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Pythagorean fuzzy shortest hyper path in a network

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Abstract

Objectives: The Pythagorean fuzzy shortest hyperpath problem balances distance minimization with network fuzziness and uncertainty, considering node lengths, membership and non-membership degrees, when accurate data is unavailable or traditional methods cannot handle uncertain information. **Methods:** Pythagorean Fuzzy Dijkstra's Algorithm uses Pythagorean fuzzy numbers as edge weights, while the A* algorithm considers fuzziness and uncertainty. Evolutionary, metaheuristic, linear programming, and fuzzy decision-making techniques can be used to solve the Pythagorean fuzzy shortest hyperpath problem. **Findings:** The Pythagorean fuzzy shortest hyper path algorithm is a useful tool for navigating complex networks under uncertainty, identifying optimal routes, quantifying uncertainty, and providing sensitivity, trade-off, and network resilience analysis. It's applicable in transportation, logistics, telecommunications, and urban planning. **Novelty:** The Pythagorean fuzzy shortest hyper path algorithm is a network pathfinding method that uses fuzzy logic principles in graph theory. It quantifies ambiguity along paths, enabling informed decision-making in uncertain environments. This algorithm is applicable across various domains, bridging theoretical concepts with practical applications.

Subject Classification: *05C90, 05C72, 62A86.*

Keywords: *Intuitionistic fuzzy hyper graph (InFHyG), Pythagorean fuzzy hyper graph (PFHyG), Pythagorean fuzzy number (PFN), Pythagorean fuzzy shortest hyper path (PFSHyP), Triangular pythagorean fuzzy number (TriPFN).*

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1. Introduction

The paper explores Pythagorean fuzzy sets in network analysis, focusing on shortest hyper path computation in edge weighted networks. Researchers aim to develop efficient navigation algorithms for robustness, adaptability, and better decision-making in uncertain environments. In order to convey ambiguity and uncertainty, Zadeh [6] created fuzzy sets, which have since been researched in a number of disciplines. Atanassov [4] created intuitionistic fuzzy sets in 1986; these sets indicate the extent to which an attribute is part of set. Since intuitionistic, higher-level fuzzy sets are more complex and therefore require more computing power. Numerous industries, including engineering, artificial intelligence, and the life and medical sciences, have used fuzzy sets.

Each edge has a membership or non-membership value, and the basic network problem is known as the shortest route problem (SPP). The fuzzy shortest route problem (SPP) initially analysed by Dubois and Prade [2], who used fuzzy numbers rather than actual numbers to assign a value to each edge. When comparing the performance evaluation of the proposed segmentation strategy in both guided and unsupervised situations with the k-mean classification algorithm, better results are found. This illustrates how our approach can be used to various categorization problems. Thus, Shortest Path Problems (SPP) are considered and often tested in a variety of scientific and engineering disciplines. Road networks, transport, channel communications, and research are a few examples. M. Asim Basha, M. Mohammed Jabarulla [12], triangle I–V the edge weights are Pythagorean fuzzy numbers, while the nodes and linkages are exact. A graphical illustration has also been provided to demonstrate the suggested technique. M. Asim Basha, M. Mohammed Jabarulla and Said Broumi [11], as a result, we created the proposed technique, which finds the shortest path between the starting and destination nodes by applying a classification method for the Neutrosophic Pythagorean fuzzy Triangular number. Finally, a sample scenario is given for confirmation. H. Garg [3], developed in a Pythagorean Fuzzy Sets. T. Malathi and P. Senthilkumar [16], an approach to fuzzy transportation problem using triacontakaidigon fuzzy number with alpha cut ranking technique are extract. Talal Ali Al-Hawary [17], they are defined in a density result for perfectly regular and perfectly edge-regular fuzzy graphs.

Amna Habib, Muhammad Akram and Cengiz Kahraman [1], we utilise an operational cortical network to illustrate the utility and efficacy of the proposed technique. Complex structures characterise the brain's structural and functional systems. We investigate the consequences of the proposed method for hierarchical clustering represented by a tree diagram. Finally, we compare the sets of results generated by different similarity metrics. M. Muhammad Akram, W. A. Dudek and F. Ilyas [8], we extend the TOPSIS technique to multicriteria grouping decisions scenarios by utilising Pythagorean fuzzy data. Experts provide feasible solutions as Pythagorean fuzzy decision matrices, and the best possibilities are identified and assessed using a modified closeness index.

Muhammad Akram, Amna Habib, and Tofigh Allahviranloo's technique [9] illustrates its benefits for enhanced coherence by comparing Pythagorean fuzzy optimal flows for network arcs, identifying trends, and evaluating the runtime of popular maximal flow methodologies. According to Shaista Habib, Aqsa Majeed, Muhammad Akram, and Mohammed M. Ali Al-Shamiri [15], we determine APSP utilising the dependability, scores, and trapezoidal picture fuzzy additive functions. We calculate the time complexity of our approach and compare it with the fuzzy approach and the traditional Floyd-Warshall methodology.

Ayesha Shareef, Ahmad N. Al-Kenani, and Muhammad Akram [7] applied Pythagorean fuzzy occurrence graphs to an only toll roadway system. Our discussion will focus on using legal fuzzy occurrence forests for decision-making, specifically where to put the digital toll collecting Protocol on only road tolls to increase toll collection. We furthermore offer an algorithm to comprehend the techniques we employ in our software. In order to show the applicability and significance of the suggested method, we then contrast it with a previous methodology that combines logical deduction with numerical data. A. Saraswathi and K. Vidhya [5], An improved search method called A* for determining the shortest route at interval-valued Pythagorean criteri. This investigation shows that the suggested approach, out of all the approaches, offers the best price and the shortest path; as a result, it might be used to find a useful scenario in an uncertain setting.

Parvathi Rangasamy, S. Thilagavathi and M.G. Karunambigai [13], the terms dual intuitionistic and intuitionistic fuzzy hypergraph are introduced. We also introduce the notions of edge strength and (α, β)-cut hypergraph in Intuitionistic Ambiguous Hypergraph. Parvathi Rangasamy, Muhammad Akram and S. Thilagavathi [14], it presents a method for prioritising intuitionistic fuzzy scores and accuracy to Create the shortest possible hyperpath in an intuitionistic fuzzy weighted hypergraph. Muhammad Akram and Anam Luqman [10], it presents techniques for constructing line graphs, dual hypergraphs, and complex hypergraphs, and investigates The use of intuitionistic fuzzy hyper graphs in the environment interface systems, chemical component grouping, and clustering.

Hyper graphs are useful for simulating computer systems, data constructions, and circuit design. They provide expressive power for complex issues. Pythagorean fuzzy numbers (PFN) are used to find the Pythagorean shortest hyper path in networks. The study presents a score-based algorithm using Pythagorean fuzzy weights on hyper edges to identify the shortest paths in networks, offering a simpler calculation technique and numerical information. This method, combined with Pythagorean fuzzy logic and hypergraph theory, offers new areas of study for network analysis.

2. Preliminaries

This section covers the fundamental definitions of the terms "hyper graph" and "Pythagorean fuzzy number," as well as how to use the accuracy function and score for Pythagorean fuzzy numbers.

2.1 *Definition: (Pythagorean fuzzy Hyper Graph)*

A PFHG, H' is an order pair $H' = (V', E')$ were,

(i) $V' = \{v_1', v_2', \dots, v_n'\}$ a finite set of vertices.

(ii) $E' = \{e_1', e_2', \dots, e_m'\}$ a family of PF subsets of V'.

(iii) $E_j' = \left\{\left(v_i, \widetilde{\mu_j}(v_i), \widetilde{\gamma}_j(v_i)\right) : \left(\widetilde{\mu_j}(v_i)\right)^2, \left(\widetilde{\gamma}_j(v_i)\right)^2 \geq 0 \; and \; 0 \leq \left(\widetilde{\mu_j}(v_i)\right)^2 + \left(\widetilde{\gamma}_j(v_i)\right)^2 \leq 1\right\},\ j = 1,2, \dots, m.$

(iv) $E_j' \neq \Phi, j = 1,2, \dots, m.$

(v) $\cup_j \, supp\ \left(E_j'\right) = V', j = 1,2, \dots, m.$

The edges (or hyper edge) E_j' are PFS of vertices. The degree of participation (member & non- member) of vertex x_i' to edges E_j' is defined by $\widetilde{\mu_j}(x_i)$. Then the other set V', E' are crisp set.

2.2 *Definition: (Triangular Pythagorean Fuzzy number) TPFN*

A TPFN, $\widetilde{P'}$ is denoted by $\widetilde{P'} = \{< x', \widetilde{\mu_{\widetilde{P'}}}(x'), \widetilde{\gamma_{\widetilde{P'}}}(x') > / \ x' \in X'\}$, where $\widetilde{\mu_{\widetilde{P'}}}(x')$ and $\widetilde{\gamma_{\widetilde{P'}}}(x')$ are TFNs. A TPFN, $\widetilde{P'}$ is given $\widetilde{P'} = \langle\left(\widetilde{a'}, \widetilde{b'}, \widetilde{c'}\right), \left(\widetilde{e'}, \widetilde{f'}, \widetilde{g'}\right)\rangle$ where,

$$\left(\widetilde{a'}, \widetilde{b'}, \widetilde{c'}\right) = \left(\mu_{\widetilde{P_1'}}(x')\right)^2, \left(\mu_{\widetilde{P_2'}}(x')\right)^2, \left(\mu_{\widetilde{P_3'}}(x')\right)^2$$

$$\left(\widetilde{e'}, \widetilde{f'}, \widetilde{g'}\right) = \left(\gamma_{\widetilde{P_1'}}(x')\right)^2, \left(\gamma_{\widetilde{P_2'}}(x')\right)^2, \left(\gamma_{\widetilde{P_3'}}(x')\right)^2.$$

2.3 *Definition: (Addition operations)*

Let $\widetilde{A'} = \langle\left(\widetilde{a_1'}, \widetilde{b_1'}, \widetilde{c_1'}\right), \left(\widetilde{e_1'}, \widetilde{f_1'}, \widetilde{g_1'}\right)\rangle$ and $\widetilde{B'} = \langle\left(\widetilde{a_2'}, \widetilde{b_2'}, \widetilde{c_2'}\right), \left(\widetilde{e_2'}, \widetilde{f_2'}, \widetilde{g_2'}\right)\rangle$ be two TriPFNs. Next, the inclusion of two TPFN, denoted by $\widetilde{A'} \oplus \widetilde{B'}$, is defined as

$$\widetilde{A'} \oplus \widetilde{B'} =$$

$$\langle\sqrt{\left(\widetilde{a_1'}^2 + \widetilde{a_2'}^2 - \left(\widetilde{a_1'}^2\widetilde{a_2'}^2\right)\right)}, \sqrt{\left(\widetilde{b_1'}^2 + \widetilde{b_2'}^2 - \left(\widetilde{b_1'}^2\widetilde{b_2'}^2\right)\right)}, \sqrt{\left(\widetilde{c_1'}^2 + \widetilde{c_2'}^2 - \left(\widetilde{c_1'}^2\widetilde{c_2'}^2\right)\right)},\rangle$$

$$\left(\widetilde{e_1'}\widetilde{e_2'}, \widetilde{f_1'}\widetilde{f_2'}, \widetilde{g_1'}\widetilde{g_2'}\right)$$

2.4 *Definition:*

Let $\widetilde{A^{-1}} = \{\langle(\widetilde{a'}, \widetilde{b'}, \widetilde{c'}), (\widetilde{e'}, \widetilde{f'}, \widetilde{g'})\rangle\}$ be a TPFN, then the SF of $\widetilde{A^{-1}}$ is an PFS, whose values correspond to membership and non-membership,

$\widetilde{S'}(\widetilde{A^{-1}_{\mu'}}) = \frac{1}{4}(\widetilde{a'} + 2\widetilde{b'} + \widetilde{c'})$ and $\widetilde{S'}(\widetilde{A^{-1}_{\gamma'}}) = \frac{1}{4}(\widetilde{e'} + 2\widetilde{f'} + \widetilde{g'})$.

3. Network Terminology

Take the network that is directed $\widetilde{P'}(\widetilde{V'}, \widetilde{E'})$. composed of a set of m linear edges. $\widetilde{E'} \subseteq \widetilde{V'} \times \widetilde{V'}$ and a finite number of nodes $\widetilde{V'} = \{1,2,3, \dots, n\}$. An ordered pair (i', j') is used to represent each edges, Where $(i', j' \in \widetilde{V'})$ and $i' \neq j'$. The SN and the DN in this network are designated by the letters s' & t', respectively. The respective DN, we specify a route, we take into consideration two nodes in this network s'& t', which stand for SN and the DNs, respectively. We build a path with alternating nodes and edges,

$\widetilde{P'_{ij}} = \{i' = i', (i'_1, i'_2), i'_2, \dots, (i'_{1-i}, i'), i'_i = j'\}$. For each value of $i' \in \widetilde{V'} - \{S'\}$, it is assumed that 3 is at least 1 path $\widetilde{P'_{S\iota}}$ in $\widetilde{P'}(\widetilde{V'}, \widetilde{E'})$. Pythagorean Distance over the Path $\widetilde{P'}$ is indicated as $d(\widetilde{P'})$, which is defined as $d(\widetilde{P'}) = \sum_{(i', j' \in \widetilde{P'})} d_{i'j'}$. $d_{i'j'}$ signifies TPFN linked with edge. (i', j'), equating the length required to navigate (i', j') from i' to j'. Figure 1 shows Triangular Pythagorean Fuzzy shortest Hyper path network. In Table 1, TriPF membership values are given.

3.1 *Note*

If a node i' and node j' are directly connected, the node i' is considered to be preceding the node j'. The direction of the path from node i' to node j'.

4. Triangular Pythagorean Fuzzy Shortest Hyper (TPFSH) path problem:

In this study, the length of an edge in a network is referred to as a PN, specifically TPFN. The SP algorithm contains 8 steps.

STEP – 1: Assume that $\widetilde{n_1} = \langle(0,0,0), (1,1,1)\rangle$ and name the SN (let's Say node 1) as $\widetilde{n_1} = [\langle(0,0,0), (1,1,1)\rangle, -]$.

STEP – 2: Determine $\widetilde{n_{j'}} = \min\{\widetilde{n_{j'}} \oplus \widetilde{n_{i'j'}}\}, j' = 2,3,4,5, \dots, n$.

STEP – 3: Label nod j' as $[\widetilde{n_{j'}}, r]$ if min occurs and corresponds to singular value of i', i.e., $i' = r$. If the smallest value occurs at numerous values of i' it means that there are multiple values of i' which means that there are many interval – valued functions. Choose any value of i' for the Pythagorean Path (PP) between the SN and node j', but the Triangular Pythagorean Fuzzy Hyper (TPFH) distance along the path is $\widetilde{n_{n'}}$.

STEP – 4: If label for the destination node (node n) is $[\widetilde{n_{n'}}, l]$. The TPFSH Distance between the SN is then $\widetilde{n_{n'}}$.

STEP – 5: Since the DN is marked as $[\widetilde{n_{n'}}, l]$. Analyse node one's label to determine the TPFSH difference between SN and DN.. Let's say it is $[\widetilde{n_{n'}}, p]$, then we will examine the label of node p, and so forth. Up till node one is obtained, repeated the same method.

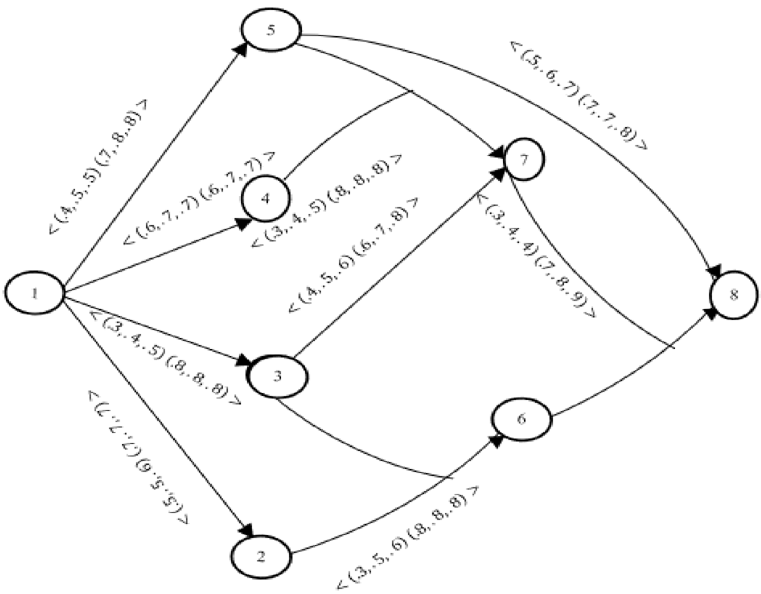

Figure 1

STEP – 6: The TPFSH path can now be created by joining all of the nodes that were produced in step – 5**.**

Remark: Given a set of TPFH numbers $\widetilde{A'_{i'}} = \{1,2,3,4, \dots, n\}$ if $\widetilde{S'}(\widetilde{A'_k}) < \widetilde{S'}(\widetilde{A'_{i'}})$, then for all i' the TriPFH the number corresponds to the minimum of $\widetilde{A'_k}$.

4.1 *Justification*

Letting $n = 8$ be the definition of the DN for node 8. Now let us take care of the beginning distance, which is $\widetilde{n'_1} = \langle(0,0,0)(1,1,1)\rangle$, also defined node 1 for $\left(\langle(0,0,0)(1,1,1)\rangle, \widetilde{\wp'}(G) = --\right)$ by that of the SN. So the network terminology can be used to assign the values of $\widetilde{\wp'_j}; j' = 2,3,4, \dots, 8$.

Table 1
TriPF membership values.

Paths	TriPF Hyper graphs
(1 – 2)	$\langle(.5,.5,.6)\ (.7,.7,.7)\rangle$
(1 – 3)	$\langle(.3,.4,.5)\ (.8,.8,.8)\rangle$
(1 – 4)	$\langle(.6,.7,.7)\ (.6,.7,.7)\rangle$
(1 – 5)	$\langle(.4,.5,.5)\ (.7,.8,.8)\rangle$
(2 – 6)	$\langle(.3,.5,.6)\ (.8,.8,.8)\rangle$
(3 – 6)	$\langle(.3,.5,.6)\ (.8,.8,.8)\rangle$
(3 – 7)	$\langle(.4,.5,.6)\ (.6,.7,.8)\rangle$
(4 – 7)	$\langle(.3,.4,.5)\ (.8,.8,.8)\rangle$
(5 – 7)	$\langle(.3,.4,.5)\ (.8,.8,.8)\rangle$
(6 – 8)	$\langle(.3,.4,.4)\ (.7,.8,.9)\rangle$
(7 – 8)	$\langle(.3,.4,.4)\ (.7,.8,.9)\rangle$
(5 – 8)	$\langle(.5,.6,.7)\ (.7,.7,.8)\rangle$

Iteration I: Enter the values of $i' = 1\ \&\ j' = 2$, suitably in step – 2 being aforementioned process, assuming the PN 2 and is node 1.

From of the $\widetilde{n_2'}$ path is,

$$\widetilde{n_2'} = \min\{\widetilde{n_1'} \oplus \widetilde{n_{12}'}\} = \min\{\langle(0,0,0)(1,1,1)\rangle \oplus \langle(.5,.5,.6)\ (.7,.7,.7)\rangle\}$$

$$\widetilde{n_2'} = \langle(.5,.5,.6)\ (.7,.7,.7)\rangle$$

According to the computation above, node 1 has a minimum and node 2 is taken to be $\langle(.5,.5,.6)\ (.7,.7,.7)\rangle$ with $\widetilde{\wp'}(G) = 1$.

Iteration II: Node 1 is PN for nod 3, so using the method described above, the distance between node 3 and the SN is,

$$\widetilde{n_3'} = \langle(.3,.4,.5)\ (.8,.8,.8)\rangle$$

Also, node 3 is, $\left(\widetilde{n_3'} = \langle(.3,.4,.5)\ (.8,.8,.8)\rangle, \widetilde{\wp'}(G) = 1\right)$.

Iteration III: Node 1 is PN for nod 4, so using the method described above, the distance between node 4 and the SN is,

$$\widetilde{n_4'} = \langle(.6,.7,.7)\ (.6,.7,.7)\rangle$$

Also, node 4 is, $\left(\widetilde{n_4'} = \langle(.6,.7,.7)\ (.6,.7,.7)\rangle, \widetilde{\wp'}(G) = 1\right)$.

Iteration IV: Node 1 is the PN for node 5, so using the method described above, the distance between node 5 and the SN is,

$$\widetilde{n_5'} = \langle(.4,.5,.5)\ (.7,.8,.8)\rangle$$

Also, node 5 is, $\left(\widetilde{n_5'} = \langle(.4,.5,.5)\ (.7,.8,.8)\rangle, \widetilde{\wp'}(G) = 1\right)$.

Iteration V: Nodes 2 and 3 serve as the PNs for node 6, after which the value of $\widetilde{n_6'}$ is provided

$$\widetilde{n_6'} = \min\{\widetilde{n_2'} \oplus \widetilde{n_{26}'}, \widetilde{n_3'} \oplus \widetilde{n_{36}'}\}$$

$$= \min \begin{matrix}\{(\langle(.5,.5,.6)\ (.7,.7,.7)\rangle \oplus \langle(.3,.5,.6)\ (.8,.8,.8)\rangle), \\ (\langle(.3,.4,.5)\ (.8,.8,.8)\rangle \oplus \langle(.3,.5,.6)\ (.8,.8,.8)\rangle)\}\end{matrix}$$

The following value for $\widetilde{n_6'}$ is obtained by using the ranking approach (based on the SF)

$$\widetilde{n_6'} = \min\{\langle(.382,.610,.770)(.64,.64,.64)\rangle\}$$

$$\widetilde{n_6'} = \langle(.382,.610,.770)(.64,.64,.64)\rangle$$

Also, node 6 is, $\left(\widetilde{n_6'} = \langle(.382,.610,.770)(.64,.64,.64)\rangle, \widetilde{\wp'}(G) = 3\right)$.

Iteration VI: Nodes 3, 4 and 5 are the PNs for node 7, hence the value of $\widetilde{n_7'}$ is provided

$$\widetilde{n_7'} = \min\{\widetilde{n_3'} \oplus \widetilde{n_{37}'}, \widetilde{n_4'} \oplus \widetilde{n_{47}'}, \widetilde{n_5'} \oplus \widetilde{n_{57}'}\}$$

The result of the rank – order method for $\widetilde{n_7'}$ is,

$$\widetilde{n_7'} = \langle(.568,.702,.760)(.56,.64,.64)\rangle$$

Also, node 7 is, $\left(\widetilde{n_7'} = \langle(.568,.702,.760)(.56,.64,.64)\rangle,\ \widetilde{\wp'}(G) = 5\right)$.

Iteration VII: Nodes 5, 6 and 7 are the PNs for node 8, hence the value of $\widetilde{n_8'}$ is provided

$$\widetilde{n_8'} = \min\{\widetilde{n_5'} \oplus \widetilde{n_{58}'}, \widetilde{n_6'} \oplus \widetilde{n_{68}'}, \widetilde{n_7'} \oplus \widetilde{n_{78}'}\}$$

The result of the rank – order method for $\widetilde{n_8'}$ is,

$$\widetilde{n_8'} = \langle(.459,.710,.835)(.448,.512,.576)\rangle$$

Also, node 8 is, $\left(\widetilde{n_8'} = \langle(.459,.710,.835)(.448,.512,.576)\rangle,\ \widetilde{\wp'}(G) = 6\right)$.

The SP is $\mathbf{1} \to \mathbf{3} \to \mathbf{6} \to \mathbf{8}$, according to the calculations above, and the separation between the SN and DNs is, $\langle(.459,.710,.835)(.448,.512,.576)\rangle$.

The SP obtained by each node from the SN using the suggested approach is shown in the Table 2:

5. Conclusion

The study explores Pythagorean fuzzy shortest hyper paths in networks using triangular Pythagorean fuzzy numbers. It introduces a score-based algorithm that assigns weights to hyper edges, simplifying computation without compromising accuracy. The research contributes to fuzzy logic and hypergraph theory, and has practical implications for applications like circuit design and transportation logistics. The algorithm's performance is compared to existing methods, demonstrating superior accuracy and computational efficiency. The findings have practical implications for real-world scenarios.

Table 2
SP and SD.

NODES	$\widetilde{n_j'}$	SP between j^{th} node & the SN
2	$\langle(.5,.5,.6)\ (.7,.7,.7)\rangle$	$1 \to 2$
3	$\langle(.3,.4,.5)\ (.8,.8,.8)\rangle$	$1 \to 3$
4	$\langle(.6,.7,.7)\ (.6,.7,.7)\rangle$	$1 \to 4$
5	$\langle(.4,.5,.5)\ (.7,.8,.8)\rangle$	$1 \to 5$
6	$\langle(.382,.610,.770)(.64,.64,.64)\rangle$	$1 \to 3 \to 6$
7	$\langle(.568,.702,.760)(.56,.64,.64)\rangle$	$1 \to 5 \to 7$
8	$\langle(\mathbf{.459},\mathbf{.710},\mathbf{.835})(\mathbf{.448},\mathbf{.512},\mathbf{.576})\rangle$	$\mathbf{1 \to 3 \to 6 \to 8}$

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