Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 27 (2024), No. 5, pp. 1185–1191
DOI : 10.47974/JIM-1951

Matrix representation of z-fuzzy soft β-covering based multigranulation fuzzy rough sets

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Abstract

In this research, fuzzy soft β-adhesion is used to generate six different kinds of z-fuzzy soft β-covering based multigranulation fuzzy rough sets (Z-FSβCBMFR). In order to speed up data processing, we explore matrix depiction of Z-FSβCBMFR.

Subject Classification: *03E72, 90B50.*

Keywords: *Fuzzy soft β-adhesion, Multigranulation fuzzy soft β-measure degree, Z-FSβCBMFR.*

1. Introduction

Fuzzy sets [11] was introduced to handle partial membership where every element is associated with a membership level that falls within the interval from 0 to 1. Rough set theory [9] is a formal methodology in data analysis, particularly useful when dealing with uncertainty. Soft set theory [8], first presented in 1999, designed to address uncertainties through a parametric perspective. Its boundaries are not fixed but depend on the parameters chosen 1. This flexibility allows soft set theory to be applied in various fields, including decision-making processes, where it can model preferences or choices that are not sharply defined in [6, 7].

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A soft covering set is a notion handled to manage uncertainty in data processing. It is an extension of classic rough set theory that uses a family of sets known as a covering to divide a universe of speech rather than equivalence relations. This hybrid approach is beneficial in areas like decision-making, pattern recognition, and data mining, where it's crucial to make informed decisions based on uncertain or incomplete data [2, 12].

Fuzzy soft rough sets integrate these three theories to handle situations where data is not only uncertain and vague but also inconsistent. It provides a richer framework for modeling complex systems where information is incomplete or imprecise in the context of approximations (from rough sets) [10].

Multigranulation rough set theory uses numerous granular structures to approximate upper and lower limits of a set. By selecting ideal granulations, multigranulation rough sets can increase the efficiency and quality of choices, notably in sectors like medical diagnosis and pattern recognition [4, 5]. Multigranulation fuzzy set theory is a sophisticated idea that combines the principles of fuzzy sets with multigranulation rough sets. It is intended to address ambiguity and vagueness in data by employing many granular levels, each represented by a fuzzy set. This method improves the capacity to analyse complicated systems in which information is fragmented and partial. Fuzzy β-neighbourhoods provide more precise approximations and decision-making, particularly in fields such as health and artificial intelligence [1].

2. Preliminaries

Definiton 2.1: [3] Let $\tilde{\mathcal{C}} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_l}\}$ with $\widetilde{C_i} \in F(\Omega)$ $(i = 1, 2, \ldots, l)$, a fuzzy covering of Ω, if $\left(\bigcup_{i=1}^{l} \widetilde{C_i}\right)(v) = 1$, for every $v \in \Omega$. Ma [3] devised a fuzzy β-covering by changing 1 with $\beta (0 < \beta \leq 1)$, we say $\tilde{\mathcal{C}} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_l}\}$ with $\widetilde{C_i} \in F(\Omega)$ $(i = 1, 2, \ldots, l)$, a fuzzy β-covering, if $\left(\bigcup_{i=1}^{l} \widetilde{C_i}\right)(v) \geq \beta$, for every $v \in \Omega$. Then $(\Omega, \tilde{\mathcal{C}})_\beta$ is known as fuzzy β-covering approximation space (FβCAS).

3. z-Fuzzy soft β-covering based multigranulation fuzzy rough set

Definition 3.1: Consider J = $(\Omega, \tilde{F}, B)_\beta$ as a FSβCAS over Ω. For any $\beta \in (0,1]$ and $M \in F(\Omega)$, we determine:

(1) The z-fuzzy soft optimistic multigranulation fuzzy rough lower approximation operator (Z-FSOMFRLAO) and the z-fuzzy soft optimistic multigranulation fuzzy rough upper approximation operator (Z-FSOMFRUAO) are defined as

$$L^{O}_{\sum_{k=1}^{n} F(B_k)}(M)(v) = \vee_{k=1}^{n} \wedge_{u\in\Omega} \{(1-\widetilde{SA}^{\beta}_{F(B_k)(v)}(u)) \vee M(u)\} \text{ and}$$

$$U^{O}_{\sum_{k=1}^{n} F(B_k)}(M)(v) = \wedge_{k=1}^{n} \vee_{u\in\Omega} \{(\widetilde{SA}^{\beta}_{F(B_k)(v)}(u)) \wedge M(u)\}, \text{ respectively.}$$

(2) The z-fuzzy soft ψ -optimistic multigranulation fuzzy rough lower approximation operator (Z- ψ -FSOMFRLAO) and the z-fuzzy soft ψ -optimistic multigranulation fuzzy rough upper approximation operator (Z- ψ -FSOMFRUAO) are defined as

$$L^{O,\psi}_{\sum_{k=1}^{n} F(B_k)}(M)(v) = \vee_{k=1}^{n} \wedge_{u\in\Omega} \{M(u) : \mathcal{D}^{\beta}_{F(B_k)}(v_i, v_j) > \psi\} \text{ and}$$

$$U^{O,\psi}_{\sum_{k=1}^{n} F(B_k)}(M)(v) = \wedge_{k=1}^{n} \wedge_{u\in\Omega} \{M(u) : \mathcal{D}^{\beta}_{F(B_k)}(v_i, v_j) > \psi\}, \text{ respectively.}$$

(3) The z-fuzzy soft $\mathcal{D}$ -optimistic multigranulation fuzzy rough lower approximation operator (Z- $\mathcal{D}$ -FSOMFRLAO) and z-fuzzy soft $\mathcal{D}$ -optimistic multigranulation fuzzy rough upper approximation operator (Z- $\mathcal{D}$ -FSOMFRUAO) are defined as

$$L^{O,\mathcal{D}}_{\sum_{k=1}^{n} F(B_k)}(M)(v) = \vee_{k=1}^{n} \wedge_{u\in\Omega} \{(1-\mathcal{D}^{\beta}_{F(B_k)(v)}(u)) \vee M(u)\} \text{ and}$$

$$U^{O,\mathcal{D}}_{\sum_{k=1}^{n} F(B_k)}(M)(v) = \wedge_{k=1}^{n} \vee_{u\in\Omega} \{(1-\mathcal{D}^{\beta}_{F(B_k)(v)}(u)) \vee M(u)\}, \text{ respectively.}$$

Here, $\widetilde{SA}^{\beta}_{F(B_k)(v)} = \{u \in \Omega : \widetilde{SN}^{\beta}_{F(B_k)(v)} = \widetilde{SN}^{\beta}_{F(B_k)(u)}\}$ is known as the fuzzy soft β -adhesion of v and $\mathcal{D}^{\beta}_{F(B_k)}(v,u) = \dfrac{\sum_{k=1}^{n}\left(\widetilde{SA}^{\beta}_{F(B_k)}(v) \wedge \widetilde{SA}^{\beta}_{F(B_k)}(u)\right)}{\sum_{k=1}^{n}\left(\widetilde{SA}^{\beta}_{F(B_k)}(v) \vee \widetilde{SA}^{\beta}_{F(B_k)}(u)\right)}$ is known as the multigranulation fuzzy soft β -measure degree.

4. Matrix representation of Z-FS β COMFRS

Definition 4.1: Consider $\tilde{F}=\{F(B_1),F(B_2),...,F(B_q)\}$ as a q-Fuzzy soft β-covering of Ω. Let $M_{\tilde{F}} = (m_{ij})_{p\times q} = (F(B_j)\ (v_i))_{p\times q}$, which is a matrix depiction of $\tilde{F}$ and the Boolean matrix $M_{\tilde{F}}^{\beta} = (r_{ij})_{p\times q}$ $\alpha\beta$-matrix representation of $M_{\tilde{F}}$ where

$$r_{ij}=\begin{cases}1, & F(B_j)(v_i)\ge\beta;\\ 0, & F(B_j)(v_i)<\beta.\end{cases}$$

Let $X=(x_{ij})_{p\times t}$ and $Y=(y_{jl})_{t\times q}$ be any two matrices. Then $M = X*Y = (m_{il})_{p\times q}$ and $N = X\cdot Y = (n_{il})_{p\times q}$ are given by $m_{il} = \wedge_{j=1}^{t}\ [(1-x_{ij}) \vee b_{jl}]$ and $n_{il} = \vee_{j=1}^{t}\ [(a_{ij}) \wedge b_{jl}]$, where $i=1,2,...,p$ and $l=1,2,...,q$.

Proposition 4.1: Consider a q-Fuzzy soft β-covering of Ω. Then

$$L^{O}_{\sum_{k=1}^{q}\tilde{F}(B_k)}(M)=\vee_{k=1}^{q}([(M^{\beta}_{\tilde{F}(B)}*M^{T}_{\tilde{F}(B)})\cap(M^{\beta}_{\tilde{F}(B)}*M^{T}_{\tilde{F}(B)})^{T}]*M),$$

$$U^{O}_{\sum_{k=1}^{q}\tilde{F}(B_k)}(M)=\wedge_{k=1}^{q}([(M^{\beta}_{\tilde{F}(B)}*M^{T}_{\tilde{F}(B)})\cap(M^{\beta}_{\tilde{F}(B)}*M^{T}_{\tilde{F}(B)})^{T}]\cdot M).$$

Proof: By using Definition 3.1, it is true.

Definition 4.2: Considering a q-Fuzzy soft β-covering of Ω. Let $\mathcal{D}^{\beta}_{F(B_k)}$ be the matrix depiction of $\mathcal{D}^{\beta}_{F(B_k)}\ (v_i,v_j)$. Let us also denote $(\mathcal{D}^{\beta}_{F(B_k)})^{\psi} = (r_{ij})_{p\times p}$ as the ψ-matrix representation of $\mathcal{D}^{\beta}_{F(B_k)}$ and is defined by,

$$r_{ij}=\begin{cases}1, & d_{ij}>\psi;\\ 0, & d_{ij}\le\psi.\end{cases}$$

Let $X=(x_{ij})_{p\times t}$, $Y=(y_{jl})_{t\times q}$ and $Z=(z_{ij})_{p\times t}$. Then $E = X\nabla Y = (e_{il})_{p\times q}$, $F = X\Delta Y = (f_{il})_{p\times q}$ and $G = X\oslash Z = (g_{ij})_{m\times l}$ are defined as $e_{il} = = \sum_{j=1}^{t}$ $[x_{ij}\wedge y_{jl}]$, $f_{il} = \sum_{j=1}^{t}\ [x_{ij}\vee y_{jl}]$, and $g_{ij}=\frac{x_{ij}}{z_{ij}}$, where $i=1,...,p$ and $j=1,...,q$. Thus, $\mathcal{D}^{\beta}_{F(B_k)}(v,u)$ can be calculated in the form of matrices.

Proposition 4.3: Consider a q-Fuzzy soft β-covering of Ω and $\psi \in [0,1]$. Then, $L^{O,\psi}_{F(B)}(M) = (\mathcal{D}^{\beta}_{F(B)})^{\psi} \triangleright M \odot 1_{p\times 1}$ and $U^{O,\psi}_{F(B)}(M) = (\mathcal{D}^{\beta}_{F(B)})^{\psi} \triangleleft M \circ 0_{p\times 1}$ where $\triangleright$ and $\triangleleft$ are given the first preference.

Proof: If $L^{O,\psi}_{F(B)}(M) = (x_i)_{p\times 1}, (\mathcal{D}^{\beta}_{F(B)})^{\psi} \triangleright M = (e_{ij})_{p\times p}$ and $(\mathcal{D}^{\beta}_{F(B)})^{\psi} = (y_{ij})_{p\times p}$ where

$$y_{ij} = \begin{cases} 1, & \mathcal{D}^{\beta}_{F(B)}(v,u) > \psi; \\ 0, & \text{otherwise.} \end{cases}$$

$$e_{ij} = \begin{cases} M(v_j), & y_{ij} = 1; \\ 1, & y_{ij} = 0. \end{cases} = \begin{cases} M(v_j), & \mathcal{D}^{\beta}_{F(B)}(v,u) > \psi; \\ 1, & \text{otherwise.} \end{cases}$$

Therefore,

$$\begin{aligned} L^{O,\psi}_{F(B)}(M)(v_i) &= x_i = \wedge_{j=1}^{p}[e_{ij} \wedge 1] = \wedge_{j=1}^{p} e_{ij}. \\ &= \wedge_{j=1}^{p}\left\{M(v_j) : \mathcal{D}^{\beta}_{F(B)}(v_i, v_j) > \psi\right\}. \end{aligned}$$

On the contrary, if $L^{O,\psi}_{F(B)}(M) = (x_i^1)_{p\times 1}, (\mathcal{D}^{\beta}_{F(B)})^{\psi} \triangleleft M = (e'_{ij})_{p\times p}$.

Hence,

$$e'_{ij} = \begin{cases} M(v_j), & y_{ij} = 1; \\ 0, & y_{ij} = 0. \end{cases} = \begin{cases} M(v_j), & \mathcal{D}^{\beta}_{F(B)}(v_i, v_j) > \psi; \\ 0, & \text{otherwise.} \end{cases}$$

Therefore,

$$L^{O,\psi}_{F(B)}(v_i) = x'_i = \vee_{j=1}^{p}[e_{ij} \vee 0] = \vee_{j=1}^{p} e_{ij} = \vee_{j=1}^{p}\{M(v_j) : \mathcal{D}^{\beta}_{F(B)}(v_i, v_j) > \psi)\}.$$

Theorem 4.1: Consider a q-Fuzzy soft β-covering of Ω and $\psi \in [0,1]$. Then,

$$L^{O,\psi}_{\sum_{k=1}^{q} F(B_k)}(M) = \vee_{k=1}^{q}((\mathcal{D}^{\beta}_{F(B_k)})^{\psi} \triangleright M \odot 1_{p\times 1}) \text{ and}$$

$U^{O,\psi}_{\sum_{k=1}^{q} F(B_k)}(M) = \wedge_{k=1}^{q}((\mathcal{D}^{\beta}_{F(B)})^{\psi} \triangleleft M \circ 0_{p\times 1})$ where $\triangleright$ and $\triangleleft$ are given the first preference.

Proof: The proof is trivial by Proposition 4.3.

Theorem 4.2: *Consider a q-Fuzzy soft* β*-covering of* Ω. *Then,*

$$L^{O,\mathcal{D}}_{\sum_{k=1}^{q} F(B_k)}(M) = \vee_{k=1}^{q}(\mathcal{D}^{\beta}_{F(B_k)} * M) \text{and } U^{O,\mathcal{D}}_{\sum_{k=1}^{q} F(B_k)}(M) = \wedge_{k=1}^{q}(\mathcal{D}^{\beta}_{F(B_k)} \cdot M).$$

Proof: The proof is trivial by Definition 3.1 and Proposition 4.1.

Conclusion

Different kinds of Z-FS β CBMFR are defined. The optimistic models of Z-FS β CBMFR are denoted using the matrix concept. In this research, the data is described by matrices, which improves the complexity of the algorithm and the efficiency of the method. Due to this, computations are easily executed by computers. Furthermore, we plan to propose a new form of Z-FS β CBMFR using N-soft set.

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Received March, 2024