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A theoretical approach to find the solution of population based mathematical model

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Abstract

In this paper, fixed point approach is used to find the existence and unique solution of linear system of equation for the impact analysis of COVID-19 population based problem using the actual data of five most affected countries. One theoretical approach for mathematical model is introduced using fixed point to find the existence and unique solution.

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Keywords: *Fixed point, Banach contraction principle, COVID-19, Linear system of equation, Mathematical model.*

1. Introduction

Over the past few years, the global population has been dramatically affected by COVID-19 pandemic. COVID-19 emerged as an infectious respiratory illness originated by a new coronavirus named SARS-CoV-2. [1]. It significantly impacted on public health, causing millions of infections and deaths globally. In Wuhan, Hubei province, China, this disease was first discovered in December 2019. As per the latest reports, there were 21,357,890 confirmed cases of COVID-19 [2] till 15 August 2020, which was an alarming situation. The effect of COVID-19 on population has been studied by various researchers using various approaches [4]. In 2021[3], the author developed two

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systems of linear equations to accurately analyze factors such as total cases, deaths, recoveries, serious cases, and testing, and by using the Gauss Jordan elimination method. In 2023 [5], the paper focuses on the construction of a fractional mathematical model with quarantine classifications and government sanctions to control the COVID-19 epidemic. In the mathematical modeling of real world process, a fundamental outcome in fixed point theory utilizes the banach contraction principle to address the arising problem. The FPT work for solving equations $Tx = x$ for mapping T defined on susbsets of normed spaces or metric space (MS) [9]. In the context of matrices, fixed-point theory includes the study of matrices having certain fixed-point characteristics. It is a very powerful mathematical tool which is used in many applications where stability of solution is needed. There are various authors who provided applications of fixed point theory [6]. In 1995 [11], to prevent overflow, the mean squared value of the Coefficient vector is estimated in the systolic array version of the QR-RLS algorithm. In 2021[10], the authors provide an in-depth examination of a new fractional COVID-19 model incorporating non-local fractional operators. and the Caputo derivative, showing the uniqueness and existence of solutions and establishing stability criteria. The Banach fixed-point theorem offers several advantages including error estimates of successive iterations and convergence rate. [12]. Detailed research on estimating fixed points could have been found in [9].

In this paper, we have used the concept of fixed point approach on a COVID-19 model on the system of linear equations. To model our data, we examine the six different factors related to COVID-19 which affected the population of the top five affected nations (i.e. total affected, total deaths, total recovered, total serious cases, overall tests, and total population) declared by the World Health Organization(WHO) and analyzing these factors [6, 7]. By fixed point approach, we find the existence and unique solution of linear system of equation for COVID-19 population based problem.

As follows, the paper is instructed. In section 2, we convert the information in binary system for the top five affected nations by COVID-19 [2] depending upon the above mentioned six different factors. In section 3, we have used the fixed point approach based on banach contraction principle for existing and uniqueness of solution for the model based on COVID-19 data.

2. Preliminaries and Mathematical Formulation

The study of matrices with certain fixed-point aspects is a part of FPT in the framework of matrices. We have used the fixed point approach based on banach contraction principle for existing and uniqueness of solution which is defined as:

Definition 2.1 Let $T: X \to X$ be a contraction mapping and (X, d) be a non-complete metric space then T has a unique fixed point x^* in X. Furthermore, it define a sequence $(x_n)_{n \in N}$ by $x_n = T(x_{n-1})$ for $n \geq 1$. Then $\lim_{n\to\infty} x_n = x^*$.

A point x is a fixed point of function f, under certain conditions which maps itself i.e. $f\,(x) = x$. In the context of matrices, if A is square matrix and the vector x is a fixed point of A, then the equation will be: $Ax = x$ which represents the condition for x to be a fixed point of the linear transformation defined by A. This can also be written as $(A - I)x = 0$, where I is the identity matrix. The information for the top five nations which were impacted by COVID-19 [2] till August 15, 2020 is provided in the table below. The following parameters are used for the studying of impact of COVID-19:

Table 1
COVID-19 Data

Country	Total Affected	Total Death	Total Recovered	Serious cases	Total Tests	Population
USA	0.0005476266 $\times 10^{10}$	0.0000171535 $\times 10^{10}$	0.0002875147 $\times 10^{10}$	0.0000017217 $\times 10^{10}$	0.0069632009 $\times 10^{10}$	0.0331240477 $\times 10^{10}$
Brazil	0.0003278895 $\times 10^{10}$	0.0000106571 $\times 10^{10}$	0.0002384302 $\times 10^{10}$	0.0000008318 $\times 10^{10}$	0.0013464336 $\times 10^{10}$	0.0212743668 $\times 10^{10}$
India	0.0002527302 $\times 10^{10}$	0.0000049148 $\times 10^{10}$	0.0001809542 $\times 10^{10}$	0.0000008944 $\times 10^{10}$	0.0028563095 $\times 10^{10}$	0.1381641319 $\times 10^{10}$
Russia	0.0000912823 $\times 10^{10}$	0.0000015z498 $\times 10^{10}$	0.0000722964 $\times 10^{10}$	0.0000002300 $\times 10^{10}$	0.0031903055 $\times 10^{10}$	0.0145942287 $\times 10^{10}$
South Africa	0.0000579140 $\times 10^{10}$	0.0000011556 $\times 10^{10}$	0.0000461734 $\times 10^{10}$	0.0000000539 $\times 10^{10}$	0.0003351111 $\times 10^{10}$	0.0059397910 $\times 10^{10}$

3. Fixed Point approach for existence and uniqueness of solution for model based on COVID-19 data:

We have arranged the COVID-19 data into a square matrix of order five. From the above table-1, the values of COVID-19 related factors have been arranged in rows for five nations [8]. For the convenience, we take 10^{10} common out from the matrix to convert it into a suitable form to apply fixed point approach. So we consider the following matrix:

$$A=\begin{bmatrix} 0.0005476266 & 0.0000171535 & 0.0002875147 & 0.0000017217 & 0.0069632009 \\ 0.0003278895 & 0.0000106571 & 0.0002384302 & 0.0000008318 & 0.0013464336 \\ 0.0002527302 & 0.0000049148 & 0.0001809542 & 0.0000008944 & 0.0028563095 \\ 0.0000912823 & 0.0000015498 & 0.0000722964 & 0.0000002300 & 0.0031903055 \\ 0.0000579140 & 0.0000011556 & 0.0000461734 & 0.0000000539 & 0.0003351111 \end{bmatrix}$$

Therefore, by using the above table, we can express as the following matrices:

$$0.0005476266\,x_1 + 0.0000171535x_2 + 0.0002875147x_3 + 0.0000017217x_4 + 0.0069632009x_5 = 0.0331240477 \quad (1)$$

$$0.0003278895x_1 + 0.0000106571x_2 + 0.0002384302x_3 + 0.0000008318x_4 + 0.0013464336x_5 = 0.0212743668 \quad (2)$$

$$0.0002527302x_1 + 0.0000049148x_2 + 0.0001809542x_3 + 0.0000008944x_4 + 0.0028563095x_5 = 0.1381641319 \quad (3)$$

$$0.0000912823x_1 + 0.0000015498x_2 + 0.0000722964x_3 + 0.0000002300x_4 + 0.0031903055x_5 = 0.0145942287 \quad (4)$$

$$0.0000579140x_1 + 0.0000011556x_2 + 0.0000461734x_3 + 0.0000000539x_4 + 0.0003351111x_5 = 0.0059397910 \quad (5)$$

These system of equations (1) - (5) can be written as:

$$x_1 = (1-0.0005476266)x_1 - 0.0000171535x_2 - 0.0002875147x_3 - 0.0000017217x_4 - 0.0069632009x_5 + 0.0331240477 \quad (6)$$

$$x_2 = -0.0003278895x_1 + (1-0.0000106571)x_2 - 0.0002384302x_3 - 0.0000008318x_4 - 0.0013464336x_5 + 0.0212743668 \quad (7)$$

$$x_3 = -0.0002527302x_1 - 0.0000049148x_2 + (1-0.0001809542)x_3 - 0.0000008944x_4 - 0.0028563095x_5 + 0.1381641319 \quad (8)$$

$$x_4 = -0.0000912823x_1 - 0.0000015498x_2 - 0.0000722964x_3 + (1-0.0000002300)x_4 - 0.0031903055x_5 + 0.0145942287 \quad (9)$$

$$x_5 = -0.0000579140x_1 - 0.0000011556x_2 - 0.0000461734x_3 - 0.0000000539x_4 + (1-0.0003351111)x_5 + 0.0059397910 \quad (10)$$

Now, by letting $\alpha_{ij} = -a_{ij} + \delta_{ij}$, where

$$\delta_{ij} = \begin{cases} 1, & \text{for } i = j \\ 0, & \text{for } i \neq j \end{cases} \quad (11)$$

System of equations (6) - (10) become equivalent to the following system:

$$x_i = \sum_{j=1}^{5} \alpha_{ij} x_j + b_j \quad (12)$$

If $x = (x_1, x_2, x_3, x_4, x_5) \epsilon \mathcal{R}^5$ and $b = (b_1, b_2, b_3, b_4, b_5) \epsilon \mathcal{R}^5$, then it becomes $x = Ax + b$. Further we can use the following transformation to find the fixed point. T: $\mathcal{R}^5 \to \mathcal{R}^5$ defined by: $T(x) = Ax + b$ (13). If T is contraction mapping, then we can apply the banach contraction principle (by definition 2.1) to get unique solution using the successive approximation approach.

4. Existence and Uniqueness of Solution for Model:

Theorem: *Let $X = R^5$ be a MS with the metric : $d_\infty(x, y) = max\,|x_i - y_i|$, where $1 \leq i \leq 5$ if, $\sum_{j=1}^{5} |\alpha_{ij}| \leq \alpha < 1$, $\forall\, i$ = 1,2,3,4,5, then the linear system of five linear equations with five unknowns has unique solution.*

Proof: It is necessary to demonstrate that the mapping T described by (13) be a contraction, since $X = \mathcal{R}^5$ with respect to metric d_∞ is complete. $d_\infty(T(x), T(y)) = \max |\sum_{j=1}^{5} \alpha_{ij}(x_i - y_i)|$

$$\leq |0.005476266(x_1 - y_1)| + |0.000171535(x_2 - y_2)| + |0.002875147$$
$$(x_3 - y_3) + |0.000017217(x_4 - y_4) + |0.069632009(x_5 - y_5)$$
$$\leq |0.005476266| d_\infty(x, y) + |0.002875147|\, d_\infty(x, y)$$
$$+|0.000017217| d_\infty(x, y) \; + |0.069632009| d_\infty(x, y)$$
$$\leq |0.005476266 + 0.000171535 + 0.002875147$$
$$+0.000017217 + 0.069632009|\, d_\infty(x, y)$$
$$d_\infty\big(T(x), T(y)\big) \leq \; 0.078172174\, d_\infty(x, y)$$

Similarly, we can obtain the other contractions also for $i = 2,3,4,5$:

$$d_\infty\big(T(x), T(y)\big) \leq 0.019242422 d_\infty(x, y)$$
$$d_\infty\big(T(x), T(y)\big) \leq 0.032958037\, d_\infty(x, y)$$
$$d_\infty\big(T(x), T(y)\big) \leq 0.033556640 d_\infty(x, y)$$
$$d_\infty\big(T(x), T(y)\big) \leq 0.004404080\, d_\infty(x, y)$$

which shows that T is a contraction mapping.

According to the banach contraction theorem (definition 2.1), T satisfy all the conditions of banach contraction principle. Therefore, the model (13) has a unique solution.

5. Conclusion

This paper provides the contraction conditions which shows that if population satisfy the conditions then the population of the COVID-19 affected countries will attain their unique solution by using fixed point approach. The main work in this paper is to apply fixed point approach for existence and uniqueness for model based on COVID-19 data. This work may be used further for finding solution of more complex real world problem.

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