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A better approximation to Sawada-Kotera via differential transform method with hybrid grey Wolf and Cuckoo algorithm

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Abstract

This article's aim is to investigate the fifth rank Sawada-kotera equation and get for approximate solution. Differential transform method was used, where the results were satisfactory and near to the optimal solution. A new hybrid modern was used between the numerical method and the hybrid algorithm hybrid gray Wolf and Cuckoo algorithm, where the approximate the shape of the solution of a polynomial and convergent series. Comparing the results with the exact solution shows the reliability of the hybrid technique and that the DTM method is an attractive, efficient and accurate method for solving systems of partial differential equations.

Subject Classification: *34A45, 65Y04.*

Keywords: *Hybrid grey Wolf and Cuckoo algorithm, Sawada-Kotera equation, Differential transform method, Partial differential equations.*

1. Introduction

Nonlinear PDE solutions can help researchers gain experience and skill for a general and complete understanding of the required process, and from the sciences in which nonlinear PDEs are found, the natural sciences including physics, mathematics, mechanical engineering and even medicine. However, due to the complexity of nonlinear PDEs and the limitations of classical mathematical techniques, we are unable to get optimal solutions to the problems. There had to be significant progress

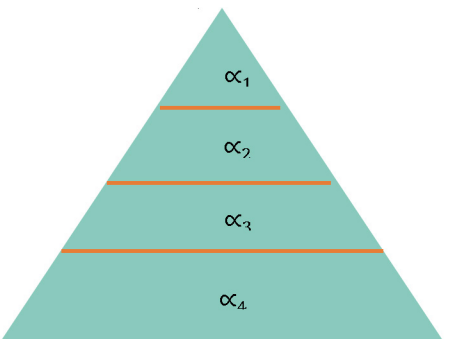
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in numerical analysis and optimal solutions for nonlinear PDEs in recent decades[1]. Since the beginning the 1970s, a wide broad of simple and direct Strategies for finding analytic solutions to PDEs have been developed [2]. However, many mathematicians, physicists and other different scientists have worked to study the optimal solutions PDEs because of their active and influential role in the interpretation of non-linear phenomena, including in physics to explain part of the models described PDEs, and the analytical methods remain to obtain optimal solutions from difficult problems[3]. NLEEs are frequently employed in theoretical research across multiple disciplines to examine an extensive array of nonlinear phenomena, and investigating the solutions of NLEEs is critical understanding the mechanisms nonlinear phenomena [4].

A new technique has been proposed in this research, which is the use of the hybrid algorithm GWO, the basis of which is to rely on the hierarchy of the nature of leadership and the method of hunting for gray wolves, 4 types of gray wolves were, namely $\alpha,\beta,\delta,\gamma$ and by implementing the main steps of hunting.

Where 29 well-known test functions were used with the CS algorithm, which works to find the best parameters of the learning strategy for the members of the decision layer α,β,γ. The main characteristic of hybrid technique (GWOCS) is the possibility of balancing between exploitation and exploration. Complex standard 10 functions are used. By applying this hybrid Technique to PDEs, we have been able to choose the best parameters for PDEs and thus reach the best optimal solutions. In this algorithm[5-7], the best values for the parameters will be chosen to be able to improve the approximate solutions of the equation. S-K, which simulates the exact solutions of the equation



Grey wolf hierarchy (dominance decreases from top down)[8]



Includes gray wolf hunting mechanism[8].

2. Mathematical Model

Nonlinear phenomena are found in many scientific and engineering specialties, chaotic, including fluid dynamics, nonlinear optics, quantum and among others. All of these phenomena are represented mathematically by nonlinear partial differential equations (PDEs), Where consider (PDEs) is the basis for representing, studying and interpreting non-linear phenomena and then finding the best and closest approximate solutions to the exact case. [9-17]. The generalized 5th order (gfKdV) equations are among these PDEs that are employed in the research of numerous significant subjects related to nonlinear physical phenomena. however, these highly nonlinear physical problems lack general exact solutions, and the gfKdV equation is exceedingly challenging. Thus far, the exact solution to the gfKdV equation has been discovered in [18] .After the equation was originally simplified to ODEs, a number of analytical and computational techniques were created. But putting these methods into practice is challenging. The KdV equation is a nonlinear PDEs that is commonly encountered in many physical applications and holds significant importance in many cases, including magneto-acoustic waves and ion acoustic waves. Many extensions and modifications to this equation have produced a variety of versions that are present in third, fifth, seventh, or higher-order differential equations. Despite the difficulties that this equation faced from the beginning and due to its importance, a new technique was proposed in order to overcome the difficulties and obtain the best approximate solutions [19]. The Sawada–Kotera equation

has been used in a number of studies, including a new supersymmetric equation, new optimal solutions of the general Riccati equation, for exact soliton solutions of the [20]. Here is the general form of the S–K equation:

$$\frac{\partial d}{\partial t}+\beta j^2\frac{\partial j}{\partial r}+\alpha\frac{\partial j}{\partial r}\frac{\partial^2 j}{\partial r^2}+\delta\frac{\partial^3 j}{\partial r^3}+\frac{\partial^5 j}{\partial r^5}=0 \tag{1}$$

Take a look at (S–K) eq. (1), where the initial condition is[21]:

$$j(n,0)=2S^2\sec h^2[S(r-c)] \tag{}$$

The exact solution to the eq. (1) with the initial condition (2) is offered in [21] as:

$$j(n,t)=2S^2\sec h^2[S(r-16k^4t)-c] \tag{3}$$

3. The Method

In order to describe the basic idea of the scalar method DTM, we define the differential transformation j(r,k) in the following form[22-24]:

$$J(l,p)=\frac{1}{l!\,p!}\left[\frac{\partial^{l+p}j(r,k)}{\partial r^l\partial k^p}\right]_{(r_o,k_o)} \tag{4}$$

where j(r,k) is the authentic function and J (l, p) is the transformed function. Then its inverse transform is defined as:

$$j(r,k)=\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}J(r,k)(r-r_o)^l(k-k_o)^p \tag{5}$$

The relations Eq. (4) and Eq. (5) imply that:

$$J(l,p)=\frac{1}{l!\,p!}\left[\frac{\partial^{l+p}j(r,k)}{\partial r^l\partial k^p}\right]_{(r_o,k)}(r-r_o)^l\ (k-k_o)^p \tag{6}$$

In fact, taking (r_o,k_o) at the origin point where the function (*r, k*)can be described with a finite series, and Eq. (5) can be written as follows:

$$J(l,p)\cong\sum_{l=0}^{i}\sum_{p=0}^{j}J(r,k)r^l\,k^p \tag{7}$$

Table 1

Operations of the two-dimensional numerical method (DTM)

Orig. function	Tran. function
$j(r,k) = u(r,k) \pm v(r,k)$	$j(l,p) = U(l,p) \pm V(l,p)$
$j(r,k) = \delta u(r,k)$	$J(l,p) = \delta U(l,p)$, where δ is constant
$j(r,k) = \frac{\partial u(r,k)}{\partial k}$	$J(l,p) = (l+1)U(l+1,p)$
$j(r,k) = \frac{\partial^{r+s} u(r,k)}{\partial r^r \, \partial k^s}$	$J(l,p) = (l+1)(l+2)\ldots (l+r)(p+1)(p+2)\ldots (p+s)U(l+r,p+s)$
$j(r,k) = \frac{\partial u(r,k)}{\partial r}\frac{\partial v(r,k)}{\partial k}$	$J(l,p) = \sum_{r=0}^{l}\sum_{s=0}^{p}(l-r+1)(p-s+1)$ $U(l-r+1,s)V(r,p-s+1)$
$j(r,k) = u(r,k) = \frac{\partial v(r,k)}{\partial r}$	$J(l,p) = \sum_{r=0}^{l}\sum_{s=0}^{p}(l-r+1)U(r,s)$ $V(l-r+1,p-s)$

4. Application

The Sawada-Kotera (SK) equation's numerical solution has been identified using the DTM approach, which is covered in this section. Results from the suggested method, both numerical and graphical, are constructive, remarkable, and accurate numerical results show the strength and effectiveness of the proposed technique. Take into consideration the (Sawada-kotera) equations, Equation (1), with the initial conditions

$$\frac{\partial d}{\partial t} + \beta j^2 \frac{\partial j}{\partial r} + \alpha \frac{\partial j}{\partial r}\frac{\partial^2 j}{\partial r^2} + \delta \frac{\partial^3 j}{\partial r^3} + \frac{\partial^5 j}{\partial r^5} = 0 \tag{8}$$

By utilize the two-dimensional transform of Eq. (1) by take the related definitions in Table1:

$$D_o = 2S^2 Sech^2[S(r-c)]$$

$$D_1 = -\beta\left[d_o(r).d_o(r).\frac{\partial}{\partial r}d_o(r)\right] - \alpha\left[\frac{\partial}{\partial r}d_o(r).\frac{\partial^2}{\partial r^2}d_o(r)\right] - \delta\left[d_o(r)\frac{\partial^3}{\partial r^3}d_o(r)\right] - \left[\frac{\partial^5}{\partial r^5}d_o(r)\right] \quad (10)$$

$$\begin{aligned} D_2 = &-\frac{\beta}{2}\left[d_o(r).d_1(r).\frac{\partial}{\partial r}d_o(r) + d_1(r).d_o(r).\frac{\partial}{\partial n}d_1(r)\right] \\ &-\frac{\alpha}{2}\left[\frac{\partial}{\partial r}d_o(r).\frac{\partial^2}{\partial r^2}d_o(r) + \frac{\partial}{\partial r}d_1(r).\frac{\partial^2}{\partial r^2}d_1(r)\right] \\ &-\frac{\delta}{2}\left[d_1(r)\frac{\partial^3}{\partial r^3}d_o(r) + d_o(r)\frac{\partial^3}{\partial r^3}d_1(r)\right] \\ &-\left[\frac{\partial^5}{\partial r^5}d_o(r) + \frac{\partial^5}{\partial r^5}d_1(r)\right] \end{aligned} \quad (11)$$

$$\begin{aligned} D_3 = &-\beta\left[d_o(r).d_2(r).\frac{\partial}{\partial r}d_o(r) + d_1(r).d_1(r).\frac{\partial}{\partial r}d_1(r) + d_2(r).d_o(r).\frac{\partial}{\partial r}d_2(r)\right] \\ &-\alpha\left[\frac{\partial}{\partial r}d_o(r).\frac{\partial^2}{\partial r^2}d_o(r) + \frac{\partial}{\partial r}d_1(r).\frac{\partial^2}{\partial r^2}d_1(r) + \frac{\partial}{\partial r}d_2(r).\frac{\partial^2}{\partial r^2}d_2(r)\right] \\ &-\delta\left[d_2(r)\frac{\partial^3}{\partial r^3}d_o(r) + d_1(r)\frac{\partial^3}{\partial r^3}d_1(r) + d_o(r)\frac{\partial^3}{\partial r^3}d_2(r)\right] \\ &-\left[\frac{\partial^5}{\partial r^5}d_o(r) + \frac{\partial^5}{\partial r^5}d_1(r) + \frac{\partial^5}{\partial r^5}d_2(r)\right] \end{aligned} \quad (12)$$

$$\begin{aligned} D_4 = &-\frac{\beta}{4}\left[d_o(r).d_3(r).\frac{\partial}{\partial r}d_o(r) + d_1(r).d_2(r).\frac{\partial}{\partial r}d_1(r) + d_2(r).d_1(r).\frac{\partial}{\partial r}d_2(r)\right. \\ &\left. + d_3(r).d_o(r).\frac{\partial}{\partial n}d_3(r)\right] \\ &-\frac{\alpha}{4}\left[\frac{\partial}{\partial r}d_o(r).\frac{\partial^2}{\partial r^2}d_o(r) + \frac{\partial}{\partial r}d_1(r).\frac{\partial^2}{\partial r^2}d_1(r) + \frac{\partial}{\partial r}d_2(r).\frac{\partial^2}{\partial r^2}d_2(r)\right. \\ &\left. + \frac{\partial}{\partial r}d_3(r).\frac{\partial^2}{\partial r^2}d_3(r)\right] \\ &-\frac{\delta}{4}\left[d_3(r)\frac{\partial^3}{\partial r^3}d_o(r) + d_2(r)\frac{\partial^3}{\partial n^3}d_1(r) + d_1(r)\frac{\partial^3}{\partial r^3}d_2(r) + d_o(r)\frac{\partial^3}{\partial r^3}d_3(r)\right] \\ &-\left[\frac{\partial^5}{\partial r^5}d_o(r) + \frac{\partial^5}{\partial r^5}d_1(r) + \frac{\partial^5}{\partial r^5}d_2(r) + \frac{\partial^5}{\partial r^5}d_3(r)\right] \end{aligned} \quad (13)$$

Where:

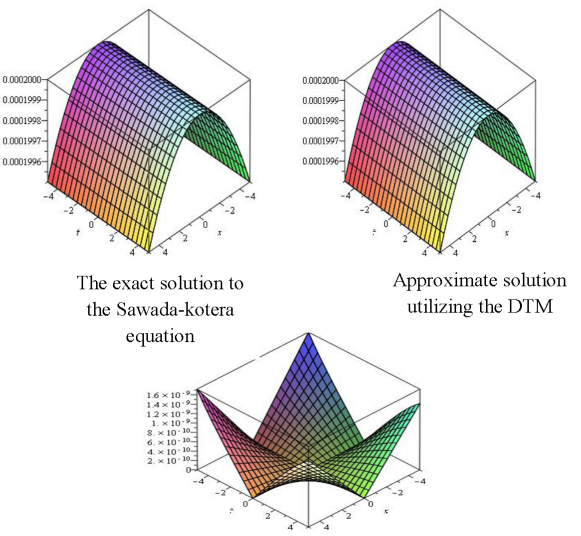
$$D = D_o + D_1 + D_2 + D_3 + D_4$$

$$D = \frac{1}{\cosh(0.01n)^{47}}(1.0 * 10^{-78}(2.0 * (10^{74}\cosh(0.01n)^{45} \ldots\ldots\ldots$$
$$+ 64.0 * 10^{65} t^4 * \cosh(0.01n)^{44} \sinh(0.01n)) \qquad (14)$$

Table (2)

Approximate solution results using numerical method (DTM), at parameters $\alpha = 15, S = 0.01, c = 0, \beta = 45, \delta = 15$

x	t	$\lvert$ Numerical Solution $\rvert_{DTM}$
6	0.1	$3.817996*10^{-11}$
7	0.2	$8.904697*10^{-11}$
8	0.3	$1.522951*10^{-10}$
9	0.4	$2.275158*10^{-10}$
10	0.5	$3.153979*10^{-10}$



The exact solution to the Sawada-kotera equation

Approximate solution utilizing the DTM

The figure shows the utter error in the DTM between the exact solution and the approximate solution

Figure 2

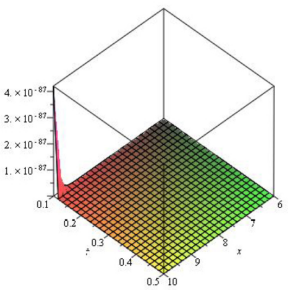
By to watch the results of the used method (DTM) with the optimal solution, it is clear that the mistake is between 10-10 and 10-11 as shown in Table 2, Figure 2.

Table (3)

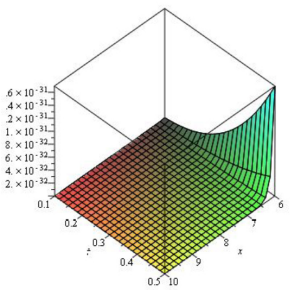
The results of the approximate solution utilizing the proposed technique between (DTM) and Algorithm (GWC) at parameters

$\alpha = -30.342, S = 4.73198, c = -4.26308, \beta = -11.5541, \delta = -24.9426$

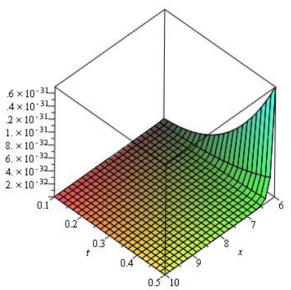
x	T	|Numerical Solution| $_{(GWC)\ Algorithm}$
6	0.1	1.356751×10^{-33}
7	0.2	8.418136×10^{-37}
8	0.3	2.204447×10^{-40}
9	0.4	4.054656×10^{-44}
10	0.5	6.145122×10^{-48}



The exact solution to the Sawada-kotera equation



An approximate solution utilizing the proposed technique



The figure shows the utter error of the proposed technique between the exact solution and the approximate solution

Figure (3)

5. Conclusions

In this paper, we present the solution approximate to a solution for partial differential equations. This goal was accomplished by applying the differential transformation method. Using the DTM, it is possible to find a very good approximate solution of the equation. It was found through the application of DTM that it is a strategy with accurate and effective results in finding accurate solutions to various types of problems. A new strategy was also proposed, which is between the numerical method DTM with a hybrid algorithm between gray Wolf and Cuckoo, where the results were impressive, very effective and able to find an exact solution to many problems. The solution to the differential equation (1), which represents f (x), was obtained and then, using the proposed intelligent technique, the roots very close to zero were obtained, representing a solution to the function f(x).

References

[1] D. Ganji, N. Jamshidi, and Z. Ganji, "HPM and VIM methods for finding the exact solutions of the nonlinear dispersive equations and seventh-order Sawada–Kotera equation," *International Journal of Modern Physics B*, vol. 23, no. 01, pp. 39-52 (2009).

[2] A. Salas, "Some solutions for a type of generalized Sawada–Kotera equation," *Applied Mathematics and Computation*, vol. 196, no. 2, pp. 812-817 (2008).

[3] Y. Guo, D. Li, and J. Wang, "The new exact solutions of the Fifth-Order Sawada–Kotera equation using three wave method," *Applied Mathematics Letters*, vol. 94, pp. 232-237 (2019).

[4] A. R. Adem et al., "Multiple soliton solutions of the Sawada-Kotera equation with a nonvanishing boundary condition and the perturbed Korteweg de Vries equation by using the multiple exp-function scheme," *Advances in Mathematical Physics*, vol. 2019, pp. 1-5 (2019).

[5] S. Mirjalili, S. M. Mirjalili, and A. Lewis, "Grey Wolf Optimizer," *Advances in Engineering Software*, vol. 69, pp. 46-61 (2014).

[6] K. A. Abed and A. A. Ahmad, "The best parameters selection using PSO algorithm to solving for Ito system by new iterative technique," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 18, no. 3, pp. 1638-1645 (2020).

[7] A. A. Ahmad, "By using a new iterative method to the generalized system Zakharov–Kuznetsov and estimate the best parameters via applied the PSO algorithm," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 19, no. 2, pp. 1055-1061 (2020).

[8] C. Muro et al., "Wolf-pack (Canis lupus) hunting strategies emerge from simple rules in computational simulations," *Behav Processes*, vol. 88, no. 3, pp. 192-197 (2011).

[9] O. Bazighifan, H. Ahmad, and S.-W. Yao, "New oscillation criteria for advanced differential equations of fourth order," *Mathematics*, vol. 8, no. 5, p. 728 (2020).

[10] A. E. Abouelregal, H. Ahmad, and S.-W. Yao, "Functionally graded piezoelectric medium exposed to a movable heat flow based on a heat equation with a memory-dependent derivative," *Materials*, vol. 13, no. 18, p. 3953 (2020).

[11] I. Ahmad et al., "Solution of multi-term time-fractional PDE models arising in mathematical biology and physics by local meshless method," *Symmetry*, vol. 12, no. 7, p. 1195 (2020).

[12] A. Yokus, H. Durur, and H. Ahmad, "Hyperbolic type solutions for the couple Boiti-Leon-Pempinelli system," *Facta Universitatis, Series: Mathematics and Informatics*, vol. 35, no. 2, pp. 523-531 (2020).

[13] A. S. Al-Obeidi and S. F. Al-Azzawi, "Hybrid synchronization of high-dimensional chaos with self-excited attractors," *Journal of Interdisciplinary Mathematics*, vol. 23, no. 8, pp. 1569-1584 (2020).

[14] M. M. Aziz and S. F. Al-Azzawi, "A modification of nonlinear feedback controller," *International Journal of Computing Science and Mathematics*, vol. 13, no. 1, pp. 64-79 (2021).

[15] A. S. Al-Obeidi and S. F. Al-Azzawi, "A novel six-dimensional hyperchaotic system with self-excited attractors and its chaos synchronisation," *International Journal of Computing Science and Mathematics*, vol. 15, no. 1, pp. 72-84 (2022).

[16] K. A. Abed, "Controlling of jerk chaotic system via linear feedback control strategies," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 20, no. 1, pp. 370-378 (2020).

[17] A. F. Qasim, K. A. Abed, and O. S. Qasim, "Optimal parameters for nonlinear Hirota-Satsuma coupled KdV system by using hybrid firefly algorithm with modified Adomian decomposition," *Journal of Mathematical & Fundamental Sciences*, vol. 52, no. 3 (2020).

[18] Y. Yamamoto and É. I. Takizawa, "On a solution of non-linear time-evolution equation of fifth order," *Journal of the Physical Society of Japan*, vol. 50, no. 5, pp. 1421-1422 (1981).

[19] H. Ahmad et al., "Modified variational iteration technique for the numerical solution of fifth order KdV-type equations," *Journal of Applied and Computational Mechanics*, (2020).

[20] H. Durur and A. Yokuş, "Hyperbolic traveling wave solutions for Sawada–Kotera equation using (1/G')-expansion method," *Afyon Kocatepe University Journal of Sciences and Engineering*, vol. 19, no. 3, pp. 615-619 (2019).

[21] H. O. Bakodah, "Modified Adomain decomposition method for the generalized fifth order KdV equations," (2013).

[22] M. M. Rashidi and E. Erfani, "Traveling wave solutions of WBK shallow water equations by differential transform method," *Advances in Theoretical and Applied Mechanics*, vol. 3, no. 6, pp. 263-271 (2010).

[23] A. Mahmoud, S. Abdelghany, and K. Ewis, "Free vibration of uniform and non-uniform Euler beams using the differential transformation method," *Asian Journal of Mathematics and Applications*, (2013).

[24] H. Günerhan et al., "Numerical solution of fractional order logistic equations via conformable fractional differential transform method," *Journal of Interdisciplinary Mathematics*, vol. 24, no. 5, pp. 1207-1220 (2021).

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