Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 27 (2024), No. 5, pp. 1109–1116
DOI : 10.47974/JIM-1939

Twin proper neutrosophic fuzzy graphs

V. Prabavathy *
Department of Mathematics
Vivekananda College, Agasteeswaram
(Affiliated to Manonmaniam Sundaranar University)
Tirunelveli
Tamil Nadu
India

B. Preethika †
Department of Mathematics
Vivekananda College, Agasteeswaram
(Affiliated to Manonmaniam Sundaranar University)
Tirunelveli
Tamil Nadu
India

Abstract

In this critique, twin proper neutrosophic fuzzy graphs and totally twin proper neutrosophic fuzzy graphs are portrayed. Collasion of these two graphs were given through exemplifications. Provided is the unavoidable circumstance under which these two graphs were coequal. The characterisation of twin correct neutrosophic fuzzy graphs with a cycle as the underlying crisp graph is studied. The applicability of the results to fully twin accurate neutrosophic fuzzy graphs is also explored.

Subject Classification: *03E72, 05C12, 05C72.*

Keywords: *Neutrosophic fuzzy graph(NFG), Regular NFG, Totally regular NFG.*

1. Introduction

The neutrosophic theory has demonstrated its superiority over other approaches and theories in handling data, especially in its capability to consider indeterminate data. In this paper, we consider nonregular graphs. The focus is

* *E-mail:* vpraba1982@gmail.com (Corresponding Author)
† *E-mail:* mail2preethika96@gmail.com

mainly on the case of graphs in which all the vertices takes any one of the two degree values. Liangsong Huang et.al., [1] introduced the concept of Regular NFGs. Researchers Muzychuk .M and Klin. M [2] investigated biregular graphs. These urge us to examine some of the features of entirely twin proper NFGs as well as twin proper NFGs. For more details see [1, 2].

2. Preliminaries

For easy reference while reading the work provided in this article, we provide some well-known definitions of fuzzy graphs and NFGs.

Definition 2.1: [4] A fuzzy graph G : (σ, μ) is a pair of functions (σ, μ), where $\sigma : V \rightarrow [0,1]$ is a fuzzy subset of a non empty set V and $\mu : V XV \rightarrow [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V, the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(V)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(V)$ is satisfied.

Definition 2.2: [4] The strength of connectedness between two vertices u and v is defined as $\mu^{\infty}(u, v) = \sup \{\mu^k(u, v) : k = 1, 2, ...\}$, where $\mu^k(u, v)=\sup \{\mu(u, u_1) \wedge \mu(u_1, u_2) \wedge ... \wedge \mu(u_{k-1}, v) : u, u_1, u_2, ..., u_{k-1}, v$ is a path connecting u and v of length k$\}$.

Definition 2.3: [3] Let G : (N, M) be a single - valued neutrosophic graph where N and M are represented by two neutrosophic sets on V and E, respectively, which satisfy the following.

$T_M(a, b) \leq \min(T_N (a), T_N (b))$, $I_M (a, b) \geq \max(I_N (a), I_N (b))$, $F_M (a, b) \geq \max(F_N (a), F_N (b))$

Here a and b are two vertices of G and (a, b) ∈ E is an edge of G. Where T_M denote the truth value, I_M denoted the indeterminacy value and F_M denoted the false value.

Definition 2.4: [3] Let G : (N, M) be an single - valued neutrosophic graph and a ∈ V be vertex of G. The degree of vertex a is the sum of the truth membership values, the sum of the indeterminacy membership values, and the sum of the membership values of falsity of all the arcs that are adjacent to vertex a .The degree of vertex a is denoted by $d(a) = (d_T (a), d_I (a), d_F (a))$ where $d_T (a) = \Sigma_{a\in V} T_M(a, b)$,$d_I (a) = \Sigma_{a\in V} I_M(a, b)$,$d_F (a)= \Sigma_{a\in V} F_M (a, b)$.

Here $d_T (a)$,$d_I (a)$, and $d_F (a)$ are the degree of the truth membership value, the degree of the indeterminacy membership value, and the degree of falsity membership value, respectively, of vertex a.

Definition 2.5: [3] Let G : (N, M) be an single - valued neutrosophic graph. G is called regular neutrosophic fuzzy graph if all the vertices of the graph have the same degree $(\hbar_1, \hbar_2, \hbar_{31})$.

Definition 2.6: [3] Let G : (N, M) be an single - valued neutrosophic graph. The total degree of the vertex a in G is defined as td(a) = (td_T (a), td_I (a), td_F (a)) where td_T (a) = d_T (a) + T(a), td_I (a) = d_I (a) + I(a), td_F (a) = d_F (a) + F(a). It can also be defined as td(a) = d(a) + N(a).

Definition 2.7: [3] Let G : (N, M) be an single - valued neutrosophic graph. G is called totally regular NFG if all the vertices of the graph have the same degree (r_1, r_2, r_3).

3. Twin Proper Neutrosophic Fuzzy Graphs

This section introduces both entirely twin proper NFGs and twin proper NFGs.

Definition 3.1: Let G_N : (N, M) be a NFG on G^*_N : (V, E). G_N is called a twin proper NFG if the degree of each vertex is either (ρ_1, ρ_2, ρ_3) or (ρ_4, ρ_5, ρ_6). Then G is said to be ((ρ_1, ρ_2, ρ_3), (ρ_4, ρ_5, ρ_6))-twin proper NFG.

Example 3.2: Deem a NFG on G^*_N : (V, E).

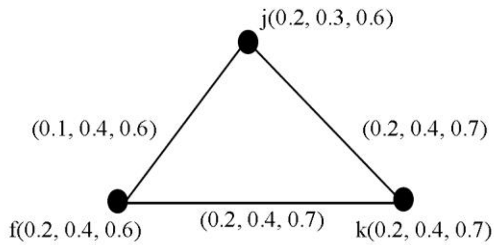

Figure 1

Here, as in figure.1 d(j) = (0.3, 0.8, 1.3), d(f) = (0.3, 0.8, 1.3), d(k) = (0.3, 0.8, 1.4). This graph is ((0.3, 0.8, 1.3),(0.3, 0.8, 1.4))−twin proper NFG.

Definition 3.3: Let G_N : (N, M) be a NFG on G^*_N : (V, E). G_N is said to be totally twin proper NFG if the total degree of each vertex is either ($\hbar_1$, $\hbar_2$, $\hbar_3$) or ($\hbar_4$, $\hbar_5$, $\hbar_6$). Then G is said to be (($\hbar_1$, $\hbar_2$, $\hbar_3$),($\hbar_4$, $\hbar_5$, $\hbar_6$))-totally twin proper NFG.

Example 3.4: Deem a NFG on G^*_N : (V, E).

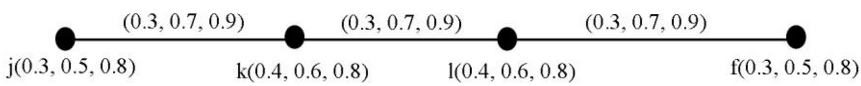

Figure 2

Here as in figure.2, td(j) = (0.6, 1.2, 0.7), td(k) = (1.0, 1.8, 2.6), td(l) = (1.0, 1.8, 2.6), td(f) = (0.6, 1.2, 1.7). This graph is ((0.6, 1.2, 0.7), (1.0, 1.8, 2.6))-totally twin proper NFG.

Remark 3.5: A twin proper NFG need not be totally twin proper NFG.

Remark 3.6: A totally twin proper NFG need not be twin proper NFG.

Remark 3.7: A twin proper NFG is also a totally twin proper NFG.

Theorem 3.8: Let G_N : (N, M) be a NFG on G^*_N : (V, E). The following conditions must hold in order for N to be a constant function.

(i) G_N is a twin proper NFG.

(ii) G_N is a totally twin proper NFG.

Proof: Intend that N is a constant function. Let N(z) =($\hbar_1, \hbar_2, \hbar_3$) be a constant for all z ∈ V . Assume that G_N is a ((ρ_1, ρ_2, ρ_3), (ρ_4, ρ_5, ρ_6)) twin proper NFG. Then, let d(z) = (ρ_1, ρ_2, ρ_3) and d(w) = (p_4, p_5, p_6) for some z, w ∈ V . Now, td(z) = d(z)+N(z) for all z ∈ V ⇒ td(z) = (ρ_1, ρ_2, ρ_3)+($\hbar_1, \hbar_2, \hbar_3$), for some z ∈ V and td(w) = d(w) + N(w), for some w ∈ V ⇒ td(w) = (ρ_4, ρ_5, ρ_6) + ($\hbar_1, \hbar_2, \hbar_3$) for some w ∈ V . Hence G_N is totally-twin proper NFG.

Thus (i) ⇒ (ii) is proved.

Now Intend G_N is a ((ρ_1, ρ_2),(ρ_3, ρ_4))-totally twin proper NFG. Then td(z) = d(z) + N(z) = (ρ_1, ρ_2, ρ_3) for some z ∈ V ⇒ d(z) + N(z) = (ρ_1, ρ_2, ρ_3), for some z ∈ V ⇒ d(z) + ($\hbar_1, \hbar_2, \hbar_3$) = ($\rho_1, \rho_2, \rho_3$), for some z ∈ V ⇒ d(z) = (ρ_1, ρ_2, ρ_3)−($\hbar_1, \hbar_2, \hbar_3$), for some z ∈ V and td(w) = d(w) + N(w) = (ρ_4, ρ_5, ρ_6) for some w ∈ V ⇒ d(w) + N(w) = (p_4, p_5, p_6),for some w ∈ V ⇒ d(w) + ($\hbar_1, \hbar_2, \hbar_3$) = ($\rho_4, \rho_5, \rho_6$), for some w ∈ V ⇒ d(w) = (ρ_4, ρ_5, ρ_6) − ($\hbar_1, \hbar_2, \hbar_3$), for some w ∈ V and so G_N is twin proper NFG. Thus (ii) ⇒ (i) is proved. Hence (i) and (ii) are equivalent.

Conversely, assume that (i) and (ii) are equivalent. That is G_N is twin proper NFG if and only if G_N is totally twin proper NFG. Intend N is not a constant function, then N(z) ≠ N(v) ≠ N(w) for some vertices u, v, w ∈ V. Let G_N be a((ρ_1, ρ_2, ρ_3),(ρ_4, ρ_5, ρ_6)) -twin proper NFG. Then d(z) = (p_1, p_2, p_3), d(v) = (ρ_1, ρ_2, ρ_3) and d(w) = (ρ_4, ρ_5, ρ_6) and so td(z) = d(z) + N(z) = (ρ_1, ρ_2, ρ_3) + N(z), td(v) = d(v) + N(v) = (ρ_1, ρ_2, ρ_3) + N(v) and td(w) = d(w) + N(w) = (ρ_4, ρ_5, ρ_6) + N(w). Since N(z) ≠ N(v) ≠ N(w),we have td(z) ≠td(v) ≠ td(w) for some vertices u, v, w. So G_N is not totally twin proper NFG which is a contradiction to our assumption.

Now let G_N be a totally twin proper NFG. Then let, td(z) = td(v) = ($\hbar_1, \hbar_2, \hbar_3$) and td(w) = ($\hbar_4, \hbar_5, \hbar_6$) ⇒ d(z) + N(z) = d(v) + N(v) = ($\hbar_1, \hbar_2, \hbar_3$) and d(w) + N(w) = ($\hbar_4, \hbar_5, \hbar_6$) ⇒ d(z) = ($\hbar_1, \hbar_2, \hbar_3$) − N(z), d(v) = ($\hbar_1, \hbar_2, \hbar_3$) − N(v) and d(w) = ($\hbar_4, \hbar_5, \hbar_6$) − N(w). Intend N(z) ≠ N(v) ≠N(w) then G_N is not twin proper NFG which is contradiction to our assumption. Hence N is a constant function.

Theorem 3.9: *If a NFG G_N : (N, M) is both twin proper neutrosophic and totally twin proper neutrosophic. Then N is a constant function.*

Proof: Let G_N be a $((\rho_1, \rho_2, \rho_3),(\rho_4, \rho_5, \rho_6))$-twin proper neutrosophic and $((ħ_1, ħ_2, ħ_3),(ħ_4, ħ_5, ħ_6))$- totally twin proper NFG . So $d(z) = (\rho_1, \rho_2, \rho_3)$ for some $z \in V$, $d(w) = (\rho_4, \rho_5, \rho_6)$ for some $z \in V$ and $td(z) = (ħ_1, ħ_2, ħ_3)$ for some $z \in V$, $td(w) = (ħ_4, ħ_5, ħ_6)$ for some $z \in V$. Now $td(z) = (ħ_1, ħ_2, ħ_3)$ for some $z \in V \Rightarrow d(z) + N(z) = (ħ_1, ħ_2, ħ_3)$ for some $z \in V \Rightarrow (\rho_1, \rho_2, \rho_3) + N(z) = (ħ_1, ħ_2, ħ_3)$ for some $z \in V \Rightarrow N(z) = (ħ_1, ħ_2, ħ_3) - (\rho_1, \rho_2, \rho_3)$ for all $z \in V$. Hence N is a constant function.

Remark 3.10: The above theorem's inverse need not be true.

Theorem 3.11: *Let G_N : (N, M) be a NFG on G^*_N : (V, E), a cycle on N vertices. If N is a constant function and* $M(e_i) = \begin{cases}(ħ_1, ħ_2, ħ_3) & i = 1,2,\dots,n-1\\(ħ_4, ħ_5, ħ_6) & i = n\end{cases}$

Then G_N is both twin proper neutrosophic and totally twin proper neutrosophic fuzzy graph.

Proof: Let $e_1, e_2, \dots, e_n$ be the edges of the cycle S_n. Let $N(z) = (\rho_1, \rho_2, \rho_3)$, for all $z \in V$ be a constant function.

Let $M(e_i) = \begin{cases}(ħ_1, ħ_2, ħ_3) & i = 1,2,\dots,n-1\\(ħ_4, ħ_5, ħ_6) & i = n\end{cases}$

Then $d(v_1) = (ħ_1, ħ_2, ħ_3) + (ħ_4, ħ_5, ħ_6) = d(v_n)$. Also $d(v_i) = 2(ħ_1, ħ_2, ħ_3)$ for $i = 2, \dots, n-1$.

So, G_N is $((ħ_1, ħ_2, ħ_3) + (ħ_4, ħ_5, ħ_6), 2(ħ_1, ħ_2, ħ_3))$ twin proper NFG Also, $td(v_1) = (ħ_1, ħ_2, ħ_3) + (ħ_4, ħ_5, ħ_6) + (\rho_1, \rho_2, \rho_3) = td(v_n)$ and $td(v_i) = 2((ħ_1, ħ_2, ħ_3) + (\rho_1, \rho_2, \rho_3))$ for $i = 2, 3, \dots, n-1$. Hence G_N is $(((ħ_1, ħ_2, ħ_3) + (ħ_4, ħ_5, ħ_6) + (\rho_1, \rho_2, \rho_3)), 2(ħ_1, ħ_2, ħ_3) + (\rho_1, \rho_2, \rho_3))$ totally twin proper NFG. Hence G_N is both twin proper neutrosophic and totally twin proper NFG.

Theorem 3.12: *Let G_N : (N, M) be a neutrosophic fuzzy graph G^*_N : (V, E),a path on 2n vertices. If M is constant function or alternative edges take same membership values then G_N is twin proper neutrosophic fuzzy graph.*

Proof: Let $e_1, e_2, \dots, e_{2n-1}$ be the edges of the path.

Case (i): M is constant function. Then $M(e_i) = (ħ_1,ħ_2,ħ_3)$ for all i.

$$d(v_i) = \begin{cases}(ħ_1, ħ_2, ħ_3) & \text{if } i \text{ is pendant vertex}\\2(ħ_1, ħ_2, ħ_3) & \text{if } i \text{ is internal vertex}\end{cases}$$

Hence G_N is $((ħ_1, ħ_2, ħ_3), 2(ħ_1, ħ_2, ħ_3))$ twin proper NFG.

Case (ii): Alternate edge take same membership values.

Let the edges $e_1, e_3, \dots, e_{2n-1}$ takes the membership values $(ħ_1, ħ_2, ħ_3)$ and the edges $e_2, e_4, \dots, e_{2n-2}$ takes the membership values $(ħ_4, ħ_5, ħ_6)$.

$$d(v_i)= \begin{cases} (ħ_1, ħ_2, ħ_3) & \text{if } i \text{ is pendant vertex} \\ (ħ_1, ħ_2, ħ_3) + (ħ_4, ħ_5, ħ_6) & \text{if } i \text{ is internal vertex} \end{cases}$$

Hence G_N is $((ħ_1, ħ_2, ħ_3),((ħ_1, ħ_2, ħ_3) + (ħ_4, ħ_5, ħ_6)))$ twin proper NFG..

Remark 3.13: The theorem mentioned above is not applicable to totally twin proper neutrosophic fuzzy graphs.

Theorem 3.14: *Let G_N : (N, M) be a NFG on G^*_N : (V, E) a cycle on n vertices. Let M be a constant function and alternative vertices takes same membership values then G_N is totally twin proper NFG.*

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle $ħ_n$ and

$$N(v_i) = \begin{cases} (\rho_1, \rho_2, \rho_3) & \text{if } i \text{ is odd} \\ (\rho_4, \rho_5, \rho_6) & \text{if } i \text{ is even} \end{cases}$$

Since M is constant $M(e_i) = (ħ_1, ħ_2, ħ_3)$, for all i. Then $d(v_i) = 2(ħ_1, ħ_2, ħ_3)$ and

$$td(v_i) = d(v_i) + N(v_i) = \begin{cases} 2(ħ_1, ħ_2, ħ_3) + (\rho_1, \rho_2, \rho_3) & \text{if } i \text{ is odd} \\ 2(ħ_1, ħ_2, ħ_3) + (\rho_4, \rho_5, \rho_6) & \text{if } i \text{ is even} \end{cases}$$

Hence G_N is $(2(ħ_1, ħ_2, ħ_3) + (\rho_1, \rho_2, \rho_3),\ 2(ħ_1, ħ_2, ħ_3) + (\rho_4, \rho_5, \rho_6))$ be a totally twin proper NFG.

Remark 3.15: The above theorem does not hold for twin proper NFG, since M is constant function, $d(z) = (\rho_1, \rho_2, \rho_3)$ for all $z \in V$.

Theorem 3.16: Let G_N : (N, M) be a NFG on G^*_N : (V, E) an even cycle on n vertices. If alternate vertices takes same membership values and alternate edges takes same membership values then G_N is totally twin proper NFG.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of the cycle C_n and

$$\text{Let } N(u_i) = \begin{cases} (ħ_1, ħ_2, ħ_3) & \text{if } i \text{ is odd} \\ (ħ_4, ħ_5, ħ_6) & \text{if } i \text{ is even} \end{cases}$$

Since alternate edges takes same membership values and G_N is an even cycle, $d(u_i) = (\rho_1, \rho_2, \rho_3)$, for all i.

$$\text{Now, } td(u_i) = \begin{cases} (ħ_1, ħ_2, ħ_3) + (\rho_1, \rho_2, \rho_3) & \text{if } i \text{ is odd} \\ (ħ_4, ħ_5, ħ_6) + (\rho_1, \rho_2, \rho_3) & \text{if } i \text{ is even} \end{cases}$$

So, G_N is $((ħ_1, ħ_2, ħ_3) + (\rho_1, \rho_2, \rho_3),(ħ_4, ħ_5, ħ_6) + (\rho_1, \rho_2, \rho_3))$- totally twin proper NFG.

Theorem 3.17: *If the ladder on n vertices take constant vertex membership value ($\hbar_1$, $\hbar_2$, $\hbar_3$) and constant edge membership value (ρ_1, ρ_2, ρ_3). Then G_N is both twin proper NFG and totally twin proper NFG.*

Proof: Let $u_1, u_2, \ldots, u_{2n}$ be the vertices of the ladder and $e_1, e_2, \ldots e_{3n-2}$ be the edges of the ladder.

Let $N(u_i) = (\hbar_1, \hbar_2, \hbar_3)$, for all i and $M(e_i) = (\rho_1, \rho_2, \rho_3)$, for all i. Then

$$d(u_i) = \begin{cases} 2(\rho_1, \rho_2, \rho_3) & \text{if } i \text{ is end vertex} \\ 3(\rho_1, \rho_2, \rho_3) & \text{if } i \text{ is internal vertex} \end{cases}$$

So, G_N is$(2(\rho_1, \rho_2, \rho_3), 3(\rho_1, \rho_2, \rho_3))$-twin proper NFG. Now,

$$td(u_i) = \begin{cases} 2(\rho_1, \rho_2, \rho_3) + (\hbar_1, \hbar_2, \hbar_3) & \text{if } i \text{ is end vertex} \\ 3(\rho_1, \rho_2, \rho_3) + (\hbar_1, \hbar_2, \hbar_3) & \text{if } i \text{ is internal vertex} \end{cases}$$

Hence G_N is $((2(\rho_1, \rho_2, \rho_3) + (\hbar_1, \hbar_2, \hbar_3), 3(\rho_1, \rho_2, \rho_3) + (\hbar_1, \hbar_2, \hbar_3))$-totally twin proper NFG.

Theorem 3.18: *Let G_N : (N, M) be a NFG on G^*_N : ((V, E), a cycle on n vertices. If*

$$M(e_i) = \begin{cases} (\hbar_1, \hbar_2, \hbar_3) & \text{if } i = 1,2,\ldots,n-1 \\ (\hbar_4, \hbar_5, \hbar_6) & \text{if } i = n \end{cases}$$

where $(\hbar_1, \hbar_2, \hbar_3) < (\hbar_4, \hbar_5, \hbar_6)$, then G_N is a fuzzy cycle.

Proof: Let $e_1, e_2, \ldots, e_n$ be the edges of the cycle $\hbar_n$. Assume that

$$M(e_i) = \begin{cases} (\hbar_1, \hbar_2, \hbar_3) & \text{if } i = 1,2,\ldots,n-1 \\ (\hbar_4, \hbar_5, \hbar_6) & \text{if } i = n \end{cases}$$

where $(\hbar_1, \hbar_2, \hbar_3) < (\hbar_4, \hbar_5, \hbar_6)$.

So there does not exists unique edge e_n such that $M(e_n) = \wedge\{M(e_i) : e_i \in E\}$. Hence G_N is a fuzzy cycle.

Theorem 3.19: *Let G_N be a NFG on G^*_N : (V, E), a path on n vertices. If M is constant function, then G_N does not have fuzzy bridge.*

Proof: Assume that G^*_N : (V, E) is a path on N vertices. Since M is constant function, G_N is twin proper neutrosophic fuzzy graph. Also, Since M is constant, removal of any edge does not reduce the strength of connectedness. Hence G_N does not have fuzzy bridge.

References

[1] M.Basher, Even vertex odd mean labeling of some cycle related graphs, *Journal of Discrete Mathematical Sciences and Cryptography,* 2321-2340 (2021).

[2] R. Joseph Paul and U. Mary, Geo chromatic number of certain graphs, *Journal of Interdisciplinary Mathematics*, 26(1), 11-16 (2023).

[3] Liangsong Huang et.al., A Study on Regular and Irregular Neutrosophic Fuzzy Graph with Real Life Application, *Mathematics* 7(6), 551- 571 (2019).

[4] A.Nagoorgani and K. Radha, On Regular Fuzzy Graphs, *Journal of Physical Sciences*, Volume 12, 33-44 (2008).

Received March, 2024