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Proactive decisions and stochastic discounting models in risk treatment operations

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Abstract

It is known the strong practical support of the consideration of fundamental probabilistic concepts as structural factors for the implementations of stochastic modeling operations. It is widely acknowledged that random sums, stochastic integrals, and stochastic models are probabilistic concepts of exceptional significance. The paper focuses on the development of stochastic models by establishing interconnections between the very important probabilistic concepts of random sums and stochastic integrals. In addition, the paper demonstrates how such stochastic models can be implemented to incorporate the crucial concept of proactivity in decision making related to risk treatment operations. Clearly, evidence is presented regarding the effectiveness of making proactive decisions in facilitating the design and implementation of risk treatment.

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1. Introduction

It is widely acknowledged that stochastic models constitute highly efficient probabilistic instruments with applications in many applied fields [10, 11, 12, 13, 20, 27, 28, 29, 32, 33, 34, 35]. Moreover, stochastic modeling constitutes a research activity of particular importance. The structural components of stochastic modeling are model formulation, model investigation, and model interpretation. The implementation of the structural components of stochastic modeling requires extensive use of fundamental probabilistic concepts with significant applications in practical situations [3, 4, 5, 9, 16, 17, 23]. In consequence, stochastic modeling is strongly related to the development of new probabilistic concepts with properties suitable for describing and analyzing real-world problems.

It is widely accepted that the development of such probabilistic concepts is significantly supported by various results in the field of probability distributions. The inclusion of continuous random variables and discrete random variables in the aforementioned probabilistic ideas presents significant challenges in formulating, investigating, and interpreting stochastic models, making it a particularly difficult research activity [22]. The present paper makes use of three extremely important probabilistic concepts, incorporating positive continuous random variables and positive discrete random variables, in order to formulate and investigate the two proposed models. More precisely, the paper employs stochastic integrals, discrete random sums, and continuous discounting operations as structural probabilistic concepts to formulate, investigate, and interpretate two stochastic models [22].

The paper's theoretical contribution establishes an interconnection between two Poisson random sums of two types of stochastic integrals. The first category of stochastic integrals represents the present value of a stochastic process [15]. The representation of the second type of stochastic integrals has not been established [21]. Consequently, the interconnection that is established by the paper introduces a new representation of the second type of stochastic integrals in continuous discounting operations. Furthermore, the establishment of the interconnection offers an indirect computational approach for studying and analyzing the probabilistic behavior of the two Poisson random sums. The paper's practical contribution lies in the interpretation of the formulated Poisson random sums and the mathematical results of the paper's theoretical contribution in portfolio valuation and risk treatment operations.

The paper is specifically focused on implementing five objectives. First objective is to formulate a random sum by employing a Poisson random variable and a stochastic integral arising in continuous stochastic discounting [15]. The conditions for the existence and assessment of the characteristic function corresponding with the stochastic integral have been provided [15]. The second objective is the formulation of a random sum through the utilization of a Poisson random variable and a stochastic integral arising in the field of infinite divisibility with both theoretical and practical applications [21, 25, 30]. Sufficient conditions for the existence and the evaluation of the corresponding characteristic function of that integral are also known [21]. The third objective includes developing a stochastic model utilizing a positive random variable and a stochastic integral that arises from continuous stochastic discounting. The use of that stochastic model for establishing the equality in distribution between the two Poisson random sums of stochastic integrals constitutes the fourth objective. Moreover, the interpretation of the four purposes for making proactive decisions in risk treatment constitutes the fifth objective.

The remaining sections are structured as follows. In Section 2 of the paper, previous research relating to the probabilistic concepts implemented is presented. In Section 3 of the paper, interconnections between Poisson random sums and two categories of stochastic integrals are established. An interpretation of the theoretical findings from the preceding section within the context of risk treatment operations is presented in Section 4. The discussion of the paper's conclusion is presented in Section 5.

2. Previous Research

The probabilistic concepts of great significance discussed in the present paper have been thoroughly investigated and implemented by several researchers in order to develop, analyze, and utilize stochastic models in various scientific disciplines [1, 24, 26, 31]. Klüppelberg and Mikosch investigated probabilities of extreme deviations for Poisson random sums. The results of the research are utilised to address issues in the fields of insurance and finance [18]. Artikis and Malliaris illustrated the present value of a random sum in portfolio management and fundraising, taking into account random timing [2]. A stochastic model by utilizing two Poisson random sums and the derivation of limit distributions of the stochastic model has been addressed by Ladoucette and Teugels [19]. Harrison represented the present value of a firm's income, in the form of a stochastic integral. The firm's cumulative income is determined by a stochastic process that has stationary and independent increments [15]. Artikis and Voudouri proposed the utilization of a convolutional model that incorporates a stochastic integral to discount continuous cash flows. They also introduced the associated features and applications [6]. Furthermore, Artikis and Artikis established a Poisson random sum of power contractions with interpretations in risk treatment operations [7].

3. Formulating and Investigating Stochastic Models

The third section contains the mathematical results of the paper. More precisely, the establishment of interconnections between random sums incorporating the Poisson random variable and two classifications of stochastic integrals is utilized as a research activity providing researchers and experts with particularly valuable information to support proactive decision making and proactive acting.

We consider the Poisson random variable N with parameter λ, the stochastic process $\{Y(t),\ t \geq 0\}$, the positive random variable $S = Y(t+1) - Y(t)$, the characteristic function $\varphi_S(u)$, the stochastic integral

$$C = \int_0^\infty e^{-t/a} dY(t),\ a > 0$$

the characteristic function

$$\varphi_C(u) = e^{a\int_0^u \frac{\log \varphi_S(w)}{w} dw}$$

and the sequence of positive random variables

$$\{C_n, n = 1, 2, \ldots\}$$

distributed as C [15].

We also consider the stochastic process $\{X(t),\ t \geq 0\}$, the positive random variable $L = X(t+1) - X(t)$, the characteristic function $\varphi_L(u)$, the stochastic integral

$$V = \int_0^1 t^{1/a} dX(t),\ a > 0$$

the characteristic function

$$\varphi_V(u) = e^{\frac{a}{u^a}\int_0^u log \varphi_L(w) w^{a-1} dw}$$

and the sequence of positive random variables

$$\{V_n, n = 1, 2, \ldots\}$$

distributed as V [21].

The last part of the present section focuses on examining the equality in distribution of the two Poisson random sums of stochastic integrals

$$V_1 + V_2 + \ldots + V_N.$$

and

$$C_1 + C_2 + \ldots + C_N.$$

Theorem: *By assuming that the random variable S is independent of the stochastic integral*

$$C = \int_0^\infty e^{-t/a} dY(t), \; a > 0$$

then the Poisson random sums

$$V_1 + V_2 + \ldots + V_N$$

and

$$C_1 + C_2 + \ldots + C_N$$

satisfy the equality in distribution

$$V_1 + V_2 + \ldots + V_N \overset{d}{=} C_1 + C_2 + \ldots + C_N \quad (1)$$

if, and only if,

$$L \overset{d}{=} S + \int_0^\infty e^{-t/a} dY(t) \quad (2)$$

is valid.

Proof: We shall only prove the sufficiency condition. The necessity condition is easily established by a reverse argument. By using characteristic functions in (2) we get

$$\varphi_L(u) = \varphi_S(u) e^{a\int_0^u \frac{log\varphi_S(w)}{w}dw}$$

or equivalently

$$log\varphi_L(u) = log\varphi_S(u) + a\int_0^u \frac{log\varphi_S(w)}{w}dw. \quad (3)$$

From (3) we get

$$u^{a-1} log\varphi_L(u) = u^{a-1} log\varphi_S(u) + au^{a-1}\int_0^u \frac{log\varphi_S(w)}{w}dw \tag{4}$$

It is readily seen that the integral equation (4) can be written in the following form

$$\frac{d}{du}\int_0^u log\varphi_L(w)w^{a-1}dw = \frac{d}{du}u^a\int_0^u \frac{log\varphi_S(w)}{w}dw. \tag{5}$$

Integration of (5) implies that

$$\int_0^u log\varphi_L(w)w^{a-1}dw = u^a\int_0^u \frac{log\varphi_S(w)}{w}dw. \tag{6}$$

From (6), with $u \neq 0$, we get that

$$\frac{a}{u^a}\int_0^u log\varphi_L(w)w^{a-1}dw = a\int_0^u \frac{log\varphi_S(w)}{w}dw \tag{7}$$

and from (7) we get that

$$\left(e^{\frac{a}{u^a}\int_0^u \log\varphi_L(w)w^{a-1}dw} - 1\right) = \left(e^{a\int_0^u \frac{\log\varphi_S(w)}{w}dw} - 1\right). \tag{8}$$

From (8) it easily follows that

$$e^{\lambda\left(e^{\frac{a}{u^a}\int_0^u log\varphi_L(w)w^{a-1}dw} - 1\right)} = e^{\lambda\left(e^{a\int_0^u \frac{log\varphi_S(w)}{w}dw} - 1\right)}. \tag{9}$$

By using random variables in (9) we get

$$V_1 + V_2 + \ldots + V_N \overset{d}{=} C_1 + C_2 + \ldots + C_N \ .$$

As a result, the present section provides a theoretical contribution by establishing an interconnection between stochastic models (1) and (2). The practical interpretation of such an interconnection is provided by the next section of the paper.

4. Random Sums of Stochastic Integrals in Modeling

Proactivity is generally recognized as a concept of particular importance in various practical disciplines. More precisely, decision making, strategic thinking, risk treatment, systemics, operational research, global governance, and econometrics are several practical disciplines strongly supported by incorporating the concept of proactivity [7, 14]. This section focuses on interpreting the mathematical results of the previous section in making proactive decisions suitable for risk treatment operations. The quantitative nature of the fundamental components of risk constitutes the primary reason for utilizing mathematical models in the structural operations of risk treatment. It is readily recognized that the occurrence time, the severity, the frequency and the duration of a risk are the fundamental measurable risk components. The presence of randomness in these components makes quite clear that the mathematical models in the structural operations of risk treatment are stochastic [8]. Moreover, it can be said that stochastic discounting models are appropriate for introducing proactivity in decision making related to risk treatment operations. The last part of this section is devoted to interpreting the probabilistic concepts and results of the previous section in stochastic discounting and making proactive decisions.

We consider the stochastic process $\{Y(t), t \geq 0\}$ and that $Y(t)$ represents the income earned by an asset with an unlimited lifespan in the time interval $[0, t]$.

If $1/a$ represents force of interest, then

$$C = \int_0^\infty e^{-t/a} dY(t)$$

indicates the present value of income obtained by the asset during its indefinite life [15]. Clearly the stochastic discounting model C provides practitioners with valuable information at the time point 0 on strategic planning and proactive decision making for risk treatment operations. It can be said that C supports the adoption of proactivity in decision making related to selections and implementations of efficient risk treatment operations.

Let N represent the number of assets in an investment portfolio used as capital reserve in order to ensure that a financial institution has the appropriate capital requirements to sustain operating losses resulting from the manifestation of risks threatening the institution. The sequence

$$\{C_n, n = 1, 2, \ldots\}$$

distributed as C denotes the present value of the income obtained by the nth asset comprising the investment portfolio [15]. Consequently,

$$C_1 + C_2 + \ldots + C_N$$

represents the sum of present values of the incomes generated by the N assets comprising the investment portfolio.

In consequence,

$$V_1 + V_2 + \ldots + V_N \overset{d}{=} C_1 + C_2 + \ldots + C_N.$$

provides an indirect interpretation in stochastic discounting of the Poisson random sum

$$V_1 + V_2 + \ldots + V_N.$$

It seems to be of some interest to mention the direct interpretation in stochastic discounting of the above Poisson random sum if L is also a Poisson random sum.

The conclusion drawn from the present section is that fundamental probabilistic concepts are useful for formulating stochastic models for proactive activities. More precisely, this section makes clear that interconnections of random sums and stochastic integrals are extremely beneficial in formulating and investigating stochastic discounting models, since they significantly improve proactive decision-making processes.

5. Conclusions

Random sums are widely accepted as strong probabilistic concepts that have significant applications across several scientific areas. It is also known that stochastic integrals constitute particularly valuable probabilistic concepts for theorists and practitioners. Incorporations of stochastic integrals in random sums are readily recognized as research activities providing strong support to various proactive decision making operations. From a theoretical standpoint, the paper establishes new connections between three key probabilistic concept categories. In addition, the paper provides new interpretations for continuous discounting operations involving two Poisson random sums incorporating two distinct stochastic integrals. The contribution of the present paper consists of interconnecting stochastic integrals and random sums in order to formulate and investigate stochastic discounting models. An application in a particular area of proactive decision making, is also considered.

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