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Applications on soft somewhere dense sets

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Abstract

Believing in the importance of generalizations related to soft open sets and their important role in generalizing many topological concepts and properties, this research paper was presented. Specially, we introduce the new class of soft somewhere dense generalized closed sets and soft somewhere dense generalized open sets in soft topological spaces (or STSs). Several properties and theorems have been explored. Moreover, we provide the comparison between this new class and some other categories of soft sets such as soft closed sets, soft regular closed sets, soft b-closed sets, soft semi-closed sets, soft pre-closed sets, soft α-closed sets, soft β-closed sets, soft g-closed sets, s*g-closed sets, soft g α b-closed sets and soft g β -closed sets with the support of number of examples and counterexamples. Hence, we can say that the presented approach is wider than all of the above mentioned categories. Therefore, this opens the way for the researchers to make further generalizations and future applications.

Subject Classification: *(2010) 54A05, 54A20, 54C10.*

Keywords: *Soft topological space, Soft somewhere dense generalized closed sets, Soft somewhere dense closed mappings.*

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1. Introduction

The definition of soft topological space (or STS) was introduced by Shabir and Naz [46]. They introduced the main concepts of STS, which are main tools to investigate more topological properties to such spaces [20, 48]. The generalization of soft open sets leads to expand more and more topological properties. Depending on this, many weaker classes of soft open sets named soft regular closed sets [47], soft semi-closed sets, soft pre-closed sets, soft α-closed sets, soft β-closed sets [21, 26] were presented. Later, a new generalization to these mentioned notions except the notions of soft β-closed sets are presented under the name of soft b-closed sets [13, 23]. After that, the the concepts of soft α^*-open [40] and soft N_p-open sets [38].

Recently, El-Shami [17] succeed to present more general soft open sets than the previous studies under the name of soft somewhere dense sets (shortly, soft sd-sets). Another kind of generalization by using the notions of soft ideals in STS [29]. Many generalizations of weaker forms of soft sets [9, 12, 15, 18, 25, 27, 45] and soft topological properties [28, 30–32, 43] via soft ideals were introduced.

Kharal and Ahmad [10] introduced the notion of soft continuity, which is investigated in [11, 14, 51]. Another types of wider soft open sets like soft g-closed sets [34], s*g-closed sets [35], soft gαb-closed sets [39] and soft gβ-closed sets [19] is presented and many of its properties is discussed.

Recently, many applications on supra (soft) topological spaces were introduced in digital plane [36] and improving the accuracy measure of a rough set [16].

El-Sheikh and Abd El-latif [24] in 2014 presented the notion of supra soft topology (shortly, SS-topology). Later, many investigated topological properties to this space were explored. To name a few: SS-regular open sets [37], SS-b-open sets [2], SS-(strongly) generalized closed sets [8, 33], SS-locally closed sets [1], and the baire categories of soft sets [49], SS-locally α-closed sets [7], SS-δ_i-open sets [3], SS-sd-sets [6], SS-sd-connected spaces [4] and different types of SS-continuous functions inspired by SS-sd-sets [5]. The approach of nearly soft β-open sets via soft ditopological spaces was presented in [44].

The motivations of ordering this paper, are to present a new notion of soft open sets named, soft sd-generalized closed sets, in section 3. After that, the comparing of our approach and many previous studies in the same direction is explored, in section 4. We show that, it is contain many of the previous categories of soft sets. Many examples and counterexamples are presented to support our suggestions. In section 5, we introduce the notion of soft sd-generalized open sets. We discuss many of its essential properties.

2. Preliminaries

In this section, we introduce the notions and terminologies which will needed in this paper.

Definition 2.1: [41] Let W be an initial universe and ζ be a set of parameters. A pair (A,ζ) is called a soft set over W, is a map from ζ into the class of all subsets of W. The class of all soft sets over W will denoted by $S(W)$.

If $A(\xi)=\varphi$ for all $\xi\in\zeta$, then (A,ζ) is called a null soft set and will denoted by $\tilde{\varphi}$.

Also, if $A(\xi)=W$ for all $\xi\in\zeta$, then (A,ζ) is called an absolute soft set and will denoted by $\tilde{W}$.

Definition 2.2: [46] The collection $\sigma\subseteq S(W)$ is called soft topology (or STS) on W if:

(1) $\tilde{W},\tilde{\varphi}\in\sigma$,

(2) The soft union of arbitrary members of σ belongs to σ,

(3) The soft intersection of finite members of σ belongs to σ.

The triplet (W,σ,ζ) is called an STS over W. The elements of σ are called soft open sets and their soft complement is called soft closed sets. The collection of all soft open (respectively, closed) sets will denoted by $SO(W)$ (respectively, $SC(W)$).

Definition 2.3: [46, 50] Let (W,σ,ζ) be an STS and $(T,\zeta)\in S(W)$. The soft closure of (T,ζ), denoted by $cl(T,\zeta)$, is the smallest soft closed superset of (T,ζ).

Also, the soft interior of (V,ζ), denoted by $int(V,\zeta)$, is the largest soft open subset of (V,ζ).

Theorem 2.4: *[10] For the soft function $f_{pu}:(W_1,\sigma_1,\zeta_1)\to(W_2,\sigma_2,\zeta_2)$, the following statements hold,*

(a) $f_{pu}^{-1}((N^{\tilde{c}},\zeta_2))=(f_{pu}^{-1}(N,\zeta_2))^{\tilde{c}}\ \forall\,(N,\zeta_2)\in S(W_2)_{\zeta_2}$.

(b) $f_{pu}(f_{pu}^{-1}((N,\zeta_2)))\tilde{\subseteq}(N,\zeta_2)\ \forall\,(N,\zeta_2)\in S(W_2)_{\zeta_2}$. If f_{pu} is surjective, then the equality holds.

(c) $(M,\zeta_1)\tilde{\subseteq} f_{pu}^{-1}(f_{pu}((M,\zeta_1)))\ \forall\,(M,\zeta_1)\in S(W_1)_{\zeta_1}$. If f_{pu} is injective, then the equality holds.

(d) $f_{pu}(\tilde{W}_1)\tilde{\subseteq}\tilde{W}_2$. If f_{pu} is surjective, then the equality holds.

Definition 2.5: [13, 19, 21, 23, 26, 47] Let (W,σ,ζ) be an STS and $(G,\zeta)\in S(W)$. Then, (G,ζ) is said to be:

(1) Soft pre open set if $(F,E)\tilde{\subseteq} int(cl(F,E))$.
(2) Soft semi open set if $(G,\zeta)\tilde{\subseteq} cl(int(G,\zeta))$.
(3) Soft α -open set if $(G,\zeta)\tilde{\subseteq} int(cl(int(G,\zeta)))$.
(4) Soft β -open set if $(G,\zeta)\tilde{\subseteq} cl(int(cl(G,\zeta)))$.
(5) Soft b-open set if $(G,\zeta)\tilde{\subseteq} cl(int(G,\zeta))\tilde{\cup} int(cl(G,\zeta))$.
(6) Soft regular-open set if $int(cl(G,\zeta))=(G,\zeta)$.

The soft complement of soft pre- (respectively, regular, semi-, α -, β -, b-) open sets is called soft pre- (respectively, regular, semi-, α -, β -, b-) closed. The set of all soft pre- (respectively, regular, semi-, α -, β -, b-) open sets is denoted by $SPO(W)$ (respectively, $SRO(W)$, $SSO(W)$, $S\alpha O(W)$, $S\beta O(W)$, $SBO(W)$) and the set of all soft pre- (respectively, regular, semi-, α -, β -, b-) closed sets is denoted by $SPC(W)$ (respectively, $SRC(W)$, $SSC(W)$, $S\alpha C(W)$, $S\beta C(W)$, $SBC(W)$).

Definition 2.6: []17 Let (W,σ,ζ) be an STS and $(K,\zeta)\in S(Z)_{\zeta}$. Then, (K,ζ) is called a soft somewhere dense set (briefly, soft sd-set) if there is $\varphi\neq(O,\zeta)\in\sigma$ such that $(O,\zeta)\tilde{\subseteq} cl[(O,\zeta)\tilde{\cap}(K,\zeta)]$.

The soft complement of a soft sd-set is said to be soft cs-dense set. The class of all soft sd-set (respectively, soft cs-dense set) will denoted by $SD(W)$ (respectively, $CS(W)$).

Definition 2.7: [22] The soft sd-closure points of a subset (T,ζ) of an STS (W,σ,ζ), denoted by $cl_{sd}(T,\zeta)$, is the smallest soft cs-dens superset of (T,ζ).

Also, the soft sd-interior of (V,ζ), denoted by $int_{sd}(V,\zeta)$, is the largest soft sd-subset of (V,ζ).

3. Characterizations of soft somewhere dense generalized closed sets

In this part of the paper, we introduce the new notion of soft somewhere dense generalized closed sets (briefly, soft sdg-closed set) and classify its main characterizations. We study the soft union (respectively, soft intersection) of any two soft sdg-closed sets with the help of examples. Moreover, the image of soft sdg-closed sets under soft continuous and soft sd-closed mappings is explored.

Definition 3.1: A soft subset (H,ζ) of an STS (W,σ,ζ) is called soft somewhere dense generalized closed set (briefly, soft sdg-closed set) if

$$cl_{sd}(H,\zeta)\tilde{\subseteq}(G,\zeta) \text{ whenever } (H,\zeta)\tilde{\subseteq}(G,\zeta) \text{ and } (G,\zeta)\in\sigma.$$

The category of all soft sdg-closed sets will denoted by $SGC_{sd}(W)$.

Example 3.2: Suppose that $W=\{w_1,w_2,w_3\}$ and $\zeta=\{\xi_1,\xi_2\}$ be the set of parameters. Let $(H_i,\zeta), i=,2,3,...,8$ be soft sets over W, defined by:

$$H_1(\xi_1)=\{w_3\},\ \ H_1(\xi_2)=\{w_1\},$$

$$H_2(\xi_1)=\{w_3\},\ \ H_2(\xi_2)=\varphi,$$

$$H_3(\xi_1)=\{w_3\},\ \ H_3(\xi_2)=\{w_1,w_3\},$$

$$H_4(\xi_1)=\varphi,\ \ H_4(\xi_2)=\{w_3\},$$

$$H_5(\xi_1)=\{w_3\},\ \ H_5(\xi_2)=\{w_3\},$$

$$H_6(\xi_1)=\{w_2\},\ \ H_6(\xi_2)=\{w_3\},$$

$$H_7(\xi_1)=\{w_2,w_3\},\ \ H_7(\xi_2)=\{w_1,w_3\},$$

$$H_8(\xi_1)=\{w_2,w_3\},\ \ H_8(\xi_2)=\{w_3\}.$$

Therefore, $\sigma=\{\tilde{W},\tilde{\varphi},(H_i,\zeta), i=,2,3,...,8\}$ is an STS over W [42]. We have the soft set (K,ζ) = $\{(\xi_1,\{w_1,w_2\}),\ (\xi_2,W)\}$ is a soft sdg-closed set. On the other hand, the soft sets (H_3,ζ) and (H_5,ζ) aren't soft sdg-closed.

Remark 3.3: The soft union (respectively, soft intersection) of any two soft sdg-closed sets need not to be a soft sdg-closed in general as shall shown in the following examples.

Examples 3.4.

(1) Suppose that $W=\{w_1,w_2,w_3,w_4\}$ and $\zeta=\{\xi_1,\xi_2\}$ be the set of parameters. Let (K_1,ζ), (K_2,ζ), (K_3,ζ), (K_4,ζ) be soft sets over W, defined by:

$$K_1(\xi_1)=\{w_2\},\ \ K_1(\xi_2)=\{w_3\},$$

$$K_2(\xi_1)=\{w_1,w_2\},\ \ K_2(\xi_2)=\{w_2,w_3\},$$

$$K_3(\xi_1)=\{w_2,w_3\},\ \ K_3(\xi_2)=\{w_1,w_3\},$$

$$K_4(\xi_1)=\{w_1,w_2,w_3\},\ \ K_4(\xi_2)=\{w_1,w_2,w_3\}.$$

Therefore, $\sigma=\{\tilde{W},\tilde{\varphi},(K_1,\zeta),(K_2,\zeta),(K_3,\zeta),(K_4,\zeta)\}$ is an STS over W. The soft sets $(C,\zeta),(D,\zeta),$ where:

$$C(\xi_1)=\{w_1,w_2\},\ \ C(\xi_2)=\{w_1,w_3\},$$

$$D(\xi_1)=\{w_3\},\ \ D(\xi_2)=\{w_2\},$$

are soft sdg-closed, but their soft union $(C,\zeta)\ \tilde{\cup}\ (D,\zeta)$ = $\{(\xi_1,$ $\{w_1,w_2,w_3\}),\ (\xi_2,\ \{w_1,w_2,w_3\})\}$ isn't soft sdg-closed.

(2) In Example 3.2, the soft sets $(A,\zeta),(B,\zeta),$ where:

$$A(\xi_1)=\{w_2\},\ \ A(\xi_2)=W,$$

$$B(\xi_1)=W,\ \ B(\xi_2)=\{w_3\},$$

are soft sdg-closed, but their soft intersection $(A,\zeta)\ \tilde{\cap}\ (B,\zeta)$ = $\{(\xi_1,\{w_2\}),\ (\xi_2,\{w_3\})\}$ isn't soft sdg-closed .

Theorem 3.5: *Let (N,ζ) be a soft sdg-closed subset of a SSTS (W,σ,ζ). If there exists $(G,\zeta)\in S(W)$ such that $(N,\zeta)\ \tilde{\subseteq}\ (G,\zeta)\ \tilde{\subseteq}\ cl_{sd}\ (N,\zeta)$, then (G,ζ) is a soft sdg-closed.*

Proof: Assume that, $(N,\zeta)\tilde{\subseteq}(O,\zeta)$ and $(O,\zeta)\in\sigma$. Since (N,ζ) is a soft sdg-closed, $cl_{sd}(N,\zeta)\ \tilde{\subseteq}\ (O,\zeta)$. Since $(G,\zeta)\ \tilde{\subseteq}\ cl_{sd}(N,\zeta)$,

$$cl_{sd}(G,\zeta)\tilde{\subseteq}cl_{sd}(N,\zeta)\tilde{\subseteq}(O,\zeta).$$

Hence,

$$cl_{sd}(G,\zeta) \tilde{\subseteq} (O,\zeta).$$

Thus, (G,ζ) is a soft sdg-closed.

Theorem 3.6: *A soft subset* (J,ζ) *of a SSTS* (W,σ,ζ) *is soft sdg-closed set if* $cl_{sd}(J,\zeta) \setminus (J,\zeta)$ *contains only null soft closed set.*

Proof: Suppose conversely, (J,ζ) be a soft sdg-closed set and (K,ζ) be a non null soft closed subset of $\tilde{W}$ such that $(K,\zeta) \tilde{\subseteq} cl_{sd}(J,\zeta) \setminus (J,\zeta)$. Follows,

$$(K,\zeta) \tilde{\subseteq} cl_{sd}(J,\zeta) \tag{1}$$

Since (J,ζ) is soft sdg-closed and $(J,\zeta) \tilde{\subseteq} (K^{\tilde{c}},\zeta) \in \sigma$, $cl_{sd}(J,\zeta) \tilde{\subseteq} (K^{\tilde{c}},\zeta)$. It follows,

$$(K,\zeta) \tilde{\subseteq} [cl_{sd}(J,\zeta)]^{\tilde{c}} \tag{2}$$

Hence,

$(K,\zeta) \tilde{\subseteq} cl_{sd}(J,\zeta) \tilde{\cap} [cl_{sd}(J,\zeta)]^{\tilde{c}} = \tilde{\Phi}$, from Equations (1) and (2).

Therefore, $(K,\zeta) = \tilde{\Phi}$, which is a contradiction. This completes our proof.

Definition 3.7: [17] A soft function $f_{pu}:(W_1,\sigma_1,\zeta_1) \to (W_2,\sigma_2,\zeta_2)$ is said to be a soft sd-closed if $f_{pu}(H,\zeta_1)$ is soft cs-dense subset of $\tilde{W}_2$ or the absolute soft set, for all soft closed subset (H,ζ_1) of $\tilde{W}_1$.

Theorem 3.8: *Let* $f_{pu}:(W_1,\sigma_1,\zeta_1) \to (W_2,\sigma_2,\zeta_2)$ *be a soft continuous and soft sd-closed function. If* (B,ζ_1) *is soft sdg-closed subset of* $\tilde{W}_1$, *then* $f_{pu}(B,\zeta_1)$ *is a soft sdg-closed subset of* $\tilde{W}_2$.

Proof: Assume that, $f_{pu}(B,\zeta_1) \tilde{\subseteq} (A,\zeta_2)$ such that $(A,\zeta_2) \in \sigma_2$. So,

$(B,\zeta_1) \tilde{\subseteq} f_{pu}^{-1}(A,\zeta_2)$ and $f_{pu}^{-1}(A,\zeta_2) \in \sigma_1$, where f_{pu} is soft continuous.

Since (B,ζ_1) is a soft sdg-closed subset of $\tilde{W}_1$, $cl_{sd}(B,\zeta_1) \tilde{\subseteq} f_{pu}^{-1}(A,\zeta_2)$, and so

$$f_{pu}[cl_{sd}(B,\zeta_1)] \tilde{\subseteq} f_{pu}(f_{pu}^{-1}(A,\zeta_2)) \tilde{\subseteq} (A,\zeta_2).$$

Since f_{pu} is soft sd-closed, $cl_{sd}(B,\zeta_1) \tilde{\subseteq} cl(B,\zeta_1)$ and $cl(B,\zeta_1)$ is soft closed subset of $\tilde{W}_1$. So, $f_{pu}[cl_{sd}(B,\zeta_1)]$ is a soft cs-dense subset of $\tilde{W}_2$ or the absolute soft set.

If $f_{pu}[cl_{sd}(B,\zeta_1)]$ is the absolute soft set, then we get the proof.

If $f_{pu}[cl_{sd}(B,\zeta_1)]$ is a soft cs-dense subset of $\tilde{W}_2$, then

$$cl_{sd}[f_{pu}(B,\zeta_1)] \tilde{\subseteq} cl_{sd}[f_{pu}[cl_{sd}(B,\zeta_1)]] = f_{pu}[cl_{sd}(B,\zeta_1)] \tilde{\subseteq} (A,\zeta_2).$$

Thus,
$f_{pu}(B,\zeta_1)$ is a soft sdg-closed subset of $\tilde{W}_2$.

4. Comparison with previous studies

Here, the reader can find the answer of the question: Why did we introduced this paper? Simply, we compare our new notion with almost all popular previous concepts of weaker forms of soft closed sets such soft closed sets [46], soft regular closed sets [47], soft b-closed sets [13, 23], soft semi-closed sets, soft pre-closed sets, soft α-closed sets, soft β-closed sets [21, 26], soft g-closed sets [34], s*g-closed sets [35], soft gα b-closed sets [39] and soft gβ-closed sets [39]. We found out that our new notion contains strictly all the above-mentioned concepts with the confirmations of the examples and counterexamples.

Theorem 4.1: *Every soft closed [46] (respectively, regular closed [47]) subset (H,ζ) of an STS (W,σ,ζ) is a soft sdg-closed.*

Proof: Assume that (H,ζ) be a soft closed subset of an STS (W,σ,ζ) such that $(H,\zeta) \tilde{\subseteq} (G,\zeta)$ and $(G,\zeta) \in \sigma$, then

$$cl_{sd}(H,\zeta) \tilde{\subseteq} cl(H,\zeta) = (H,\zeta) \tilde{\subseteq} (G,\zeta).$$

Therefore, (H,ζ) is a soft sdg-closed.

If (H,ζ) is soft regular closed set. Then, it is soft closed, and hence we get the proof.

Remark 4.2: The converse of Theorem 4.1 need not to be true in general. The following example shall confirm our claim.

Example 4.3: Suppose that $W = \{w_1, w_2, w_3, w_4\}$ and $\zeta = \{\xi_1, \xi_2\}$ be the set of parameters. Let (H_j,ζ), $j = 1,2,3,\ldots,11$ be soft sets over W, defined by:

$$H_1(\xi_1) = \{w_2\},\ \ H_1(\xi_2) = \{w_2\},$$

$$H_2(\xi_1) = \{w_2\},\ \ H_2(\xi_2) = \varphi,$$

$$H_3(\xi_1) = \{w_1\},\ \ H_3(\xi_2) = \{w_1\},$$

$$H_4(\xi_1) = \{w_1, w_2\},\ \ H_4(\xi_2) = \{w_1, w_2\},$$

$$H_5(\xi_1)=\{w_1,w_2,w_3\},\ \ H_5(\xi_2)=\{w_1,w_3\},$$

$$H_6(\xi_1)=\{w_1,w_2,w_4\},\ \ H_6(\xi_2)=\{w_1,w_2,w_3\},$$

$$H_7(\xi_1)=\{w_1,w_2\},\ \ H_7(\xi_2)=\{w_1\},$$

$$H_8(\xi_1)=\{w_1,w_2,w_3\},\ \ H_8(\xi_2)=\{w_1,w_2,w_3\},$$

$$H_9(\xi_1)=W,\ \ H_9(\xi_2)=\{w_1,w_2,w_3\},$$

$$H_{10}(\xi_1)=\{w_1,w_2\},\ \ H_{10}(\xi_2)=\{w_1,w_2,w_3\},$$

$$H_{11}(\xi_1)=\{w_1,w_2\},\ \ H_{11}(\xi_2)=\{w_1w_3\}.$$

Therefore, $\sigma=\{\tilde{W},\tilde{\varphi},(H_j,\zeta),j=1,2,3,\ldots,11\}$ is an STS over W [47]. Hence, the soft set (F,ζ), where:

$F(\xi_1)=\varphi,\ \ F(\xi_2)=\{w_2,w_3\}$, is a soft sdg-closed set, but it neither soft closed nor soft regular closed.

In [17], the author proved that the concept of soft cs-dense sets is wider than the notions of soft b-closed [13, 23] (respectively, semi-closed, pre-closed, α-closed, β-closed [21, 26]) sets. In the next theorem we shall prove that, our new notion soft sdg-closed is more wider than this notion, and hence it will be the widest collection comparing with all the previous studies in this direction.

Theorem 4.4: *Every soft cs-dense subset* (H,ζ) *of an STS* (W,σ,ζ) *is a soft sdg-closed.*

Proof: Assume that (H,ζ) be a soft cs-dense subset of an STS (W,σ,ζ) such that $(H,\zeta)\,\tilde{\subseteq}\,(G,\zeta)$ and $(G,\zeta)\in\sigma$. Then, $cl_{sd}(H,\zeta)=(H,\zeta)\,\tilde{\subseteq}\,(G,\zeta)$. Therefore, (H,ζ) is a soft sdg-closed.

Remark 4.5: The converse of Theorem 4.4 need not to be true in general. The following example shall confirm our claim.

Example 4.6: Suppose that $W=\{w_1,w_2,w_3,w_4\}$ and $\zeta=\{\xi_1,\xi_2\}$ be the set of parameters. Let (H_1,ζ), (H_2,ζ), (H_3,ζ), (H_4,ζ) be soft sets over W, defined by:

$$H_1(\xi_1)=\{w_1\},\ \ H_1(\xi_2)=\varphi,$$

$$H_2(\xi_1)=\{w_1,w_2\},\ \ H_2(\xi_2)=\{w_3,w_4\},$$

$$H_3(\xi_1)=\{w_3,w_4\},\ \ H_3(\xi_2)=\{w_1,w_2\},$$

$$H_4(\xi_1)=\{\{w_1,w_3,w_4\}\},\ \ H_4(\xi_2)=\{w_1,w_2\}.$$

Therefore, $\sigma=\{\tilde{W},\tilde{\varphi},(H_1,\zeta),(H_2,\zeta),(H_3,\zeta),(H_4,\zeta)\}$ is an STS over W[17]. Hence, the soft set (F,ζ), where:

$F(\xi_1)=\{\{w_1,w_3,w_4\}\}$, $F(\xi_2)=W$, is a soft sdg-closed set, but it isn't soft cs-dense.

Definition 4.7: A soft subset (H,ζ) of an STS (W,σ,ζ) is called:

(1) [34] Soft g-closed set if $cl(H,\zeta)\tilde{\subseteq}(G,\zeta)$ whenever $(H,\zeta)\tilde{\subseteq}(G,\zeta)$ and $(G,\zeta)\in\sigma$.

(2) [39] Soft gα b-closed set if $bcl(H,\zeta)\tilde{\subseteq}(G,\zeta)$ whenever $(H,\zeta)\tilde{\subseteq}$ (G,ζ) and (G,ζ) is soft α-open set, where $bcl(H,\zeta)$ is the soft intersection of all soft b-closed supersets of (H,ζ).

(3) [35] s*g-closed set if $cl(H,\zeta)\tilde{\subseteq}(G,\zeta)$ whenever $(H,\zeta)\tilde{\subseteq}(G,\zeta)$ and (G,ζ) is soft semi-open set.

(4)[19] Soft gβ-closed set if $\beta cl(H,\zeta)\tilde{\subseteq}(G,\zeta)$ whenever $(H,\zeta)\tilde{\subseteq}(G,\zeta)$ and $(G,\zeta)\in\sigma$, where $\beta cl(H,\zeta)$ is the soft intersection of all soft β-closed supersets of (H,ζ).

Theorem 4.8: *Every soft g-closed (respectively, s*g-closed, soft g β-closed, soft g α b-closed) subsets (A,ζ) of an STS (W,σ,ζ) is a soft sdg-closed.*

Proof: We prove the case of soft gα b-closed set. The other case is similar. Assume that (A,ζ) be a soft gα b-closed subset of an STS (W,σ,ζ) such that $(A,\zeta)\tilde{\subseteq}(G,\zeta)$ and $(G,\zeta)\in\sigma$, then (G,ζ) is soft α-open set. Since (A,ζ) is soft gα b-closed, $cl_{sd}(A,\zeta)\tilde{\subseteq}bcl(H,\zeta)\tilde{\subseteq}(G,\zeta)$. Therefore, (H,ζ) is a soft sdg-closed.

Remark 4.9: The converse of Theorem 4.8 need not to be true in general. The following examples shall confirm our claim.

Examples 4.10:

(1) In Example 3.2, the soft set (F,ζ), where:

$F(\xi_1)=\{w_2\}$, $F(\xi_2)=\{w_1,w_3\}$, is a soft sdg-closed set, but it isn't soft g-closed.

(2) Suppose that $W=\{w_1,w_2,w_3\}$ and $\zeta=\{\xi_1,\xi_2\}$ be the set of parameters. Let (F_1,ζ), (F_2,ζ), (F_3,ζ), (F_4,ζ), (F_5,ζ), (F_6,ζ) be soft sets over W, defined by:

$$F_1(\xi_1)=\{w_1\},\ \ F_1(\xi_2)=\{w_1\},$$

$$F_2(\xi_1)=\{w_1,w_2\},\ \ F_2(\xi_2)=\{w_1,w_2\},$$

$$F_3(\xi_1)=\{w_2\},\ \ F_3(\xi_2)=\{w_2\},$$

$$F_4(\xi_1)=\{w_2,w_3\},\ \ F_4(\xi_2)=\{w_2,w_3\},$$

$$F_5(\xi_1)=\{w_1,w_3\},\ \ F_5(\xi_2)=\{w_1,w_3\},$$

$$F_6(\xi_1)=\{w_3\},\ \ F_6(\xi_2)=\{w_3\}.$$

Therefore, $\sigma=\{\tilde{W},\tilde{\varphi},(F_1,\zeta),(F_2,\zeta),(F_3,\zeta),(F_4,\zeta),(F_5,\zeta),(F_6,\zeta)\}$ is an STS over W [39]. Hence, the soft set (M,ζ), where:

$M(\xi_1)=\{w_1,w_2\}$, $M(\xi_2)=\{w_1,w_2\}$, is a soft sdg-closed set, but it neither soft gα b-closed nor s*g-closed.

(3) In (2), the soft set (N,ζ), where:

$N(\xi_1)=\{w_1,w_2\}$, $N(\xi_2)=\{w_1,w_3\}$, is a soft sdg-closed set, but it isn't soft gβ-closed.

Corollary 4.11: *Depending on the above discussion and [[17],Proposition 2.8], we have the following implications for an STS (W,σ,ζ), which aren't reversible.*

$$\begin{tikzcd}
SRC(W) \arrow[r] & SC(W) \arrow[r] \arrow[d] & S\alpha C(W) \arrow[r] \arrow[dl] & SSC(W) \arrow[r] \arrow[dr] & S\beta C(W) \arrow[r] & CS(W) \arrow[r] & SGC_{sd}(W) \\
 & SPC(W) \arrow[rrr] & & & SBC(W) \arrow[u] & &
\end{tikzcd}$$

Diagram 1

The relationships among the new class of soft sdg-closed sets $SGC_{sd}(W)$ and other previous studies

5. Soft somewhere dense generalized open sets and relationships

In this section, the properties of the concept of soft somewhere dense generalized open set (briefly, soft sdg-open set) is discussed. Moreover, the relationships with the other previous types of weaker forms of soft open sets were introduced.

Definition 5.1: A soft subset (O,ζ) of an STS (W,σ,ζ) is called soft somewhere dense generalized open set (briefly, soft sdg-open set) if its relative soft complement $(O^{\tilde{c}},\zeta)$ is soft sdg-closed. The family of all soft sdg-open sets will denoted by $SGO_{sd}(W)$.

Example 5.2: In Example 3.2, the soft set $(K^{\tilde{c}},\zeta)$ is soft sdg-open and the soft sets $(H_3^{\tilde{c}},\zeta),(H_5^{\tilde{c}},\zeta)$ aren't soft sdg-open .

Note 5.3: The soft intersection (respectively, soft union) of any two soft sdg-open sets need not to be a soft sdg-open in general. In Examples 3.4,

(1) The soft sets $(C^{\tilde{c}},\zeta),(D^{\tilde{c}},\zeta)$, where:

$$C^{\tilde{c}}(\xi_1)=\{w_3,w_4\},\ C^{\tilde{c}}(\xi_2)=\{w_2,w_4\},$$

$$D^{\tilde{c}}(\xi_1)=\{w_1,_2,w_4\},\ D^{\tilde{c}}(\xi_2)=\{w_1,w_3,w_4\},$$

are sdg-open, but their soft intersection $(C^{\tilde{c}},\zeta)\ \tilde{\cap}\ (D^{\tilde{c}},\zeta)$ = $\{(\xi_1, \{w_4\}), (\xi_2, \{w_4\})\}$ isn't soft sdg-open.

(2) The soft sets $(A^{\tilde{c}},\zeta),(B^{\tilde{c}},\zeta)$, where:

$$A^{\tilde{c}}(\xi_1)=\{w_1,w_3,w_4\},\ \ A^{\tilde{c}}(\xi_2)=\varphi,$$

$$B^{\tilde{c}}(\xi_1)=\varphi,\ B^{\tilde{c}}(\xi_2)=\{w_1,w_2,w_4\},$$

are soft sdg-open, but their soft union $(A^{\tilde{c}},\zeta)\ \tilde{\cup}\ (B^{\tilde{c}},\zeta)$ = $\{(\xi_1, \{w_1,w_3,w_4\}), (\xi_2, \{w_1,w_2,w_4\})\}$ isn't soft sdg-open .

Theorem 5.4: *A soft subset (O,ζ) of an STS (W,σ,ζ) is soft sdg-open set if and only if $(H,\zeta)\ \tilde{\subseteq}\ int_{sd}(O,\zeta)$ whenever $(H,\zeta)\ \tilde{\subseteq}\ (O,\zeta)$ and $(H,\zeta)\in\sigma^c$.*

Proof: Necessity: Assume that (O,ζ) is a soft sdg-open subset of $\tilde{W}$ and $(O,\zeta)\tilde{\subseteq}(H,\zeta)$ such that $(H,\zeta)\in\sigma^c$. It follows that,

$$(H^{\tilde{c}},\zeta)\tilde{\subseteq}(O^{\tilde{c}},\zeta),\ (H^{\tilde{c}},\zeta)\in\sigma.$$

Since $(O^{\tilde{c}},\zeta)$ is soft sdg-closed, $cl_{sd}(H^{\tilde{c}},\zeta)\tilde{\subseteq}(O^{\tilde{c}},\zeta)$. Hence,

$$(O,\zeta)\tilde{\subseteq}[cl_{sd}(H^{\tilde{c}},\zeta)]^{\tilde{c}}=int_{sd}(H,\zeta).$$

Sufficient: Suppose that $(O^{\tilde{c}},\zeta) \tilde{\subseteq} (G,\zeta)$ s.t $(G,\zeta) \in \sigma$. So, $(G^{\tilde{c}},\zeta) \tilde{\subseteq} (O,\zeta)$ and $(G^{\tilde{c}},\zeta) \in \sigma^{c}$. From the necessary condition,

$$(G^{\tilde{c}},\zeta) \tilde{\subseteq} int_{sd}(O,\zeta).$$

Hence,

$$(G,\zeta) \in \sigma$$

Therefore,

$(O^{\tilde{c}},\zeta)$ is a soft sdg-closed set, and hence (O,ζ) is a soft sdg-open.

Theorem 5.5: *Let (P,ζ) be a soft sdg-open subset of an STS (W,σ,ζ). If there exists a soft set (Q,ζ) such that $int_{sd}(P,\zeta) \tilde{\subseteq} (Q,\zeta) \tilde{\subseteq} (P,\zeta)$, then (Q,ζ) is a soft sdg-open.*

Proof: Assume that $(H,\zeta) \tilde{\subseteq} (Q,\zeta)$ and $(H,\zeta) \in \sigma^{c}$. Since $(Q,\zeta) \tilde{\subseteq} (P,\zeta)$, $(H,\zeta) \tilde{\subseteq} (P,\zeta)$. Since (P,ζ) is a sdg-open, $(H,\zeta) \tilde{\subseteq} int_{sd}(P,\zeta)$. Follows,

$$(H,\zeta) \tilde{\subseteq} int_{sd}(P,\zeta) \tilde{\subseteq} int_{sd}(Q,\zeta).$$

Therefore,

$$(H,\zeta) \tilde{\subseteq} int_{sd}(Q,\zeta).$$

Hence, (P,ζ) is a sdg-open set.

Definition 5.6: A soft function $f_{pu}:(W_1,\sigma_1,\zeta_1) \to (W_2,\sigma_2,\zeta_2)$ is said to be soft sd*-open if $f_{pu}(G,\zeta_1)$ is a soft sd-subset of $\tilde{W}_2$, for all non-empty soft sd-subset (G,ζ_1) of $\tilde{W}_1$.

Theorem 5.7: *If a soft function $f_{pu}:(W_1,\sigma_1,\zeta_1) \to (W_2,\sigma_2,\zeta_2)$ is soft sd*-open, then $f_{pu}(int_{sd}(E,\zeta_1)) \tilde{\subseteq} int_{sd}[f_{pu}(E,\zeta_1)]$ for all soft subset (E,ζ_1) of $\tilde{W}_1$.*

Proof: Let f_{pu} be a soft sd*-open, then $f_{pu}(int_{sd}(E,\zeta_1))$ is a soft sd-subset of $\tilde{W}_2$. Hence,

$$f_{pu}(int_{sd}(E,\zeta_1)) = int_{sd}[f_{pu}(int_{sd}(E,\zeta_1))] \tilde{\subseteq} int_{sd}[f_{pu}(E,\zeta_1)].$$

Theorem 5.8: *Let $f_{pu}:(W_1,\sigma_1,\zeta_1) \to (W_2,\sigma_2,\zeta_2)$ be a soft continuous, soft sd*-open and surjective function. If (M,ζ_1) is a soft sdg-open subset of $\tilde{W}_1$, then $f_{pu}(B,\zeta_1)$ is a soft sdg-open subset of $\tilde{W}_2$.*

Proof: Suppose that $(K,\zeta_2)\tilde{\subseteq} f_{pu}(M,\zeta_1)$ such that $(K,\zeta_2)\in\sigma_2^c$ and (M,ζ_1) is a soft sdg-open. It follows,

$$f_{pu}(M^{\tilde{c}},\zeta_1)\tilde{\subseteq}(K^{\tilde{c}},\zeta_2), (K^{\tilde{c}},\zeta_2)\in\sigma_2.$$

So,

$$(M^{\tilde{c}},\zeta_1)\tilde{\subseteq} f_{pu}^{-1}[f_{pu}(M^{\tilde{c}},\zeta_1)]\tilde{\subseteq} f_{pu}^{-1}(K^{\tilde{c}},\zeta_2),$$

Since f_{pu} is soft continuous, $f_{pu}^{-1}(K^{\tilde{c}},\zeta_2)\in\sigma_1$.

Since (M,ζ_1) is a soft sdg-open, $(M^{\tilde{c}},\zeta_1)$ is a soft sdg-closed, and hence

$$[int_{sd}(M,\zeta_1)]^{\tilde{c}} = cl_{sd}(M^{\tilde{c}},\zeta_1)\tilde{\subseteq} f_{pu}^{-1}(K^{\tilde{c}},\zeta_2).$$

Hence,

$$f_{pu}^{-1}(K,\zeta_2)\tilde{\subseteq} int_{sd}(M,\zeta_1).$$

Since f_{pu} surjective and soft sd*-pen,

$(K,\zeta_2)= f_{pu}[f_{pu}^{-1}(K,\zeta_2)]\tilde{\subseteq} f_{pu}[int_{sd}(M,\zeta_1)]\tilde{\subseteq} int_{sd}[f_{pu}(M,\zeta_1)]$ from Theorem 5.7,

Thus, $f_{pu}(M,\zeta_1)$ is a soft sdg-open.

Theorem 5.9: *Every soft open (respectively, regular open) subset* (S,ζ) *of an STS* (W,σ,ζ) *is a soft sdg-open.*

Proof: Assume that $(F,\zeta)\tilde{\subseteq}(S,\zeta)$ such that $(F,\zeta)\in\sigma^c$ and $(S,\zeta)\in\sigma.$. So, (S,ζ) is soft sd-set, and hence $int_{sd}(S,\zeta)=(S,\zeta)$. Hence,

$$(F,\zeta)\tilde{\subseteq} int_{sd}(S,\zeta).$$

Thus, (S,ζ) is a soft sdg-open.

If (H,ζ) is soft regular open set. Then, it is soft open, and hence we get the proof.

The converse of this theorem is not true in general as can explored from Example 4.3 that the soft set $(F^{\tilde{c}},\zeta)$ is a soft sdg-open set, but it neither soft open nor soft regular open.

Theorem 5.10: *Every soft sd-subset* (H,ζ) *of an STS* (W,σ,ζ) *is a soft sdg-open.*

Proof: Assume that (H,ζ) be a soft sd-subset of an STS (W,σ,ζ) such that $(C,\zeta)\ \tilde{\subseteq}\ (H,\zeta)$ and $(C,\zeta)\in\sigma^c$, then $int_{sd}(H,\zeta) = (H,\zeta)$, and hence (C,ζ) $\tilde{\subseteq}\ int_{sd}(H,\zeta) = (H,\zeta)$. Hence , (H,ζ) is a soft sdg-open.

Remark 5.11: The converse of Theorem 5.10 need not to be true in general. In Example 4.6, the soft set $(F^{\tilde{c}},\zeta)$ is a soft sdg-open set, but it isn't soft sd-set.

Theorem 5.12: *Every soft g-open (respectively, s*g-open, soft g β -open, soft g α b-open) subsets (A,ζ) of an STS (W,σ,ζ) is a soft sdg-open.*

Proof: It is direct from the proof of Theorem 4.8.

Remark 5.13: The converse of Theorem 5.12 need not to be true in general. For more confirmation:

(1) In Examples 4.10 (1), the soft set $(F^{\tilde{c}},\zeta)$, where: $F(\xi_1)=\{w_1,w_3\}$, $F(\xi_2)=\{w_2\}$, is a soft sdg-open set, but it isn't soft g-open.

(2) In Examples 4.10 (2), the soft set $(M^{\tilde{c}},\zeta)$, where: $M^{\tilde{c}}(\xi_1)=\{w_3\}$, $M^{\tilde{c}}(\xi_2)=\{w_3\}$, is a soft sdg-open set, but it neither soft g α b-open nor s*g-open.
Also, the soft set $(N^{\tilde{c}},\zeta)$, where: $N^{\tilde{c}}(\xi_1)=\{w_3\}$, $N^{\tilde{c}}(\xi_2)=\{w_2\}$, is a soft sdg-open set, but it isn't soft g β -open.

Corollary 5.14: *Depending on the above arguments, we have the following implications for an STS (W,σ,ζ), which aren't reversible.*

$$\begin{tikzcd}
SRO(W) \arrow[r] & SO(W) \arrow[r] \arrow[d] & S\alpha O(W) \arrow[r] \arrow[dl] & SSO(W) \arrow[r] \arrow[dr] & S\beta O(W) \arrow[r] & SD(W) \arrow[r] & SGO_{sd}(W) \\
 & SPO(W) \arrow[rrr] & & & SBO(W) \arrow[u] & &
\end{tikzcd}$$

Diagram 2

The relationships among the new class of soft sdg-open sets $SGO_{sd}(W)$ and other previous notions

6. Conclusion and future extensions

In this manuscript, a new wider class of soft open sets named soft somewhere dense generalized closed sets is presented. Several characterizations and results regarding to this notion are given and explored. The relations between our novel approach and their topologically corresponding notions have been presented. To declare the divergences between our new class and other old generalizations, we have provided many examples and counterexamples. Our coming studies is to use the provided notion in this paper to discuss and generalize other topological properties like connectedness, compactness and separation axioms. Moreover, the improvement of the accuracy measures of subsets in information systems via the presented category will be considered.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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