

Odd vertex even edge root square mean labelling graphs

V. Senthilkumar *

Department of Mathematics

Cardamom Planters Association College

Bodinayakanur 625582

Tamil Nadu

India

K. Venkatesan

Department of Mathematics

Sri Malolan College of Arts and Science

Madurantagam 603306

Tamil Nadu

India

Abstract

Let G be a graph with p -vertices and q edges. A graph G is said to be odd vertex even edge root square mean labelling if there exist an injective map $f : V(G) \rightarrow \{1, 2, 3, \dots, 2q+1\}$ such that the induced edge labels are even and distinct which can be obtained by $f^*(e) = \lfloor \sqrt{(f(u)^2 + f(v)^2)/2} \rfloor$ or $\lceil \sqrt{(f(u)^2 + f(v)^2)/2} \rceil$. A graph that allows such labelling is called an odd vertex even edge root square mean labelling graph. This paper introduces this new labelling type and discusses several related theorems concerning path graphs.

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1. Introduction

In this paper, consider only a simple and connected graph. All the terminology of the graph referred in [1-4]. Labelling is assigning members to the vertex set and edge set of graph based on the mathematical condition. Graph labelling were first introduced in the late 1960. The concepts of mean labelling,

* E-mail: 1984senthilvkumar@gmail.com

root square mean labelling and super root square mean labelling were discussed in [3, 4, 6,7, 8]. For a detail survey of graph labelling discussed by Gallian[1]. In this paper we introduce a new type of labelling called odd vertex even edge root square mean labelling and also discuss few theorem related to path graph. This work also will be extended to fuzzy graphs using reference [9-17]. Further we discuss a few family of graphs which obeys the above labelling patterns.

2. Preliminary

Definition 2.1 [5]: A graph G with p vertices and q edges, is a mean graph if there is an injective function f from the vertex set to $\{1, 2, 3, \dots, q\}$ when each edge uv is labelled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)+1$ is odd then the labelling edges are distinct.

Definition 2.2 [5]: A graph G with p vertices and q edges is called a Root Square Mean graph if is possible to label the vertices $v \in V$ with distinct labels $f(x)$ from $1, 2, 3, \dots, q+1$ in such a way that when each edge $e = uv$ is labelled with $f^*(e) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ or $\left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ then each labels are distinct.

Definition 2.3 [4]: The corona graphs is obtained by taking one copy of path P_n and n copies of K_1 then joining the i^{th} vertex of P_n with an edge to every vertex in the i^{th} copy of K_1 . It is denoted by $P_n \odot K_1$.

Definition 2.4 [6]: $TW(P_n)$ is a graph which is obtained from a path by identifying $k_{1,2}$ to all the vertices of the path except one pendent vertex(Twing graph)

A path with a least vertices is connected and has two terminal vertices while all other vertices have degree 2.

3. Main result on odd vertex even edge root square mean labelling

This section explores various standard graphs related to paths and their corresponding results.

Theorem 3.1: *A path P_n is an odd vertex even edge root square mean labelling graph.*

Proof: Let G be a Path P_n , where $V(G) = \{u_i : 1 \leq i \leq n\}$ and $E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1\}$ Clearly, G admits an odd vertices even edge root square mean labelling, since we can assign labels to the vertices and edges in a sequential manner. Therefore, G is classified as an odd vertex even edge root square mean graph.

Illustration: Figure 1 shows $G = P_5$ is odd vertex even edge root square mean labelling.



Figure 1

Odd vertex even edge root square mean labelling of P_5

Theorem 3.2: $P_n \odot K_1$ is an odd vertex even edge root square mean labelling graph.

Proof: Let $G = P_n \odot K_1$

The vertex set and edge set of G is defined as follows;

$$V(G) = \{ u_i, u'_i ; 1 \leq i \leq n \} \text{ and}$$

$$E(G) = \{ u_i u'_i ; 1 \leq i \leq n \text{ and } u'_i u'_{i+1} : 1 \leq i \leq n-1 \}$$

We define a map $f: V(G) \rightarrow \{1, 2, 3, \dots, 2n+1\}$.

The labelling pattern of vertices can be defined as follows;

$$f(u_i) = \begin{cases} 4i - 1 ; i \leq n \text{ and } i \text{ is even} \\ 4i - 3 ; 1 \leq i \leq n \text{ and } i \text{ is odd} \end{cases}$$

$$f(u'_i) = \begin{cases} 4i - 1 ; i \leq n \text{ and } i \text{ is odd} \\ 4i - 3 ; 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

Then the induced edge labels are

$$f'(u_i u'_i) = 4i - 2 : 1 \leq i \leq n \text{ and}$$

$$f'(u'_i u'_{i+1}) = 4i : 1 \leq i \leq n-1$$

Thus $G = P_n \odot K_1$ admit odd vertex even edge root square mean labelling graph.

Illustration: Figure 2 illustrate the $G = P_4 \odot K_1$ is odd vertex even edge root square mean labeling

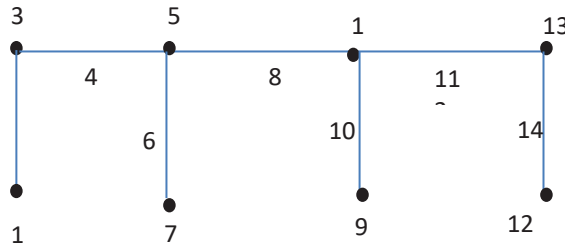


Figure 2
 $G = P_4 \odot K_1$

Theorem 3.3: $P_n \odot K_{1,2}$ is an odd vertex even edge root square mean labelling graph.

Proof: Graph $P_n \odot K_{1,2}$ is obtained by a path on n vertices attaching $K_{1,2}$ to each vertex of P_n .

Let $G = P_n \odot K_{1,2}$

Let $V(G)$ and $E(G)$ be the vertex and edge set of G are defined as follow

$$V(G) = \{u_i, u'_i \text{ and } u_i^2 : i \leq i \leq n\} \text{ and}$$

$$E(G) = \{u_i u_i^1, u_i u_i^2 : i \leq i \leq n \text{ and } u_i u_{i+1} : 1 \leq i \leq n-1\}$$

We define a map $f: V(G) \rightarrow \{1, 2, \dots, 2q-1\}$ by $f(u_i) = 6i - 3 : i \leq i \leq n$

$$f(u'_i) = 6i - 5 : i \leq i \leq n \text{ and } f(u_i^2) = 6i - 1 : i \leq i \leq n.$$

Then the included edge labels are $f'(u_i u_{i+1}) = 6i : 1 \leq i \leq n-1$

$$f'(u_i u'_i) = 6i - 4 : 1 \leq i \leq n \text{ and } f'(u_i u_i^2) = 6i - 2 : 1 \leq i \leq n.$$

Hence $G = P_n \odot K_{1,2}$ is an odd vertex even edge root square mean labelling graph.

Illustration: Figure 3 illustrate the $G = P_3 \odot K_{1,2}$ is odd vertex even edge root square mean labeling

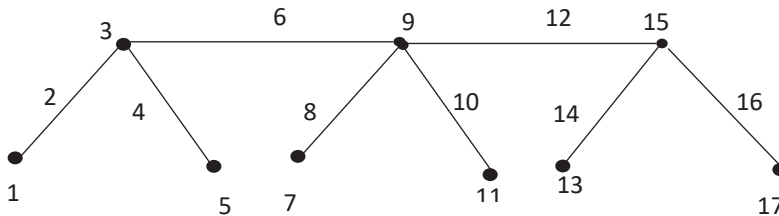


Figure 3
 $G = P_3 \odot K_{1,2}$

Theorem 3.4: The graph SL_n is an odd vertex even edge mean square labelling graph.

Proof: Let $G = SL_n$

Let the vertex set $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and edge set

$$E(G) = \begin{cases} u_i u_{i+1} : 1 \leq i \leq n-1 \\ v_i v_{i+1} : 1 \leq i \leq n-1 \\ v_{i+2} v_i : 2 \leq i \leq n-1 \end{cases}$$

Define a map $f: V(G) \rightarrow \{1, 2, \dots, 2q-1\}$ by $f(u_1) = 1$, $f(v_n) = 2q + 1$, further

$$f(u_i) = 6i - 9 : 2 \leq i \leq n, f(v_i) = 6i - 1 : 1 \leq i \leq n-1$$

Then the induced edge labels are defined as follows;

$$f'(v_i v_{i+1}) = 6i + 2 : 1 \leq i \leq n-2,$$

$$f'(v_{n-1}v_n) = 2q - 1: 1 \leq i \leq n - 2 \text{ and} \\ f'(u_{i+1}v_i) = 6i - 2: 1 \leq i \leq n - 1.$$

Hence SLn admits odd vertex even edge mean square labelling graph.

Theorem 3.5: Let G_p be a graph which is obtained from a path by attaching a pendent edge to each internal vertices of the path. Then G_p is an odd vertex even edge root mean square labelling graph.

Proof: Let $V(G) = \{u_i: 1 \leq i \leq n \text{ and } u'_i: 2 \leq i \leq n - 1 \text{ and}$

$$E(G) = \begin{cases} u_i u_{i+1}: 1 \leq i \leq n - 1 \\ u_i u'_i: 2, 3, 4, 5 \dots n - 2 \end{cases}$$

be the vertex set and edge set of G respectively.

Define a map $f: V(G) \rightarrow \{1, 3, \dots, 2q + 1\}$ by $f(u)$ where $f(u)$ are discussed in the following two cases.

Case 1: (for n is even)

If i is odd then $f(u_i) = 4i - 3$ and $f(u'_i) = 4i - 5: i = 3, 5, \dots$

If i is even then $f(u_i) = 4i - 5$ and $f(u'_i) = 4i - 3: i = 2, 4, \dots$

Then the induced edge labels are defined as follows :

$$f'(u_i u'_i) = 4i - 4: i = 2, 3, \dots, n - 1 \text{ and } f'(u_i u_{i+1}) = 4i - 2: i = 1, 2, 3, \dots, n - 1$$

Case 2: (for n is odd)

If i is odd then $f(u_i) = 4i - 3, i = 1, 3, \dots, n$ and $f(u'_i) = 4i - 5, i = 3, 5, \dots, n - 2$

If i is even then $f(u_i) = 4i - 5, i = 2, 4, \dots, n - 1$ and $f(u'_i) = 4i - 3, i = 2, 4, \dots, n - 1$.

Then the induced edge labels are defined as follows;

$$f'(u_i u'_i) = 4i - 4: i = 2, 4, \dots, n - 1 \text{ and } f'(u_i u_{i+1}) = 4i - 2: i = 1, 2, 3, \dots, n - 1.$$

Hence f is an odd vertex even edge root square mean labelling of graph G .

Therefore, G is an odd vertex even edge root square mean labelling of graph with respect to f .

Theorem 3.6: $TW(P_n)$ is an odd vertex even edge root square mean labelling of graph.

Proof: Let $G = TW(P_n)$

$$\text{Let } V(G) = \begin{cases} u_i & : 1 \leq i \leq n \\ u'_i \text{ and } u_i^2 & : 2 \leq i \leq n \end{cases} \text{ and}$$

$$E(G) = \begin{cases} u_i u_{i+1} & : 1 \leq i \leq n - 1 \\ u_i u'_i \text{ and } u_i u_i^2 & : 2 \leq i \leq n \end{cases}$$

be the vertex set and edge set of G respectively. We define a map $f: V(G) \rightarrow \{1, 3, \dots, 2q - 1\}$ by $f(u_1) = 1; f(u'_i) = 2(i + j) - 1, i = 2$ and $j = 1, 2; f(u'_i) = 6i - 5, i = 3, 4, \dots, n; f(u_i^2) = 6i - 9, i = 3, 4, \dots, n$

Then the induced edge labels are defined as follows;

$$f'(u_i u'_i) = 6i - 8, f'(u_i u''_i) = 6i - 6; i = 2, 4, \dots, n-1 \text{ and } f'(u_i u_{i+1}) = 6i - 4; i = 1, 2, 3, \dots, n-1.$$

Hence G is an odd vertex even edge root square mean labelling of graph since f is an odd vertex even edge root square mean labelling of graph G .

Illustration: Figure 4 illustrate the $G = TW(P_4)$ is odd vertex even edge root square mean labeling

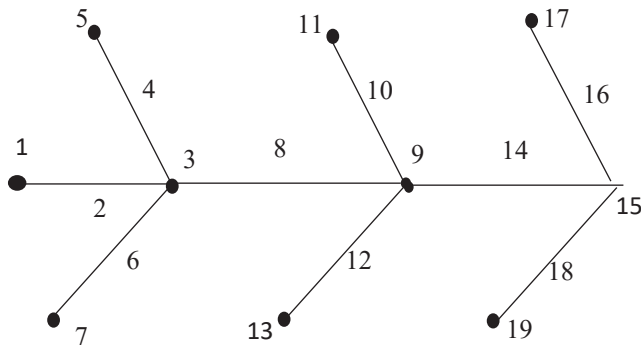


Figure 4
Graph $G = TW(P_4)$

Application:

Telecommunication Network:

Mean labelling in telecommunication network involves assigning labels to nodes and edge based on the average signal strength or latency between connected nodes. These labels efficient routing and helps in optimizing data transmission, ensuring smooth communication within the network.

Social network:

Mean labelling is utilized in social network analysis to assign labels to individual within a network based on the average strength of their connections or interactions. This aids in identifying key influencers or central figures in the network, which can be crucial for targeted Marketing or understanding social dynamics.

Transportation network:

In transportation network, mean labelling assigns labels to nodes and edges representing average travel times, congestion levels, or transportation costs. This assists in route planning, traffic management and optimizing systems for enhanced efficiency.

Bio-informatics:

Mean labelling is applied in biological networks like protein interaction networks. Labels are assigned to vertices or edges based on average properties such as interaction strength or similarity scores, adding in the analysis of complex biological systems and identifying biological elements.

Internet and Web graphs:

Mean labelling is used to analysis the structure and behaviour of the internet or wen pages. Labels may represent average page rank, connectively levels, or relevance scores, enabling the understanding of web page importance and helping search engine relevant results.

Image Processing:

In image processing, mean labelling assigns labels to pixels or regions based on the average pixel intensities or colour information. This is often used in image segmentation, where identifying regions with similar characters is crucial for various image analysis tasks.

Conclusion

The paper introduces a new type of graph labelling called “odd vertex even edge root square mean labelling”. This paper discusses theorems related to path graphs, a specific type of graphs. It explores and discusses families of graphs that conform to the proposed labelling patterns.

References

- [1] J.A.Gallian, ‘A dynamic survey of graphs labelling’,The electronic journal of combinatorics,18 (2015).
- [2] B.Gayathri and R.Gobi, ‘(k, d) even mean labelling of corona graphs, *International Journal of Mathematics and Soft Computing*, Vol.(1), 17-23, Aug (2011).
- [3] B.Gayathri and R.Gobi, ‘Necessary condition for mean labelling, *International Journal of Engineering Science, Advanced Computing and Bio Technology*’, Vol. 4(3), 43-52, July-Sep. (2013).
- [4] R.Ramdani, A.Y Fanisa, S. Gumilar, S Sabini, ‘Root square Mean labelling of some graphs obtained from paths ‘ *Journal of Physics Conference Series* (2021).
- [5] S.Somasundaram and R.Ponraj, ‘Mean labelling of graphs’, National academy science letters, 26, 210-213 (2003).

- [6] S. Sandiya, S. Somasundaram and S.Arun, 'Root square mean labelling graphs' *Internal Journal Contemporary Math. Science*, Vol.9(14), page no. 667-676 (2014).
- [7] K. Thirugnasambandam and K. Venkatesan, 'Super root square mean labelling of graphs', *International Journal of Mathematics and Soft Computing*, Vol. 5(2), page no. 189-198 (2025).
- [8] Senthilkumar.V, Ponnappan. C.Y., and Selvam. A., A note on Domination in fuzzy graph $G(\sigma, \mu)$ using Strong arc, *Journal of Science and Computation*, Vol. 5(6), page. no. 84-92 (2018).
- [9] Senthilkumar.V, Ponnappan. C.Y., Note on strong support vertex covering of fuzzy graph $G(\sigma, \mu)$ by strong arc, *Advances and applications in Mathematics*, Mili publication, Vol. 18, issue 11 (2019).
- [10] Senthilkumar.V, Ponnappan. C.Y., Introduction to strong support domination in fuzzy graphs by strong arc, *Advances in Mathematics: Scientific Journal* 8, no. 3, 724- 735 (2019).
- [11] Senthilkumar.V, Global strong domination in α - Cut diminish fuzzy graph by α - Cut strong arc. *Bulletin of pure and Applied Sciences-E Maths & Statistics*, Vol.41E (1), Page 81-87 June (2022).
- [12] Ponnappan, C.Y., Senthilkumar.V., Domination in Cartesian product fuzzy graphs by Strong arc, *Bulletin of pure and applied mathematics*, Vol(2), 37E (Math& Stat), 321-336 (2018).
- [13] Ponnappan, C.Y., Senthilkumar.V., Case study of non- strong arc in Cartesian product fuzzy graphs, *Annls of Pure and Applied Mathematics*, Vol. 18, no 1, 15-25, July (2018).
- [14] Ponnappan, C.Y., Senthilkumar.V., Study of Domination in (Union and Join) of fuzzy graphs, *Mathematical Sciences International Research Journal*, volume 7, issue 2, IMRF journal publication, Sep. (2018).
- [15] Senthilkumar. V., Types of Domination in Intuitionistic fuzzy graph by strong arc an effective arc. *Bulletin of pure and applied mathematics*, Vol (2), 37E (Math& Stat), 490-498 (2018).
- [16] Ponnappan, C.Y., Senthilkumar., A selvam , A note on intuitionistic fuzzy graph by strong arc, *JASC*, Vol 5 (3) 957-968, March (2019).
- [17] Senthilkumar.V, Ponnappan. C.Y, Independent strong support domination & Independent anti – strong support domination in fuzzy graphs by strong Arc, *Advances in Mathematics : Scientific Journal* 8, no. 3, 714-723 (2019).