$

$$\times \frac{\Gamma_k(\theta+\eta n)\Gamma_k(\theta+\eta n-\xi+\varrho)}{\Gamma_k(\theta+\eta n-\xi)\Gamma_k(\theta+\eta n+\sigma+\varrho)} \frac{\Gamma_k(w+\eta n)\Gamma_k(e)}{\Gamma_k(w+\eta n+e)}\left(-ckm^{\eta/k}\right)^n \quad (23)$$

Expressing RHS in terms of k-Wright function ${}_p\psi_q^k$ can derive the desired outcome, which is shown in (20).

Theorem 2.3.2: *Let* $\sigma, \xi, \varrho, \theta, \mathrm{u}, \alpha, \beta, \delta \in C$, $k \in R_e{}^{+}(0, \infty)$*and* $R_e(\theta) > max[R_e(-\xi), R_e(-\varrho)]$ *and* $R_e(\mathrm{u}) > 0, R_e(\alpha) \geq -1, R_e(\beta) > 0, R_e(\delta) > 0.$ *Therefore, the subsequent fractional integral is valid.*

$$B_k\left(\left(I_{-,k}^{\sigma,\xi,\varrho}\left(t^{-(\theta/k)} J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}: c(\mathfrak{z}t)^{-\eta/k}\right)\right)(m); w, e\right) = \frac{m^{-(\theta+\xi/k)}\Gamma_k(\delta)\Gamma_k(w)}{\Gamma_k(\beta)}$$

$$\times\ {}_4\psi_5^k\left[\begin{matrix}(\beta, qk), (\theta + \xi, \eta), (\theta + \varrho, \eta), (w, -\eta)\\ (\alpha+1, \mathrm{u}), (\delta, pk), (\theta, \eta), (\theta + \xi + \sigma + \varrho, \eta), (w - \eta, e)\end{matrix} : -ckm^{-\eta/k}\right] \quad (24)$$

Proof: The proof closely resembles that of theorem 2.3.1. Hence, we'll omit the specifics.

Theorem 2.3.3: *Let* $\sigma, \xi, \varrho, \theta, \mathrm{u}, \alpha, \beta, \delta \in C, k \in {R_e}^{+}(0, \infty)$ *such that* $R_e(\theta) > max[0, R_e(-\sigma-\xi-\varrho)]$ *and* $R_e(\mathrm{u}) > 0, R_e(\alpha) \geq -1, R_e(\beta) > 0, R_e(\delta) > 0$. *Therefore, the subsequent fractional derivative is valid:*

$$B_k\left(\left(D_{0^+,k}^{\sigma,\xi,\varrho}\left(t^{(\theta/k)-1} J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}: c(\mathfrak{z}t)^{\eta/k}\right)\right)(m); w, e\right) = \frac{x^{(\theta+\xi/k)-1}\Gamma_k(\delta)\Gamma_k(e)}{\Gamma_k(\beta)} \times {}_4\psi_5^k\left[\begin{matrix}(\beta, qk), (\theta, \eta), (\theta+\xi+\varrho+\sigma, \eta), (w, \eta)\\(\alpha+1, \mathrm{u}), (\delta, pk), (\theta+\varrho, \eta), (\theta+\xi, \eta+1-k), (w+e, \eta)\end{matrix}: -cm^{(\eta+1/k)-1}\right] \tag{25}$$

Proof: By employing the LHS of equation (25) and applying (41), we have.

$$= \frac{1}{k}\int_0^1 \mathfrak{z}^{w/k-1}(1-\mathfrak{z})^{e/k-1}\left(D_{0^+,k}^{\sigma,\xi,\varrho}\left(t^{(\theta/k)-1} J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}: c(\mathfrak{z}t)^{\eta/k}\right)\right)(m)\,d\mathfrak{z} \tag{26}$$

which, under the conditions of Theorem 2.1.3, employing (1) and altering the sequence of integration and summation, yields

$$= \sum_{n=0}^{\infty} \frac{(\beta)_{qn,k}(-c)^n}{\Gamma_k(un+\alpha+1)(\delta)_{pn,k}}\left(D_{0^+,k}^{\sigma,\xi,\varrho}\left(t^{\theta+\eta n/k-1}\right)(m)\right) \times \frac{1}{k}\int_0^1 \mathfrak{z}^{(w+\eta n)/k-1}(1-\mathfrak{z})^{e/k-1}\,d\mathfrak{z} \tag{27}$$

By utilizing Lemma 1 and replacing eqn (2) into (19), we can derive the following result.

$$= \frac{m^{(\theta+\xi/k)-1}\Gamma_k(\delta)\Gamma_k(e)}{\Gamma_k(\beta)}\sum_{n=0}^{\infty}\frac{\Gamma_k(\beta+qnk)}{\Gamma_k(\mathrm{u}n+\alpha+1)\Gamma_k(\delta+pnk)} \times \frac{\Gamma_k(\theta+\eta n)\Gamma_k(\theta+\eta n+\xi+\varrho+\sigma)}{\Gamma_k(\theta+\eta n+\varrho)\Gamma_k(\theta+\eta n+\xi+n-nk)}\frac{\Gamma_k(w+\eta n)\Gamma_k(e)}{\Gamma_k(w+e+\eta n)}\left(-cm^{(\eta+1/k)-1}\right)^n \tag{28}$$

Expressing RHS in terms of k-wright function ${}_p\psi_q^k$ can derive the desired outcome, which is shown in (26).

Theorem 2.3.4: *Let* $\sigma, \xi, \varrho, \theta, \mathrm{u}, \alpha, \beta, \delta \in C, k \in R_e, R_e(\theta) > max[R_e(-\sigma-\varrho), R_e(\varsigma-nk+n)]$ *and* $R_e(\mathrm{u}) > 0, R_e(\alpha) \geq -1, R_e(\beta) > 0, R_e(\delta) > 0$. *Therefore, the subsequent fractional derivative is valid:*

$$B_k\left(\left(D_{-,k}^{\sigma,\xi,\varrho}\left(t^{-(\theta/k)} J_{k,\alpha,q,p}^{\mathrm{u},\beta,\delta}: c(\mathfrak{z}t)^{-\eta/k}\right)\right)(m); w, e\right) = \frac{m^{(-\theta+\xi/k)}\Gamma_k(\delta)\Gamma_k(e)}{\Gamma_k(\beta)} \times {}_4\psi_5^k\left[\begin{matrix}(\beta, qk), (\theta-\xi, \eta+k-1), (\theta+\sigma+\varrho, \eta), (w, -\eta)\\(\alpha+1, \mathrm{u}), (\delta, pk), (\theta, \eta), (\theta-\xi+\sigma, \eta), (w+e, -\eta)\end{matrix}: -cm^{(-\eta+1/k)-1}\right] \tag{19}$$

Proof: The proof closely resembles that of theorem 2.3.1. Hence, we'll omit the specifics.

Conclusion

Some theorems have been derived with the Saigo k-operator which is extensively utilized to establish inequalities such as Chebyshev type inequalities, Grüss type inequalities, Andreev-Korkin identity leading to the derivation of various theorems. The outcomes obtained through this process have been further utilized to extend several new and intriguing results, often utilizing other special functions.

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