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# Nano (1, 2, 3) generalized closed sets in nano tritopological space

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**Abstract**

This Paper work entitled "Nano (1,2,3) Generalized Closed Sets in Nano Tritopological Space" introduces a new class of sets in Nanotritopological Space and some of its developments are discussed.

***Subject Classification:*** *05C10, 55M15, 55R37, 57Q99.*

***Keywords:*** $N_{(1,2,3)\,g}$ *closure,* $N_{(1,2,3)\,g}$ *interior,* $N_{(1,2,3)\,g}$ *Closed.*

## 1. Introduction

In, 1970 N.Levine [5] significant understanding Generalized closed sets in topological structure. Bhuvaneswari [1],[2],[3] has been initiated and explore Nano Generalized closed set in Nano Topology. Martin Kovar[6] is credited with pioneering the investigating of Tritopological space. Palaniammal and Hameed [4] examined disunion analyzed the foundational rules in tri topological spaces and provided the definitions for 1,2,3 open sets within this framework. Tapi U.D. et al. [7] [8] discuss the introduction of the application to tri open sets and outline some of their fundamental properties. Zaman T. Sameer [9] introduce concepts related to the v-open set, including v-closure, v-interior, and v-exterior. This paper delves into the introduction and exploration of a novel category of functions termed Nano (1,2,3) generalized closed functions (abbreviated as $N_{(1,2,3)}$gcf) within the context of Nano Tri Topological spaces, and further examines their advancements and connections with established functions.

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## 2. Preliminaries

**Definition 2.1:** Let (U, $\tau_{R_{1,2}}(X)$) be a Nano bitopological space. A subset A of (U, $\tau_{R_{1,2}}(X)$) is called Nano (1,2)* generalized closed set if $N_{\tau_{1,2}}$Cl(A) where A⊆V and V is Nano (1,2)* open.

**Definition 2.2:** A subset A of a space X is called **123 open** set if A$\in \tau_1 \cup \tau_2 \cup \tau_3$and the complement of 123 open set is 123 closed set .

## 3. Nano (1, 2,3) Generalized Closed Sets in Nano Tri Topological Space

In this paper we define a novel category for a sets known as Nano (1,2,3) generalized closed sets in Nano Tritopological spaces & thoroughly examine the various properties related to this category.

**Definition 3.1:** Let (P, $N_{\tau_{(1,2,3)}}(X)$) and (Q, $N_{\tau'_{(1,2,3)}}(Y)$) be two Nano Tritopological spaces. A mapping f: (P, $N_{\tau_{(1,2,3)}}(X)$) → (Q, $N_{\tau'_{(1,2,3)}}(Y)$) is Nano (1,2,3)generalizedclosedsets (shortly $N_{\tau_{(1,2,3)}}gc(X)$) function on P if it satisfies the condition that the preimage of every Nano (1,2,3) closed set in (Q, $N_{\tau'_{(1,2,3)}}(Y)$) is N(1, 2,3) closed in (P, $N_{\tau_{(1,2,3)}}(X)$).

**Example 3.2:** U = {11, 13, 15, 17}, U/R$_1$ = {{11}, {13}, {15,17}}, X$_1$ = {11,15}, $\tau_{R_1}$={$\cup, \emptyset$,{11},{15, 17}, {11,15,17}}, U/R$_2$ = {{15}, {17}, {11,13}}, X$_2$ = {13, 17}, $\tau_{R_2}$= {$U, \emptyset$, {17}}, {11,13}, {11,13,17}, U/R$_3$ = {13} {15} {11,17}}, X$_3$ = {11,13}, $\tau_{R_3}$={, $\emptyset$, {15}, {11,17}, {11,13,17}}, $\tau_{R_{(1,2,3)}}(X)$ = {$U, \emptyset$,{11}, {13}, {17}, {11,13}, {11,17}, {13,17}, {11,13,17}, {11,15,17}}. Nano (1, 2, 3) closedsets = {$U, \emptyset$, {13}, {15}, {11,13}, {13,15}, {15,17}, {11,13,15}, {11,15,17}, {13,15,17}}. Nano (1,2,3) generalized closedsets R = {U,∅,{11}, {13}, {15}, {17}, {11,13}, {11,15}, {11,17}, {13,15}, {13,17}, {15,17}, {11,13,15}, {11,13,17}, {11,15,17}, {13,15,17}}.

**Theorem 3.3**: *If both P & Q are N(1,2,3) generalized closed set, then the union of P& Q forms another N(1,2,3)generalized closedset.*

***Proof:*** Let the closure of P union Q is the union of the closures of P and Q, both contained within Y, N(1,2,3)generalizedclosedsets. Then $N_{\tau_{(1,2,3)}}cl(P) \subseteq Y$ where P Subset Y and Y is N(1,2,3) open and $N_{\tau_{(1,2,3)}}cl(P) \subseteq Y$ where Q Subset Y and Y is N(1,2,3) open. Then $N_{\tau_{(1,2,3)}}cl(P \cup Q) = N_{\tau_{(1,2,3)}}cl(P) \cup N_{\tau_{(1,2,3)}}cl(Q) \subseteq Y$. Which implies that the set P union Q is also a N(1,2,3) generalized closed set.

**Theorem 3.4:** *If P is a closed set in N(1,2,3) (U, $N_{\tau_{(1,2,3)}}(X)$), then P is also considered a Generalized closed set under the N(1,2,3) confirms that P is indeed a closed set in this space.*

***Proof:*** let P is a closed set in the N(1,2,3), then demonstrating that the closure of the interior of A under the N(1,2,3) is a subset of P. $N_{\tau_{(1,2,3)}}Cl(A)) = P$. To prove that $N_{\tau_{(1,2,3)}}Cl(N_{\tau_{(1,2,3)}}Int(A)) \subseteq P$ is indeed a closed set in this space that P is a N(1,2,3) closed set. $N_{\tau_{(1,2,3)}}Cl(N_{\tau_{(1,2,3)}}Int(A)) = N_{\tau_{(1,2,3)}}Int(A) \subseteq P$. Hence P is a N(1,2,3) closed set.

**Example 3.5:** U = {5,6,7,8}, U/$R_1$ = {{5}, {8}, {6,7}}, $X_1$ = {5,6}, $\tau_{R_1}$ = { $U$, $\emptyset$, {5}, {5,6,7}, {6,7}}, U/$R_2$ = {{6}, {7}, {5,8}}, $X_2$ = {7,8}, $\tau_{R_2}$ = {$U$, $\emptyset$, {7}, {5,7,8}, {5,8}}, U/$R_3$ = {{5} {6} {7,8}}, $X_3$ = {6,7}, $\tau_{R_3}$ = {$U$, $\emptyset$,{6}, {6,7,8}, {7,8}}. N(1,2,3) closedset = {$U$,$\emptyset$, {5}{6}, {8}, {5,6}, {5,8}, {6,7}, {5,6,8}, {5,7,8}, {6,7,8}}. Nano (1, 2, 3) generalized closed set = {$U$,$\emptyset$, {5} {6}, {7},{8}, {5,6}, {5,7}, {5,8}, {6,7}, {6,8}, {7,8}, {5,6,7}, {5,6,8}, {5,7,8}, {6,7,8}}

**Theorem 3.6:** *Prove that when P is identified as a closed set within the N(1,2,3) (U, $N_{\tau_{(1,2,3)}}(X)$), it follows that P also qualifies as a generalized closed set under the N (1,2,3).*

***Proof:*** Given that P is a closed set under the N(1,2,3) in U and is contained within an open set V, which is also open under N(1,2,3) in U. Since P is N(1,2,3) closed. $N_{\tau_{(1,2,3)}}Cl(A)) = P \subseteq V$.That is $N_{\tau_{(1,2,3)}}Cl(A) \subseteq N_{\tau_{(1,2,3)}}Cl(A)) \subseteq V$, it can be concluded that P qualifies as a N(1,2,3)generalized closed set.

**Theorem 3.7:** *Combining two N(1,2,3) generalized closed sets through an infinitary union in the N(1,2,3) (U, $N_{\tau_{(1,2,3)}}(X)$) results in yet another N(1,2,3) generalized closedset within the same space.*

***Proof:*** Given two N(1,2,3) generalized closed sets P and Q in the Nano tritopological space (U, $N_{\tau_{(1,2,3)}}(X)$) , which is also open under N(1,2,3) in U. Since P⊆V and Q⊆V. Then we have P Union Q Subset V . P and Q are N(1,2,3) generalized closed sets in (U, $N_{\tau_{(1,2,3)}}(X)$), $N_{\tau_{(1,2,3)}}Cl(P) \subseteq V$ and $N_{\tau_{(1,2,3)}}Cl(Q)$ ⊆V. Now $N_{\tau_{(1,2,3)}}pCl(P \cup Q)$ = $N_{\tau_{(1,2,3)}}pCl(P) \cup$ $N_{\tau_{(1,2,3)}}pCl(Q)$ ⊆V. Thus we have $N_{\tau_{(1,2,3)}}Cl(P \cup Q) \subseteq V$. ∃ an N(1,2,3) open set V in U contains both P and Q, then their union, P∪Q, is also a N(1,2,3)generalizedclosedset within (U, $N_{\tau_{(1,2,3)}}(X)$)

**Theorem 3.8:** *If P ⊆ (U, $N_{\tau_{(1,2,3)}}(X)$) and satisfies the condition that no nonempty N(1,2,3) generalized closed set exists within the difference $N_{\tau_{(1,2,3)}}$Cl(A)-A , then P qualifies as a N(1,2,3) generalized closed set within the same space.*

***Proof:*** Suppose A is a N(1,2,3)generalizedclosedset. If $N_{\tau_{(1,2,3)}}$Cl(A) is contained within a set P, where P is a subset of an open set V under N(1,2,3), and X is a closed subset of $N_{\tau_{(1,2,3)}}$Cl(A) $\subseteq P$, then it follows that P is contained within the complement of some set Y P$\subseteq$ $Y^c$ and $Y^c$ is N (1,2,3) open . Since P is a N(1,2,3)generalizedclosedset, the closure of A, $N_{\tau_{(1,2,3)}}$Cl(A) $\subseteq X^c$ or X $\subseteq \left[N_{\tau_{(1,2,3)}}Cl(A)\right]^C$. That is X $\subseteq N_{\tau_{(1,2,3)}}$Cl(A) and X$\subseteq \left[N_{\tau_{(1,2,3)}}Cl(A)\right]^C$ implies that X $\subseteq \emptyset$. So X is empty.

**Theorem 3.9:** *If P be a N(1,2,3) generalizedclosed $\subseteq$ (U, $N_{\tau_{(1,2,3)}}(X)$). If P$\subseteq Q \subseteq N_{\tau_{(1,2,3)}}Cl(P)$, then Q is also a N(1,2,3) generalizedclosed $\subseteq$ (U, $N_{\tau_{(1,2,3)}}(X)$)*

***Proof:*** Consider (U, $N_{\tau_{(1,2,3)}}(X)$) as a N(1,2,3) open set containing a N(1,2,3)generalizedclosed subset . If Q is a subset of U and P is contained within Q, then P is also contained within U. Given A is a N(1,2,3)generalizedclosedset, and $N_{\tau_{(1,2,3)}}Cl(P) \subseteq U$. Given Q$\subseteq N_{\tau_{(1,2,3)}}Cl(P)$ we have $N_{\tau_{(1,2,3)}}Cl(Q) \subseteq N_{\tau_{(1,2,3)}}Cl(P)$. We have $N_{\tau_{(1,2,3)}}Cl(Q) \subseteq U$, whenever Q$\subseteq U$, $N_{\tau_{(1,2,3)}}Cl(Q)$ is also within U, as U is N(1,2,3) open . This implies that Q is also a N(1,2,3)generalizedclosedsubset of $N_{\tau_{(1,2,3)}}(X)$.

**Example 3.10:** U = {5,6,7,8} U/$R_1$ = {{5}, {8}, {6,7}}, $X_1$ = {5,6}, $\tau_{R_1}$ = {∪, ∅, {5} {5,6,7} {6,7}}, U/$R_2$ = {{6}, {7}, {5,8}}, $X_2$ = {7,8}, $\tau_{R_2}$ = {∪, ∅, {7}, {5,7,8}, {5,8}}, U/$R_3$ = {{5} {6} {7,8}}, $X_3$ = {6,7}, $\tau_{R_3}$= {$U$, ∅,{6}, {6,7,8}, {7,8}}, $N_{\tau_{(1,2,3)}}(X)$ = {∪,∅,{5}, {6}, {7}, {5,8}, {6,7}, {7,8}, {5,6,7}, {5,7,8}, {6,7,8}}. N(1,2,3) closedsets = {U,ϕ,{5}, {6}, {8}, {5,6}, {5,8}, {6,7}, {5,6,8}, {5,7,8} {6,7,8},}. N(1,2,3) generalized closedsets R = { U, ∅,{5}, {6}, {7}, {8}, {5,6}, {5,7}, {5,8}, {6,7}, {6,8}, {7,8}, {5,6,7}, {5,7,8}, {5,6,8}, {6,7,8}}. Here{5,6}∩{5,7}∩{5,8}={5},{6,7}∩{6,8}={6},{5,6}{7,8}={7} which is again a N(1,2,3) generalizedclosedset.

**Theorem 3.11:** *Let P & Q be the subsets of a topological space (U, $N_{\tau_{(1,2,3)}}(X)$). If N(1,2,3)int(P) ⊆Q⊆P and if P is N(1,2,3)g-open, then Q is N(1,2,3)g-open*

***Proof:*** Let N(12,3)int(P) ⊆Q⊆P, then $A^c \subseteq B^c \subseteq$N(1,2,3)cl($A^c$) where $A^c$ is N(1,2,3) g-closed set. Hence $B^c$ is also N(1,2,3)g-closed set. Thus B is N(1,2,3) g-open set.

**Theorem 3.12:** *Given that P is a N(1,2,3) generalized closed set and it is contained within Q P⊆Q⊆ $N_{\tau_{(1,2,3)}}$Cl(P), then Q also qualifies as a N(1,2,3) generalized closed set.*

***Proof:*** If ⊆V, where V is an open set under N(1,2,3) in X, and if P⊆Q implies P⊆V. Since P is contained within V, and P is N(1,2,3) generalized closed, $N_{\tau_{(1,2,3)}}$Cl(P) ⊆V. Furthermore, if Q⊆ $N_{\tau_{(1,2,3)}}$Cl(P) implies $N_{\tau_{(1,2,3)}}$Cl(Q) ⊆ $N_{\tau_{(1,2,3)}}$Cl(P). Thus $N_{\tau_{(1,2,3)}}$Cl(Q) ⊆$V$ and so Q is $N_{\tau_{(1,2,3)}}$generalized closed set.

**Theorem 3.13:** *If P is a N(1,2,3)generalizedclosed ⊆ (U, $N_{\tau_{(1,2,3)}}(X)$), & $Q$ ⊆$P$ which itself is a subset of U, then Q is also a N(1,2,3)generalizedclosedset relative to P and hence to U as well.*

***Proof:*** Let Q⊆V, where V is a set open under N(1,2,3) in U, Then Q⊆P∩V. Consequently, $N_{\tau_{(1,2,3)}}$Cl(P)(Q) ⊆ P∩V. It follows that P∩ $N_{\tau_{(1,2,3)}}$Cl(Q). Since P is $N_{\tau_{(1,2,3)}}$g closed in U. we have $N_{\tau_{(1,2,3)}}$Cl(P) ⊆VU$N_{\tau_{(1,2,3)}}$Cl(Q) and so $N_{\tau_{(1,2,3)}}$Cl(Q) ⊆ $V$. Then Q is N(1,2,3) generalized closed relative to V.

**Theorem 3.14:** *If P is a subset of the Nano tritopological space (U, $N_{\tau_{(1,2,3)}}(X)$) and is classified as a N(1,2,3) closed set, then it follows that P also qualifies as a N(1,2,3) generalized closed set within the same space.*

***Proof:*** Suppose P is a N(1,2,3) closed set in the Nano tritopological space (U, $N_{\tau_{(1,2,3)}}(X)$) and P ⊆ $U$, U is N(1,2,3) an open in U. Since P is N(1,2,3) closed. $N_{\tau_{(1,2,3)}}Cl(A) = P \subseteq G$. That is $N_{\tau_{(1,2,3)}}Cl(A) \subseteq G$. Where G is N(1,2,3) generalized pre closed set.

**Theorem 3.15:** *If a subset P of (U, $N_{\tau_{(1,2,3)}}(X)$) is classified as a N(1,2,3) generalizedclosedset and it is contained within Q, which itself is contained within the closure of P, then Q also qualifies as a N(1,2,3) generalizedclosedset within the same space.*

***Proof:*** Consider P as a N(1,2,3) generalizedclosedset such that it is contained within Q, which in turn is contained within the closure of A. If Q⊆P where G is N(1,2,3) open in $N_{\tau_{(1,2,3)}}(X)$. Then P⊆Q implies P⊆G. Since P is N(1,2,3) g-closed, N(1,2,3)cl(A) ⊆$G$. Additionally, if Q⊆N(1,2,3)cl(A) implies N(1,2,3)cl(Q) ⊆N(1,2,3)cl(P). Thus N(1,2,3)cl(B) ⊆G and Q is N(1,2,3) g-closed in (U, $N_{\tau_{(1,2,3)}}(X)$).

**Example 3.16:** U = {11, 13, 15, 17}, U/$R_1$ = {{11}, {13}, {15,17}}, $X_1$ = {11,15}, $\tau_{R_1}$= {U, ∅, {11}, {15, 17}, {11,15,17}}, U/$R_2$ = {{15}, {17}, {11,13}}, $X_2$ = {13,17}, $\tau_{R_2}$ = {$U$, ∅, {17}}, {11,13}, {11,13,17}, U/$R_3$ = {13} {15} {11,17}}, $X_3$ = {11,13}, $\tau_{R_3}$= {, ∅, {15}, {11,17}, {11,13,17}}, $\tau_{R_{(1,2,3)}}(X)$ = {$U$, ∅, {11}, {13}, {17}, {11,13}, {11,17}, {13,17}, {11,13,17}, {11,15,17}}. Nano (1,2,3) closedsets = {$U$, ∅, {13}, {15}, {11,13}, {13,15}, {15,17}, {11,13,15}, {11,15,17}, {13,15,17}}. Nano (1,2,3) generalized closedsets R =

{U,∅,{11}, {13}, {15}, {17}, {11,13}, {11,15}, {11,17}, {13,15}, {13,17}, {15,17}, {11,13,15}, {11,13,17}, {11,15,17}, {13,15,17}}.

U = {5,6,7,8}, U/$R_1$ = {{5}, {8}, {6,7}}, $X_1$ = {5, 6}, $\tau_{R_1}$= {$U$,∅ , {5}, {5,6,7}, {6,7}}, U/$R_2$ = {{6}, {7}, {5,8}}, $X_2$ = {7,4}, $\tau_{R_2}$ = {$U$, ∅, {7}, {5,7,8}, {5,8}}, U/$R_3$ = {{5}{6}{7,8}}, $X_3$ = {6,7}, $\tau_{R_3}$ = {$U$, ∅, {6}, {6,7,8}, {7,8}}. N(1,2,3) closedset = {$U$,∅, {5}{6}, {8}, {5,6}, {5,8}, {6,7}, {5,6,8}, {5,7,8}, {6,7,8}}. Nano (1,2,3) generalized closed set Q = {$U$,∅, {5}{6}, {7}, {8}, {5,6}, {5,7}, {5,8}, {6,7}, {6,8}, {7,8}, {5,6,7}, {5,6,8}, {5,7,8}, {6,7,8}}. Let P = {U,∅,{11}, {13}, {15}, {17}, {11,13}, {11,15}, {11,17}, {13,15}, {13,17}, {15,17}, {11,13,15}, {11,13,17}, {11,15,17}, {13,15,17}}. Q = {$U$,∅, {5}{6}, {7}, {8}, {5,6}, {5,7}, {5,8}, {6,7}, {6,8}, {7,8}, {5,6,7}, {5,6,8}, {5,7,8}, {6,7,8}}. Then P and Q are N(1,2,3) g- closed sets in (U, $N_{\tau_{(1,2,3)}}(X)$) such that P⊆Q. Also N(1,2,3) cl(P) = {$U$, ∅,{13}, {15}, {11,13}, {13,15}, {15,17}, {11,13,15}, {11,15,17}, {13,15,17}}. Therefore P⊆Q ⊄N(1,2,3) cl(A).

**Theorem 3.17:** *If P ⊆U is N(1,2,3)g-open ⇔Q⊆N(1,2,3)int(P) ⇒ Q is N(1,2,3)closed and Q⊆P.*

***Proof:*** Suppose P is a N(1,2,3) generalized open set, and if P is contained within U where it is also N(1,2,3) closed, then the complement of P in U, denoted as U-P, forms a N(1,2,3) generalized closed set which is contained within an open set U-Q. Consequently, the closure of U-P is contained within U-Q, and U minus the interior of A is contained within U-Q. This Q⊆N(1,2,3)int (P). Conversely, if Q is a N(1,2,3) closed set contained within the interior of A and also contained within P, then the complement of the interior of A in U, denoted as U-Nint(A), is contained within U-Q. Hence, the closure of U-P is contained within U-P, indicating that U-P is a N(1,2,3)generalizedclosedset and P is a N(1,2,3) generalized open set.

## Conclusion

This paper develops into the properties pertaining to Nano (1,2,3) generalizedclosedsets in Nano tritopological Space. Future research efforts are anticipated to explore various applications and extend these findings.

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